4894. B. Ticknor

# THE PRINCIPLES

OF

# HYDROSTATICS:

DESIGNED

FOR THE USE OF STUDENTS

IN THE

UNIVERSITY.

21

BY THE

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THE FIFTH EDITION.

CAMBRIDGE:

*Printed by J. Smith, Printer to the University;*

AND SOLD BY DEIGHTON & SONS, AND NICHOLSON & SON, CAMBRIDGE;

AND J. MAWMAN, LUDGATE-STREET, LONDON.

1820

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# THE PRINCIPLES

## OF

# HYDROSTATICS.

### DEFINITIONS.

(Art. 1.) The science which treats of the nature and properties of Fluids has been usually divided into the following branches; *Hydrostatics*, which comprises the doctrine of the equilibrium of *non-elastic* fluids, as water, mercury, &c.; *Hydraulics*, which relates to the motion of those fluids; and *Pneumatics*, which treats of the properties of the different kinds of airs. But these are now all included under the general term *Hydrostatics*.

(2.) A *Fluid* is a body whose parts are put in motion one amongst another by any force impressed; and which, when the impressed force is removed, restores itself to it's former state*.

Fluids may be divided into *elastic* and *non-elastic*. An *elastic* fluid is one, whose dimensions are diminished by increasing the pressure, and increased by diminishing the pressure upon it; of which description are the different kinds of airs. A *non-elastic* fluid is one, whose dimensions are not, at least as to sense, affected by any increase of pressure, as water, mercury, &c. As many bodies, by cold, from a state of fluidity become

* By it's *former state*, we do not mean that every article is re-instated in it's former situation, but that the whole body recovers it's former dimensions and figure.

solids, such bodies are fluids so long as their surfaces, when disturbed, will restore themselves to their former position. The definition supposes a *partial* pressure; for if the fluid be incompressible, under an equal and general pressure none of the parts will be moved. Different fluids have different degrees of fluidity, according to the facility with which the particles are moved one amongst another. Water and mercury are the most perfect non-elastic fluids. Many fluids have a very sensible degree of tenacity, and are therefore called imperfect fluids. Besides the fluids which come under this definition, there are others, as the electric and magnetic fluid, light, and fire according to the opinion of some, &c. but these are not the objects of Hydrostatics.

(3.) The *specific gravity* of a body is it's weight, compared with the weight of another body whose magnitude is the same.

(4.) The *density* of a body is as the quantity of matter contained in a given space, and therefore (*Mech.* Art. 26.) in proportion to it's weight, when the magnitude is the same.

(5.) Cor. 1. Hence, the *specific gravity* of a body is in proportion to it's *density.*

A cubic inch of pure mercury is about fourteen times heavier than a cubic inch of water; the specific gravity and density of the former are therefore about fourteen times that of the latter. As the weight of a body is in proportion to it's quantity of matter (*Mechanics,* Art. 26.) the specific gravity and density of a body are also in proportion to it's quantity of matter, when the magnitude is the same.

(6.) Cor. 2. If the magnitude of a body be increased, the density remaining the same, the quantity of matter, and consequently the weight, will be increased in the same proportion. Hence, if the magnitude $M$, and density $D$, both vary, the quantity of matter, and consequently the weight of the body, will vary as $M \times D$, by the composition of ratios.

## SCHOLIUM.

(7.) We know so little of the nature and constitution of fluids, that the application of the general principles of motion to the investigation of the effects produced by their action, is subject to great uncertainty. That the different kinds of airs are constituted of particles endued with repulsive powers, is manifest from their expansion, when the force with which they are compressed is removed. The particles being kept at a distance by their mutual repulsion, it is easy to conceive that they may move very freely amongst each other, and that this motion may take place in all directions, each particle exerting it's repulsive power equally on all sides. Thus far we are acquainted with the constitution of these fluids; but with what degree of facility the particles move, and how this may be affected under different degrees of compression, are circumstances of which we are totally ignorant. With respect to the nature of those fluids which are denominated liquids, we are still less acquainted. If we suppose their particles to be in contact, it is very difficult to conceive how they can move amongst each other with such extreme facility, and produce effects in directions opposite to the impressed force, without any sensible loss of motion. To account for this, the particles have, by some, been supposed to be perfectly smooth and spherical. If we were to admit this supposition, it would yet remain to be shown how it would solve all the phænomena, for it is by no means self-evident that it would. If the particles be not in contact, they must be kept at a distance by some repulsive power. But it is manifest that these particles attract each other, from the drops of all perfect fluids endeavouring to form themselves into spheres. We must therefore admit in this case both powers, and that where one power ends the other begins, agreeably to

Sir I. Newton's* idea of what takes place, not only in respect to the constituent particles of bodies, but to the bodies themselves. The incompressibility of liquids (for I know no decisive experiments which have proved them to be compressible) seems most to favour the former supposition, unless we admit, in the latter hypothesis, that the repulsive force is greater than any human power which can be applied. The expansion of water by heat, and the possibility of actually converting it into two permanent elastic fluids, according to some late experiments, seem to prove that a repulsive power exists between the particles, for it is hard to conceive that heat can actually create any such new powers, or that it can of itself produce any such effects.

A fluid being composed of an indefinite number of corpuscles, we must consider it's action, either as the joint action of all the corpuscles, estimated as so many distinct bodies, or we must consider the action of the whole as a mass, or as one body. In the former case, the motion of the particles being subject to no regularity, or at least to none that can be discovered by any experiments, it is impossible, from this consideration, to compute the effect; for no calculation of effects can be applied, when produced by causes which are subject to no law. And, in the latter case, the effects of the action of one fluid upon another differ so much, in many respects, from what would be it's action as a solid body, that a computation of it's effects can by no means be deduced from the same principles. In Mechanics, no equilibrium can take place between two bodies of different weights, unless the lighter acts at some mechanical advantage; but in Hydrostatics, a very small weight of fluid may, without it's acting at any mechanical advantage whatever, be made to balance a weight of any magnitude. In Mechanics, bodies act only in the direction of gravity; but the property which fluids have of acting equally in all

* See his *Optics*, Que. 31.

directions, produces effects of such an extraordinary nature, as to surpass the power of investigation. The indefinitely small corpuscles, of which a fluid is composed, probably possess the same powers, and would be subject to the same laws of motion, as bodies of finite magnitudes, could any two of them act upon each other by contact; but this is a circumstance which certainly never takes place in any of the aërial fluids, and probably not in any liquids. Under the circumstances, therefore, of an indefinite number of bodies acting upon each other by repulsive powers, or by absolute contact, under the uncertainty of the friction which may take place, and of what variation of effects may be produced by different degrees of compression, the conclusions deduced from any *theory* must be subject to considerable errors, except from *that* which is founded upon such experiments, as include in them the consequences of those principles which are liable to any degree of uncertainty.

Sir I. Newton seems to have been well aware of all these difficulties, and therefore, in his *Principia*, he has deduced his laws of resistance, and the principles upon which the times of emptying vessels are founded, entirely from experiment. He was too cautious to trust to theory alone, under all the uncertainties to which, he appears to have been sensible, it must be subject. He had, in a preceding part of that great work, deduced the general principles of motion, and applied them to the solution of problems which had never before been attempted; but when he came to treat of fluids, he saw it was necessary to establish his principles upon experiments; principles, not indeed mathematically true, like his general principles of motion before delivered, but, under certain limitations, sufficiently accurate for practical purposes. The principles therefore upon which we reason in this branch of philosophy, must be considered as depending upon direct experiments adapted to the particular cases, rather than upon the general principles of equilibrium and motion, as applied in Mechanics.

# Sect. I.

### ON THE

## PRESSURE of NON-ELASTIC FLUIDS.

### Prop. I.

*Fluids press equally in all directions.*

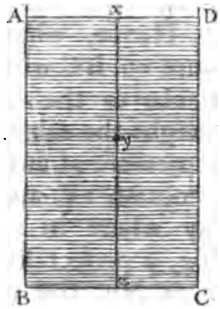
(8.) For if any liquid be put into a vessel, and glass tubes, straight, or bent at any angle, be put into the fluid, the fluid will rise in all the tubes to the height of the surface of the liquid in the vessel. Also, a vessel is found to empty itself in the same time through an hole at the same depth, whether the fluid spouts downwards, horizontally, upwards, or in any other direction.

(9.) That the pressure upwards is equal to the pressure downwards, is one of the most extraordinary properties of fluids, and can be conceived to arise only from the perfect freedom with which the particles move amongst each other, which freedom of motion is, probably, owing to the particles being kept at a distance by a repulsive power residing in them. This is one remarkable difference between fluids and solids, solids pressing only downwards, or in the direction of gravity.

### Prop. II.

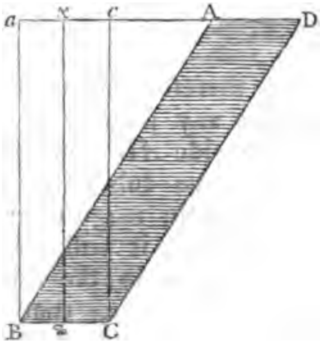
*The pressure upon any particle of a fluid, whose density is uniform, is in proportion to it's perpendicular distance from the surface of the fluid.*

(10.) Case 1. Let *ABCD* be a vessel filled with such a fluid, and draw $xz$ perpendicular to the horizon. Now the pressure is in proportion to the weight, or number of incumbent particles; and as the density of




the fluid is uniform, and consequently all the particles are at the same distance from each other, we have $xy : xz$ :: the number of particles incumbent upon $y$ : the number incumbent upon $z$ :: the pressure on $y$ : the pressure on $z$.

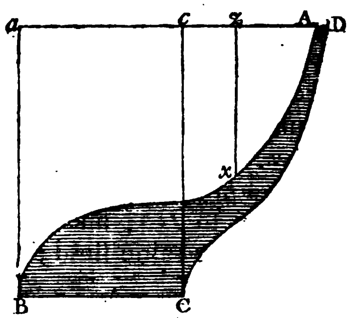
Case 2. Hence, if the sides of the vessel *ABCD* be not perpendicular to the bottom, the pressure upon any point of *BC* will still be in proportion to it's perpendicular depth. For produce *DA* to $a$, and draw $Ba$, $Cc$ perpendicular to $Da$; and conceive *DaBC* to be a vessel filled with the same fluid; then the pressure




on $z$ is as $zx$, there being actually that perpendicular depth of the fluid incumbent upon it. Now instead of

supposing the fluid $ABCD$ to be kept in it's position by the other part of the fluid $AaB$, conceive it to be kept in that position by the side $AB$ of the vessel, and the effect must remain the same; for it can manifestly make no difference by what body the fluid $ABCD$ is kept in it's place. Hence, whatever be the form of the sides, the pressure on $BC$ will be the same as if the sides were perpendicular to the surface of the fluid, and the perpendicular height the same.

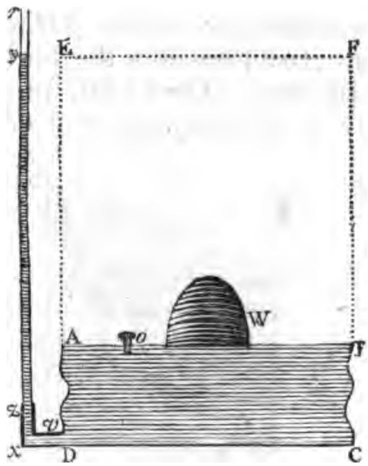
(11.) That the pressure at any point $x$ is equivalent to the weight of a column $xz$, appears from this experiment, that, if any where in $AB$, a hole be made perpendicular to $Aa$, a perfect fluid will spout upwards very nearly as far as $Aa$, the defect being no more than what may be supposed to arise from the resistance of the air, and the falling back of the upper parts of




the fluid. The want therefore of an actually incumbent column $xz$ of fluid at any point $x$ of $AB$, is supplied by the reaction of the side $AB$ downwards against the fluid. Also, let any two tubes be connected, and however they may differ in magnitude or inclination, if a fluid be put into one, it will always rise to the same perpendicular altitude in the other. These experiments therefore also prove, that fluids press in proportion to their perpendicular depths, and that the pressure is not to be estimated by the weight.

(12.) Upon these two principles, that fluids press

equally in all directions, and in proportion to their perpendicular depths, depends the explanation of the following experiment, called the *hydrostatical Paradox*. *AB, CD* are two round parallel boards, connected by leather like a pair of bellows; *zxv* is a brass pipe entering into the cavity, and into which the glass tube *zr* is fixed. Through an orifice at *o* a quantity of water is poured in, to keep the top and bottom at some distance from each other, and then it is stopped by a screw. Now let a very large weight *W* be laid upon the top *AB*, and the water in the tube will rise to some height *xy*, and there it will rest, the small weight of water in *xy* balancing the weight *W*,



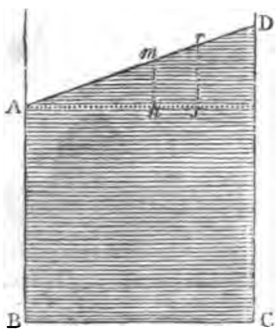

which may be 1000 or 10000 times greater than the weight of that water, according to the size of the tube; and this weight *W* will always be found equal to the weight of a cylinder *ABFE* of fluid, whose surface *EF* is upon a level with *y*. The principles before mentioned will thus explain this extraordinary circumstance. The fluid at *x*, the bottom of the tube, is pressed with a force proportional to the perpendicular altitude *xy*; this pressure is communicated horizontally

in the direction $xDC$ to all the particles, and the pressure so communicated acts equally in all directions; the pressure therefore downwards upon the bottom $DC$ is just the same as it would be, if $DEFC$ were a cylinder of water; and it is manifest that if we take away the part $ABFE$, and place an equal weight $W$ to act upon the other part $ABCD$, the effect will remain the same, or there will be an equilibrium.

## Prop. III.

*The surface of every fluid at rest, is horizontal.*

(13.) For conceive the surface $AD$ not to be horizontal, and draw $Ans$ parallel to the horizon, and $mn$, $rs$ perpendicular to it. Then (Art. 10.) the pressures at $s$ and $n$ are as $rs$ and $mn$; and (Art. 8.) fluids



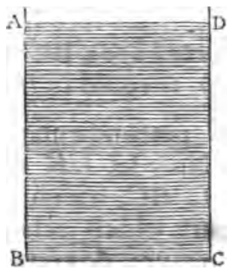

pressing equally in all directions, the particle at $n$ would be driven towards $A$; the fluid therefore would not remain at rest. But when $AD$ becomes horizontal, these pressures become equal, and the fluid remains at rest.

(14.) Cor. In like manner it appears, that if there be two fluids in the same vessel which do not mix, their common surface will be parallel to the horizon.

### Prop. IV.

*If the sides of a vessel* ABCD, *filled with a fluid, be perpendicular to it's bottom which is parallel to the horizon, the pressure upon the bottom will be equal to the weight of the fluid.*

(15.) For the reaction of the sides of the vessel against the fluid, being perpendicular to the sides, or parallel to the bottom, it can neither increase nor diminish the pressure of the fluid against the bottom; also, the force of gravity acting parallel to the sides, they can take off no part of the weight of the fluid, nor increase




it's pressure upon *BC*; the pressure therefore upon the bottom must be simply the weight of the fluid.

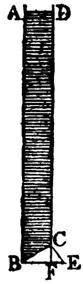
(16.) Cor. Hence, (Art. 6.) in different vessels of this description, containing different fluids, the pressures are as the areas of the bottoms × depths × specific gravities, because the magnitude = the area of the bottom × depth.

### Prop. V.

*Let the bottom* BC *be oblique to the sides, and also so small, that the whole may be conceived to be of the same depth; then the pressure perpendicular to* BC *will be as* BC × *depth.*

(17.) For the number of particles in contact with *BC* is as *BC*. Also, fluids press equally in all directions, and in proportion to their depths (Art. 8. 10.),

and the whole pressure on $BC$ must be as the number




of particles × the pressure of each; hence, the pressure perpendicular to $BC$ is as $BC \times$ depth.

### Prop. VI.

*The pressure exerted upon* BC downwards, *or in the direction of gravity, is equal to the weight of the fluid.*

(18.) Draw $CE$ perpendicular to $BC$, $BE$ to $BA$, and $CF$ to $BE$. Now let $CE$ represent the pressure ($P$) perpendicular to $BC$, which resolve into $CF$, $FE$, then $CF$ is that part which acts downwards; also, let $p$ represent the pressure downwards, $\pi$ the pressure which would act upon $BF$ as the bottom, or the weight of the fluid by Art. 15. Hence,

$$p : P :: CF : CE :: \text{(by sim. trian.)}\ BF : BC,$$
$$P : \pi :: BC \times \text{depth} : BF \times \text{depth} :: BC : BF,$$
$$\therefore p : \pi :: BF \times BC : BC \times BF;$$

hence, $p = \pi$.

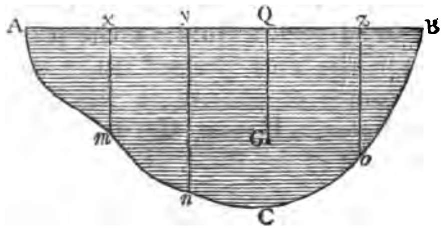
The same may be inferred directly, from the reasoning in Art. 15.

### Prop. VII.

*The pressure of a fluid against any surface, in a direction* perpendicular *to it, varies as the area of the surface multiplied into the depth of it's centre of gravity below the surface of the fluid.*

(19.) Let $ABC$ be a vessel, $AB$ the surface of the fluid; $ACB$ the surface pressed, $G$ it's centre of gra-

vity, and divide $ACB$ into an indefinite number of parts $m$, $n$, $o$, &c. and draw $GQ$, $mx$, $ny$, $oz$, &c. perpendicular to $AB$. Now every part of the indefinitely small surface $m$ may be conceived to be at the same perpendicular depth $mx$; also, (Art. 10.) the pressure at $m$ is in proportion to it's depth, and that pressure is exerted equally in all directions; hence, the pressure




on $m$ perpendicular to that surface is as $m \times mx$; consequently the whole pressure exerted on $ACB$, in a direction perpendicular to the surface at every point, will be as $m \times mx + n \times ny + o \times oz + \&c.$ But (by *Mechanics*, Art. 172.) if we consider $m$, $n$, $o$, &c. as representing weights in proportion to their respective magnitudes, and the surface $ACB$ to represent a proportional weight, then $m \times mx + n \times ny + o \times oz + \&c. = ACB \times GQ$; hence, the whole pressure perpendicular to the surface varies as $ACB \times GQ$.

(20.) Cor. 1. The same pressure is equal to the weight of a cylinder of the same fluid, the area of whose bottom is equal to the surface $ACB$, and altitude $GQ$. For the areas pressed, and the depths of the centres of gravity, are equal in the two cases, therefore the pressures are equal. But (Art. 15.) the pressure on the bottom of a cylinder is equal to the weight of the fluid contained in it; consequently the pressure on the surface $ACB$ is equal to the same weight.

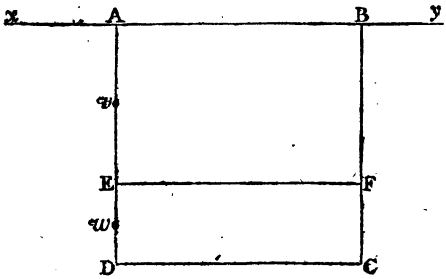
(21.) Cor. 2. The pressures of different fluids against different surfaces, will be as the areas × depths of their centres of gravity × specific gravities of the fluids. For it is manifest that, *cæteris paribus*, the pressures must be as the weights, or as the specific gravities

(Art. 3.); thus, if the same vessel be filled with mercury and water, the specific gravity, or weight, of the former, will be 14 times that of the latter, and consequently the pressure must be 14 times greater. Hence, for different vessels, combining this ratio with that in Art. 19. we have the whole pressure as above.

(22.) Cor. 3. The pressure against the side of a cubical vessel filled with a fluid $= \frac{1}{2}$ the pressure against the bottom; for the area pressed is the same, and the depth of the centre of gravity in the former case $=$ $\frac{1}{2}$ that in the latter. Hence, the pressure against the side $= \frac{1}{2}$ the weight of the fluid.

(23.) Cor. 4. Let a cylinder, the altitude of which is $a$, and diameter of it's base $d$, be filled with a fluid; then if $p = 3{,}14159$ &c. the area of the base $= \frac{1}{4}pd^2$, and the area of the sides $= pda$; hence, the pressure on the bottom : the pressure on the side :: $\frac{1}{4}pd^2a$ : $\frac{1}{2}pda^2$ :: $d$ : $2a$. If two equal cylinders be filled, one with mercury and the other with water, then, the pressure on the base of the former : the pressure on the sides of the latter :: $14d$ : $2a$ :: $7d$ : $a$.

(24.) Cor. 5. Let $xy$ be the surface of a fluid, $ABCD$




a ▭ perpendicular to it; draw $EF$ parallel to $AB$, and bisect $AE$ in $v$, and $ED$ in $w$; then $Av$, $Aw$ will be the depths of the centres of gravities of $ABFE$ and $EFCD$. Hence, the pressures on these ▭s are as $ABFE \times Av$ : $EFCD \times Aw$ :: $AE \times Av$ : $ED \times$

$Aw :: AE \times \frac{1}{2} AE : \overline{AD - AE} \times \frac{AD + AE}{2} :: AE^2 : AD^2 - AE^2.$

## Prop. VIII.

*If a vessel be filled with a fluid, the pressure on any part : the whole weight of the fluid :: the area of that part × the depth of it's centre of gravity : the solid content of the fluid.*

(25.) By Art. 15. the pressure on the base of a cylinder filled with fluid = it's weight; also, the weight of the same fluid is in proportion to it's solid content. Let a vessel of any form be filled with a fluid, and let $A$ = any part of it's surface, $D$ = the depth of it's centre of gravity, $P$ = the pressure upon it, and $W$ = the whole weight of the fluid, $S$ = it's solid content; and let a cylinder, whose base = $a$ and altitude $d$, be filled with the same fluid, and let $p$ be the pressure on it's bottom, $w$ the weight of the fluid, $s$ it's solid content; then by Art. 19.

$$\left.\begin{array}{rl} & P : p :: A \times D : a \times d \\ \text{Also,} & w : W :: s : S \end{array}\right\} \text{but by Art. 15. } w = p; \text{ also } s = a \times d; \text{ hence,}$$

$$P : W :: A \times D : S.$$

(26.) Cor. 1. If a cone, standing on its base, be filled with a fluid, and $A$ = the base, we have $S = \frac{1}{3} A \times D$; consequently the pressure on the base = three times the weight of the fluid.

(27.) Cor. 2. If a hollow sphere be filled with a fluid, the pressure $P$ against the whole internal surface ($A$) = three times the weight of the fluid; for the centre of gravity of the surface is in the centre of the sphere, whose depth $D$ below the upper point of the fluid must therefore be equal to the radius $R$ of the sphere; and $S = \frac{1}{3} A \times R = \frac{1}{3} A \times D$.

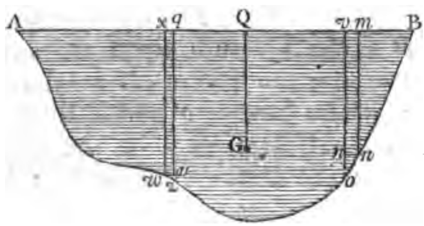
(28.) Hitherto we have considered the whole pressure of a fluid against any surface in a direction *perpendi-*

*cular* to every point of it; but in this case, the effect on one part may partly destroy the effect on another, by their not acting in the same direction. Since we do not therefore thus get the whole joint effect in any direction, let us next consider what is the whole pressure against any plane in the direction of gravity.

## Prop. IX.

*The pressure of a fluid* downwards *against the sides and bottom of any vessel, is equal to the weight of the whole fluid, provided there be, over every part of the sides and bottom, a perpendicular column of the fluid reaching to the surface.*

(29.) Let $AwzonB$ be such a vessel filled with a fluid

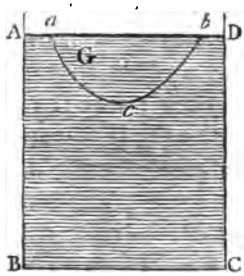



whose surface is $AB$, perpendicular to which, conceive $vmno$, $qxwz$, &c. to be indefinitely small prismatic columns into which the whole is divided; then (Art. 18.) the pressure downwards of every column is equal to the weight of the column of fluid; hence, the whole pressure downwards is equal to the whole weight of the fluid.

(30.) Cor. 1. As the pressure of every column downwards is equal to it's gravity, the joint effect of all the columns, or of the whole fluid, is the same as the gravity of the whole, if it had been solid, and consequently (*Mechanics*, Art. 160.) the effect is the same as if all the power was concentrated in the centre of gravity.

(31.) Cor. 2. If $ABCD$ be a vessel of fluid, and $acb$ be any part $G$ of the fluid, it's action downwards must be equal to the reaction of the fluid under it up-

wards; but the effect of $G$ downwards, is, (*Mech.*

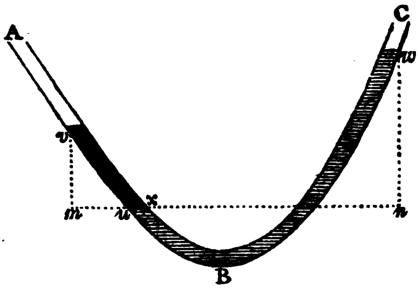



Art. 160.) just the same as if the whole effect took place at it's centre of gravity; therefore the effect of the reaction of the fluid under $G$ must be the same as if it took place at the same point.

## Prop. X.

*If two fluids communicate in a bent tube, their perpendicular altitudes above the plane where they meet, are inversely as their specific gravities.*

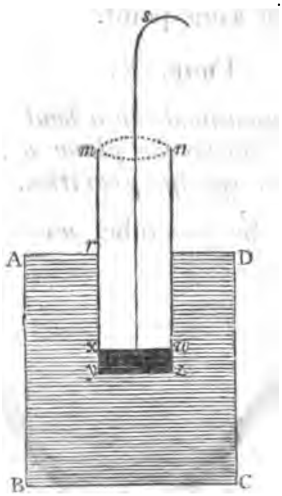
(32.) Let $ABC$ be the tube, $ux$ the plane where




the two fluids meet, standing at $v$ and $w$; draw $muxn$ parallel to the horizon, and $vm$, $wn$ perpendicular to it. Let $S$, $s$ represent the specific gravities of the two fluids in $vu$, $uw$; then the area of the section $ux$ being common to both fluids, the pressure of each fluid at that section will (Art. 21.) be as it's perpendicular depth × it's specific gravity; but as the fluids are at rest, their pressures must be equal; hence, $S \times vm = s \times wn$, therefore $S : s :: wn : vm$.

(33.) Cor. 1. Hence, the same fluid will stand at the same altitude on each side; for if $S=s$, then $wn=vm$. If therefore a pipe convey a fluid from a reservoir, it can never carry it to a place higher than the surface of the fluid in the reservoir; but it may convey it to that height. The ancients not being aware of this property, conveyed water in pipes, only down hill. To convey water therefore to a place but a little below the water in the reservoir, having a valley between, they build aqueducts, instead of carrying a pipe down the hill and then up again.

(34.) Cor. 2. If $ABCD$ be a vessel of fluid, $mxwn$




a hollow cylinder, to whose bottom a cylindrical body $wxyz$, of greater specific gravity than the fluid, may be so closely fitted, that the fluid cannot enter; then if this body be kept in that position by the string $s$, and the whole be immersed perpendicularly in the vessel, until $yr$ be to $yx$ as the specific gravity of the body to that of the fluid, the body will remain suspended without the assistance of the string. For, we may consider the body $wxyz$ just the same as if it were a fluid of the same specific gravity, and conse-

quently it will rest when the altitudes of the body and fluid above $yz$ are inversely as their specific gravities.

## Prop. XI.

*The ascent of a body in a fluid of greater specific gravity than itself, arises from the pressure of the fluid upwards against the under surface of the body.*

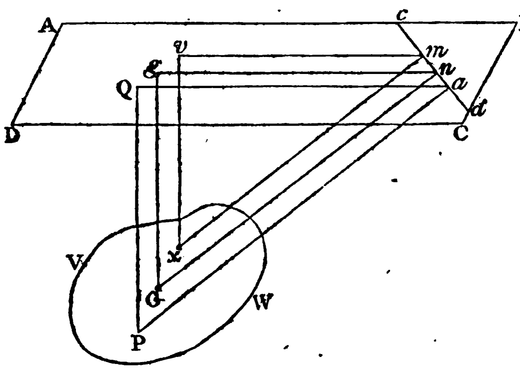
(35.) For if the body be placed upon the bottom of the vessel in which the fluid is, and be so closely fitted to it, that no part of the fluid can get under it, it will remain at rest; but if it be lifted up, so that the fluid can get under it, it will immediately rise.

Def. The *centre of pressure* is that point of a surface, against which any fluid presses, to which if a force equal to the whole pressure were applied in a contrary direction, it would keep the surface at rest.

## Prop. XII.

*To find the centre of pressure of a plane surface.*

(36.) Let $ABCD$ be the surface of the fluid, $VW$ the plane, which being produced, let $cd$ be it's




intersection with the surface, $P$ the centre of pressure, and $G$ the centre of gravity; and conceive the whole plane to be divided into an indefinite number

of indefinitely small parts, of which one is $x$; draw $PQ$, $Gg$, $xv$ perpendicular to the surface, and $Pa$, $Gn$, $xm$ perpendicular to $cd$; and join $Qa$, $gn$, $vm$; then it is manifest, that the triangles $PQa$, $Ggn$, $xvm$ are similar. Now the pressure on $x$ perpendicular to $VW$ is (Art. 17.) as $x \times xv$; and (*Mechanics*, Art. 92.) it's effect to turn the plane about $cd$ is as $x \times xv \times xm$; but (sim. trian.) $Gn : Gg :: xm : xv = xm \times \frac{Gg}{Gn}$; hence, the effect of the pressure at $x$ to turn the plane about $cd$ is as $x \times xm^2 \times \frac{Gg}{Gn}$; therefore the whole effect is as the *sum of all the* $x \times xm^2 \times \frac{Gg}{Gn}$. But if $A$ = the area of $VW$, the pressure on $VW$ is as $A \times Gg$; therefore the effect of that pressure at $P$, to turn the plane about $cd$, is as $A \times Gg \times Pa$. Hence, $A \times Gg \times Pa =$ *sum of all the* $x \times xm^2 \times \frac{Gg}{Gn}$; consequently $Pa = \frac{\textit{sum of all the } x \times xm^2}{A \times Gn}$. Hence it appears (*Fluxions*, Art. 63.), that $P$ is at the same distance from $cd$ as the centre of percussion is, $cd$ being the axis of suspension. They do not however, in general, lie in the same line, that is, in the line $nG$; for the efficacy of the pressure at $x$, to turn the plane about $nG$, is as $x \times xv \times mn$, or (since $xv$ varies as $xm$) as $x \times xm \times mn$; but the *sum of all the* $x \times xm \times mn$ is not generally $= 0$, therefore the whole pressure will not necessarily balance itself upon the line $Gn$. The situation of the line $aP$ must therefore be determined, by making the *sum of all the* $x \times xm \times mn = 0$.

The centres of pressure and percussion do not therefore in general coincide, taking the centre of percussion in it's usual acceptation*.

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* The centre of percussion has always been defined to be that point in

# Sect. II.

## On the SPECIFIC GRAVITIES of BODIES.

### Prop. XIII.

*The weight of a body varies as it's magnitude and specific gravity conjointly.*

(37.) For it is manifest, that if you vary the magnitude of any body in the ratio of $M : m$, continuing it's specific gravity the same, you will alter the quantity of matter; and consequently the weight, in the same ratio. If you alter the specific gravity of the body in the ratio of $S : s$, continuing it's magnitude the same, you will alter the weight in the same ratio, (Art. 3.). Hence, by the composition of ratios, if you alter both the magnitude and specific gravity, you will alter the weight in the ratio of $M \times S : m \times s$.

As the weight of a body is in proportion to it's quantity of matter, the quantity of matter is as the magnitude and specific gravity conjointly, or as the magnitude and density conjointly, the specific gravity and density varying in the same ratio (Art. 5).

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in the line $nG$ at which all the motion of the body would be destroyed, estimating the motion of the body about the line $cd$; and the computations have been always made upon this principle. But the body, after it's action against that centre, may still have a tendency to turn about the line $nG$. If therefore we were to define the centre of percussion to be that point at which the *whole* motion of the body would be destroyed, the centres of pressure and percussion would not, in general, coincide; in which case, the position of the line $aP$ must be computed on the above principle.

(38.) A cubic foot of rain water weighs 1000 ounces avoirdupoise, the specific gravity of which call $s$; let $W$ be the weight of another body whose magnitude in cubic feet is $M$, and specific gravity $S$. Hence, $W$ : 1000 :: $M \times S$ : 1 foot $\times s$; if therefore we assume $s =$ 1000 as a standard with which we may compare the specific gravities of other bodies, we have $W = M \times S$. To reduce it to the measure in cubic inches, corresponding to the specific gravity of water represented by unity, we have, 1728 : 1 :: 1000 : ,5787 the weight of a cubic inch of rain water; hence, $W$ : ,5787 :: $M \times S$ : 1 inch $\times$ 1 ($s$), therefore, $W =$ ,5787 $M \times S$, $M$ being the magnitude in cubic inches, and the specific gravity of water unity. Now a troy ounce : avoirdupoise ounce :: 480 : 437,5, for an avoirdupoise ounce contains 437,5 grains troy. Hence, to reduce $W$ to troy weight, as the troy ounce is the greater, the weight expressed in troy ounces will be less in the same proportion; therefore, 480 : 437,5 :: ,5787 $M \times S$ : ,52746 $M \times S = W$ the weight in troy ounces, = 253,18 $M \times S$ grains. If $M = 1$, $S = 1$, which gives 253,18 grains troy for the weight of a cubic inch of rain water*.

## Prop. XIV.

*If a body float on a fluid, it displaces as much of the fluid as is equal to the weight of the body.*

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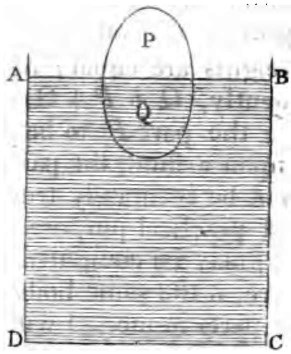
* Hence, we may very accurately determine the diameter $d$ of a sphere whose specific gravity is $s$, that of water being unity. For the content of a sphere whose diameter $= 1$ is 0, 5236; therefore, 1 : 0,5236 :: 253,18 grains (the weight of 1 cubic inch) : 132,428 grains, the weight of a sphere of water whose diameter is 1 inch. Hence, since the weights are as the magnitudes and specific gravities conjointly, and the magnitudes of spheres are as the cubes of their diameters, we have 132,428 $s d = w$ the weight, consequently, $d = \overline{\frac{w}{s}}\Big|^{\frac{1}{3}} \times ,19612$.

(39.) For the body $P+Q$ is supported by the pressure of the fluid upwards against the part immersed; also,




the same pressure supported a quantity of fluid equal in bulk to $Q$, before the body was immersed, that space being then occupied by the fluid; and as the same pressure must sustain the same weight, when there is an equilibrium, the weight of the body must be equal to the weight of a quantity of fluid equal in bulk to $Q$.

### Prop. XV.

*If a body float on a fluid, the centres of gravity of the body and of the fluid displaced, must, when the body is at rest, be in the same vertical line.*

(40.) For (Art. 31.) the effect of the pressure of the fluid upwards is the same as if the whole took place at the centre of gravity of the fluid displaced; if therefore this action of the fluid upwards against the body in a vertical line, do not pass through the centre of gravity of the body, it must, (*Mech.* Art. 164.) give the body a rotatory motion.

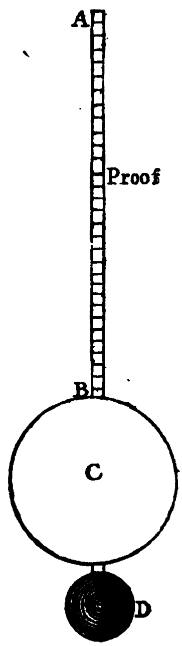
### Prop. XVI.

*If a body float on a fluid, the part* (Q) *immersed : the whole body* (P+Q) :: *the specific gravity* (s) *of the body : the specific gravity* (S) *of the fluid.*

(41.) By Art. 38. the weight of the body is $\overline{P+Q} \times s$, and the weight of a quantity of fluid equal to $Q$ is $Q \times S$, the magnitudes being expressed in cubic feet, and the specific gravity of water being 1000; but (Art. 39.) their weights are equal; hence, $\overline{P+Q} \times s = Q \times S$; consequently, $Q : P+Q :: s : S$.

We here suppose the part $P$ to be *in vacuo*. But when bodies float upon a fluid, the part $P$ being in the air, this rule will not be accurately true; near enough so, however, for all practical purposes. The effect of the air, if necessary, may be computed by Art. 54.

(42.) Cor. Hence, if the same body float upon two different fluids, the parts immersed will be inversely as the specific gravities of the fluids; for in this case $P+Q$ and $s$ being constant, $Q$ varies inversely as $S$. Upon this principle is constructed the HYDROMETER, an instrument for measuring the specific gravities of such fluids, as do not differ much in their specific




gravities. $AB$ is a graduated stem fixed to a hollow

globe *C*, which is annexed to another sphere *D*, into which mercury or shot is put, in order to make the instrument sink in the fluid, and keep it vertical. Now let us suppose the whole bulk of the instrument to be represented by 4000, and each of the divisions by 1 of such parts, and let the whole length of the stem contain 50 of such parts; then if this instrument be put into one fluid, and it sinks to 30, and in another, it sinks to 20, the parts immersed will be 3970 and 3980 respectively; and the specific gravities of these two fluids, being inversely as the parts immersed, will be as 3980 : 3970. As this can only be applied to those fluids which come within the extent of the scale, the instrument generally consists of two stems, one of which can be taken off and the other put on, the stems being adapted to fluids of different specific gravities, one measuring what the other will not.

Mr. Nicholson has lately made a considerable improvement in this instrument, by placing a small brass cup on the top of the stem, into which small weights may be put, so as to sink it into different fluids to the *same* point of the stem. In this case, the part immersed being the same, the specific gravities of the fluids will be as the whole weights, which are known, from knowing the weight of the instrument, and the weights added.

This instrument is also used to find whether spirits be above or below proof. Proof spirits consist, half of pure spirits, called alcohol, and half of water; the instrument put into this mixture, sinks to *Proof* upon the scale*. Now as water is heavier than the spirits, if there be more water than spirits, the specific gravity will be greater than that of proof spirits, and consequently the hydrometer will not sink so far as proof, or the surface of the liquor stands below proof, and the strength of the spirits is *below* proof: but if there

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* According to *Desaguliers*, a gallon of proof spirits weighs 7lb. 12oz.

be less water than spirits, the specific gravity will be less than that of proof spirits, and therefore the hydrometer will sink below proof, or the surface of the fluid will stand above proof, and the spirits are *above* proof. This is the instrument which the Officers of Excise generally use when they examine liquors, in order to determine their strength, for the purpose of ascertaining the duty.

## Prop. XVII.

*The weight which a body loses when wholly immersed in a fluid, is equal to the weight of an equal bulk of the fluid.*

(43.) First, let the body be of the *same* specific gravity as the fluid, and then it is manifest that it will remain at rest, because (Art. 3.) it is of the same weight as an equal bulk of the fluid, and therefore the pressure upwards of the fluid beneath will support the body equally as it supported the fluid which occupied the space before the body was put in; in this case, the body is said to have lost all it's weight, or the weight of an equal bulk of fluid. Now let the specific gravity, and consequently the weight of the body, be increased; then the pressure of the fluid upwards against the body still continuing the same, that action must still take off the same weight from the body, that is, the weight of an equal bulk of fluid.

When we say that a body loses part of it's weight in a fluid, we do not mean that it's absolute weight is less than it was before, but that it is partly supported by the reaction of the fluid under it, so that it requires a less power to support it.

(44.) Cor. 1. Hence, when a body is weighed in air, in order to get it's absolute weight, we must add to it the weight of an equal bulk of air. If, for example, a body, whose magnitude is one cubic foot, weigh 1500 ounces troy in air, we must add 21 penny-

weights to it, which is the weight of a cubic foot of air, and it gives 1500 oz. 21 dwts. the real weight of the body, or it's weight *in vacuo*.

Hence, to take the advantage of buying by weight, we must attend to the bodies which are to be weighed, whether their specific gravities be greater or less than that of the weights. For instance, in buying gold, we ought to buy it when the air is lightest, or when the barometer stands the lowest; since the weight itself being greater than the gold which balances it, the gold has lost less of it's real weight than the weight has; and less, the rarer the air is. In buying diamonds, we ought to buy when the air is heaviest, or when the barometer is highest; for the diamond being greater than the weight, it loses more of it's weight than the weight does; and more, as the air is denser. The same for other bodies, whose specific gravities are greater or less than that of the weights.

A bulk of gold equal to a lb. of brass weights, weighs 2lb. When the barometer stands at 28 in., the gold loses 1 grain, and the weight loses 2 grains. When the barometer stands at 31 in., the former loses $\frac{1}{10}$th of a grain more, and the latter $\frac{2}{10}$ths; which being $\frac{1}{10}$th of a grain more than the gold loses, we must add that weight to the brass weights to restore the equilibrium; we therefore call the gold $\frac{1}{10}$th of a grain more than it is, which in value is $\frac{1}{4}$th of a penny.

If we balance diamonds with a 2lb. weight of brass; when the barometer stands at 28 in., the brass loses 2 grains, and the diamonds 6. When the barometer stands at 31 in., the weight loses $\frac{2}{10}$ths of a grain more, and the diamonds $\frac{6}{10}$ths more, so that $\frac{4}{10}$ths of a grain must be added to the diamonds to restore the equilibrium; we must therefore call the diamonds $\frac{4}{10}$ths less than its real weight.

(45.) Cor. 2. Hence also it follows, that if two bodies of the same weight in air, be put into a denser fluid, the smaller body will preponderate, since it loses a less weight than the other. And if they

weigh equally in any fluid, and then be brought into a rarer medium, the greater will preponderate, because having lost more weight in the denser fluid than the other body, when carried into a rarer fluid it will regain more weight, and therefore will weigh more in the lighter fluid.

## Prop. XVIII.

*A body immersed in a fluid, ascends or descends with a force equal to the difference between it's own weight and the weight of an equal bulk of fluid, the resistance of the fluid not being considered.*

(46.) Let $W$ be the weight of the body, $w$ the weight of an equal bulk of the fluid. Now we may consider the body as descending by it's own weight $W$, and (Art. 43.) as opposed in it's descent by $w$; hence, when $W$ is greater than $w$ the body descends, and when $W$ is less than $w$ it must ascend, and, in both cases, by the difference between $W$ and $w$, as they oppose each other.

(47.) Cor. Let the specific gravity of the fluid : that of the body :: 1 : $b$; then, (Art. 3.) $w : W :: 1 : b$; hence, $w = \frac{W}{b}$, consequently the relative gravity, or weight of the body in the fluid, $= W - \frac{W}{b}$.

Hence, if we can make a body heavier or lighter than an equal bulk of water, it will descend or ascend in water. On this principle little glass images are made to descend or ascend in water. These images have a small hole in their heel, with a small quantity of water in them, so as to make them just float on the water. These are put into a glass jar of water, not filled quite up to the top, covered with a piece of bladder, closely tied about the neck of the jar. Then upon pressing the bladder with the hand, the air at the top of the jar is compressed, and this forces the water

up into the images, which making them heavier than the water, they descend, and the air within them will be condensed. By the removal of the hand, the air at the top of the jar returns to its original state, and the condensed air in the images will drive out the water which had been forced in, and the images will then ascend. To get water into the image, take some water into the mouth; put the legs of the image into the mouth, suck the air out of the image, and then let the water in the mouth flow to the image, and it will enter into it, and a proper quantity may be adjusted to make the image just float. If they vary a little in their specific gravities, they will descend one after the other.

## Prop. XIX.

*The weight* (w) *which a body loses when immersed in a fluid : it's whole weight* (W) :: *the specific gravity of the fluid* (s) : *the specific gravity of the body* (S).

(48.) For (Art. 43.) $w$ is the weight of a bulk of fluid equal to the bulk of the body, the weight of which is $W$; but (Art. 3.) the weights of equal bulks are as their specific gravities; consequently $w : W :: s : S$.

*Ex.* If a body weigh 12lb. in air, and 7lb. in water, then 5lb is the weight lost; hence, 5 : 12 :: the specific gravity of water : the specific gravity of the body. Thus we compare the specific gravities of bodies and fluids of less specific gravities.

(49.) Cor. 1. Hence, $S = \frac{s \times W}{w}$; if therefore $s$ be given, that is, if *different* bodies be weighed in the *same* fluid, then will $S$ vary as $\frac{W}{w}$.

*Ex.* If a body $A$ weigh 9lb. in air, and 7lb. in any fluid, and another body $B$ weigh 13lb. in air, and 6lb. in the same fluid, then the specific gravity of $A$ : that

of $B :: \frac{9}{2} : \frac{13}{7} :: 63 : 26$. Thus we compare the specific gravities of two bodies.

(50.) Cor. 2. Hence also, $s = \frac{S \times w}{W}$; if therefore $S$ and $W$ be given, that is, if the *same* body be weighed in *different* fluids, then will $s$ vary as $w$.

*Ex.* If a body lose 6lb. in one fluid, and 5 in another, the specific gravities of these fluids are as 6 : 5. Thus we compare the specific gravities of two fluids.

If one of the fluids be mercury, the body must be either gold or platina, these being the only two metals which will sink in mercury. It is better, however, in this case, to compare the mercury with water (Art. 46.), weighing the mercury in the water by putting it into a small glass vessel suspended from the scale, first balancing the vessel in water.

The effect of the air in diminishing the weight of a body, has not here been considered.

Cor. 3. Hence also, if two bodies of different specific gravities be connected by a rod, and balanced on it by a string, when put into water they will not balance, but the lesser body will preponderate, it losing a less weight in proportion to it's bulk than the greater does. Hence an heterogeneous body being immersed in a fluid, the place of its centre of gravity will be changed.

(51.) If the body which is weighed in the fluid be wood, it should first be well rubbed over with grease, or varnished, to prevent it from imbibing any of the fluid.

(52.) Since the specific gravities of fluids vary when their temperatures vary, in comparing the specific gravities of different fluids, we must first reduce them to some one temperature, as a standard. This temperature is arbitrary; and it must be observed, that, from the *different* expansions of fluids for the *same* variation of temperature, the proportion of the specific

gravities at different degrees of temperature will be different. Many solid bodies are also subject to a variation of their specific gravities, from the variation of their temperatures.

## Prop. XX.

*To find the specific gravity of a body Q, which is lighter than the fluid in which it is weighed.*

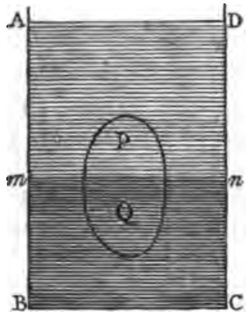
(53.) Connect $Q$ with another body $P$ which is heavier than the fluid, so that together they may sink; let the weight of $P$ in the fluid be $a$, the weight of $P+Q$ in the fluid be $b$, the weight of $Q$ out of the fluid, be $d$, and the weight of a bulk of fluid equal to $Q$, be $e$. Now, as $Q$ is of less specific gravity than the fluid, it will (Art. 46.) ascend with the force $e-d$. Also, as $Q$ by itself would ascend, when it is connected with $P$, they will together have a less descending power in the fluid than $P$ of itself would have, by the power of $Q$'s ascent; now the weight of $P$ in the fluid, or it's descending power, is $a$, and the weight of $P+Q$, or it's descending power in the fluid, is $b$; therefore the difference $a-b$ gives the ascending power of $Q$; hence, $e-d=a-b$, and $e=a-b+d$; but (Art. 3.), the weights of equal bulks are as the specific gravities; hence, the specific gravity of $Q$ : the specific gravity of the fluid :: $d : a-b+d$.

## Prop. XXI.

*If a lighter fluid rest upon a heavier, and their specific gravities be as* a : b, *and a body, whose specific gravity is* c, *rest with one part* P *in the upper fluid, and the other part* Q *in the lower, then* P : Q :: b—c : c—a.

(54.) The body will rest when it has displaced as much of the two fluids as is equal in weight to itself, for the reason given in Art. 39. Now (Art. 38). the weight of the body is $c \times \overline{P+Q}$; also, the

weight of the lower fluid displaced is $b \times Q$, and of the upper fluid, $a \times P$; hence, $a \times P + b \times Q = c \times \overline{P+Q} = c \times P + c \times Q$; therefore, $b \times Q - c \times Q =$




$c \times P - a \times P$, or $\overline{b-c} \times Q = \overline{c-a} \times P$; hence, $P : Q :: b-c : c-a$.

(55.) Cor. Hence, $Q : P+Q :: c-a : b-a$, and if $a$ be so small that it may be neglected, then $Q : P+Q :: c : b$, as in Art. 41.

## Prop. XXII.

*If* a *and* b *be the specific gravities of two fluids to be mixed together,* P *and* Q *their magnitudes, and* c *the specific gravity of the compound, then* P : Q :: b—c : c – a.

(56.) By Art. 38. the weight of $P$ is $a \times P$, the weight of $Q$ is $b \times Q$, and the weight of the compound is $c \times \overline{P+Q}$; but the weight of the compound must be equal to the sum of the weights of the two parts; hence, (as in Art. 54.) $P : Q :: b-c : c-a$.

Cor. Hence, if $a$ and $b$ be given, and the magnitudes $P$, $Q$, we get the specific gravity $c$ of the compound; for as $a \times P + b \times Q = c \times \overline{P+Q}$, we have $C = \dfrac{a \times P + b \times Q}{P+Q}$.

(57.) It is here supposed, that the magnitude $P+Q$, of the compound, is equal to the sum of the magnitudes

of the two parts when separate. But it very often happens, that the magnitude of the mixture is less than the sum, owing, probably, partly to the constituent particles of the different fluids being of different magnitudes, and partly to their chemical affinity. This is called a *penetration of dimensions*. Thus, for instance, if a pint of water and a pint of oil of vitriol be mixed together, the mixture will not make a quart. The specific gravity of the compound is manifestly increased by this circumstance; and it will be increased in the same proportion as the bulk is diminished.

Ex. Let the specific gravity of gold be 19, of silver 11, and of the compound 14; then the magnitude of the silver in the mixture : the magnitude of the gold :: 19 − 14 : 14 − 11 :: 5 : 3.

A certain quantity of gold having been given by King Hiero to make him a crown, the Artist secreted part of the gold, and substituted the same weight of silver. This being suspected, Archimedes was employed to discover the cheat; but it is not related in what manner he did it, except that by going into a bath, the rising of the water suggested to him the method of finding the magnitudes of irregular bodies.

(58.) If we want to find the proportion of the *weights* of each body, we must take the ratios of their magnitudes, and of their respective specific gravities conjointly; hence, the weights of $P$ and $Q$ are as $a \times \overline{b-c} : b \times \overline{c-a}$. In the above Example, therefore, the weight of the silver : the weight of the gold :: 11 × 5 : 19 × 3 :: 55 : 57.

## TABLE OF SPECIFIC GRAVITIES.

| | |
|---|---|
| Refined gold - - | 19637 |
| English guinea - - | 17793 |
| Mercury - - - | 14019 |
| Lead - - - | 11325 |
| Refined silver - - | 11087 |

| | |
|---|---|
| Standard silver | 10535 |
| Bismuth | 9700 |
| Copper of Japan | 9000 |
| Copper of Sweden | 8843 |
| Hammered brass | 8349 |
| Cast brass | 8100 |
| Turbeth mineral | 8235 |
| Cinnebar, factitious | 8200 |
| Cinnebar, natural | 7300 |
| Elastic steel | 7820 |
| Soft steel | 7738 |
| Iron | 7645 |
| Pure tin | 7471 |
| Glass of antimony | 5280 |
| A pseudo topaz | 4270 |
| A diamond | 3400 |
| Chrystal glass | 3150 |
| Island chrystal | 2720 |
| Rock chrystal | 2658 |
| Common glass | 2620 |
| Fine marble | 2704 |
| Stone of mean gravity | 2500 |
| Selenites | 2252 |
| Sal gemmæ | 2143 |
| Nitre | 1900 |
| Alabaster | 1875 |
| Dry ivory | 1825 |
| Brimstone | 1800 |
| Dantzic vitriol | 1715 |
| Allum | 1714 |
| Borax | 1714 |
| Calculus humanus | 1700 |
| Oil of vitriol | 1700 |
| Oil of tartar | 1550 |
| Bezoar | 1500 |
| Honey | 1450 |
| Gum arabic | 1375 |
| Spirit of nitre | 1315 |
| Aqua fortis | 1300 |

| | |
|---|---|
| Pitch - - - - | 1150 |
| Spirit of salt - - | 1130 |
| Crassamen of the human blood | 1126 |
| Spirit of urine - - | 1120 |
| Human blood - - | 1054 |
| Amber - - - | 1030 |
| Serum of human blood - | 1030 |
| Milk - - - | 1030 |
| Urine - - - - | 1030 |
| Dry box wood - - | 1030 |
| Sea water - - - | 1030 |
| Common water - - | 1000 |
| Camphire - - - | 996 |
| Bees' wax - - - | 955 |
| Linseed oil - - | 932 |
| Dry oak - - - | 925 |
| Oil olive - - - | 913 |
| Spirit of turpentine - - | 864 |
| Rectified spirit of wine - | 866 |
| Dry ash - - - | 800 |
| Dry maple - - - | 755 |
| Dry elm - - - | 600 |
| Dry fir - - - | 550 |
| Cork - - - | 240 |
| Air - - - - | $1\frac{1}{4}$ |

The density of steam from boiling water, is 14000 times less than that of water, or about $16\frac{1}{3}$ times rarer than air.

(59.) The specific gravity of rain water being here represented by 1000, and a cubic foot of rain water weighing 1000 ounces avoirdupoise, the numbers against each substance represent the weight of a cubic foot thereof in avoirdupoise ounces. The specific gravities are subject to a small degree of variation, arising from the variation of temperature of the air.

The scales which are made use of to weigh bodies, in order to determine their specific gravities, are called the *Hydrostatic Balance.*

## Prop. XXIII.

*Given the weight* W *of a body whose specific gravity is known by the table, to find it's magnitude* M.

(60.) Let $a$ = the specific gravity of the body found in the table, which represents the weight of a cubic foot in avoirdupoise ounces; then as the weight is in proportion to the magnitude, when the specific gravity is given (Art. 37.), we have $a : W :: 1$ foot $: M$ the magnitude in cubic feet.

*Ex.* If a piece of dry oak weigh 1720 ounces, what is it's magnitude?

Here, $a = 925$, $W = 1720$; hence, $925 : 1720 :: 1 : \frac{1720}{925} = 1{,}8594$ cubic feet.

Cor. 1. As $W = aM$, if the magnitude be given we can find the weight.

*Ex.* If the magnitude of a piece of iron be 1,35 cubic feet, what is it's weight?

Here, $a = 7645$, $M = 1{,}35$; hence, $W = 1{,}35 \times 7645 = 10320{,}75$ ounces.

Cor. 2. Hence we find the diameters of capillary tubes. Weigh the tube; then fill it with mercury and weigh it again, and the difference gives the weight ($w$) of the mercury in ounces. Measure it's length ($l$) in feet. Let $a$ = specific gravity of mercury, that of water being 1000. Then $a : w :: 1 : \frac{w}{a}$ the magnitude of the mercury in cubic feet. Let $d$ = diameter of the tube in feet; then $0{,}7854\, d^2 l = \frac{w}{a}$; and $d = \sqrt{\frac{w}{0{,}7854\, la}}$.

# SECT. III.

## ON THE RESISTANCE OF FLUIDS.

(61.) THE resistance of a body moving in a fluid arises from the *inertia*, the *tenacity*, and *friction* of the fluid, admitting the particles to be in contact. The latter cause, granting it to exist, is probably very small; and the second is, in most fluids, inconsiderable when compared with the inertia. The resistance therefore, which we shall here consider, is that arising from the inertia of the fluid.

### PROP. XXIV.

*If a plane surface move in a fluid with a velocity* V *in a direction perpendicular to it's plane, the resistance, within certain limits of the velocity, varies as* $V^2$.

(62.) For the resistance must vary as the number of particles which the plane strikes in a given time, multiplied into the force of each against the plane. Now the number of particles which the plane strikes in a given time, must evidently be in proportion to $V$; also, the force of each particle is as $V$; and as action and reaction are equal and contrary, the reaction of every particle of the fluid against the plane must be as $V$; hence, the resistance varies as $V \times V$, or as $V^2$. This is found, by experiment, to be very nearly true, when the velocity is small.

(63.) This proof supposes, that after the plane strikes a particle, the action of that particle immediately ceases, and the particle itself to be, as it were, annihilated; but the particles, after they are struck, must necessarily be made to diverge and act upon the particles behind, which makes some difference between this theory and experiment. Also, by increasing the velocity of the body, the action of the fluid behind it, to impel it in the direction of it's motion, will be diminished, and consequently the retardation will, on this account, be increased. Mr. ROBINS found, from experiment, that when a bullet moves with the velocity of sound, or with a greater velocity (in which case, a vacuum is left behind the body, and the pressure forwards from behind then ceases), the resistance is always greater than this law gives it. When bodies descend in fluids, such as water, the resistance is very nearly as $V^2$, because the body can never acquire a velocity beyond a certain limit. We will therefore, in the Articles here given upon resistances, suppose the resistance to vary as $V^2$. This law of resistance was established by Sir I. NEWTON, from a variety of experiments; see the *Principia*, Vol. II. Prop. 31. Scholium; also Mr. PARKINSON'S *Hydrostatics*, page 26.

Mr. ROBINS found by experiment, that when a bullet moves with a velocity exceeding 200 feet per second, the retardation increases faster; and the deviation from the law stated in the proposition increases rapidly with the velocity, owing to the condensation of the air before the ball, and rarefaction behind. He found that a ball projected with a velocity of 1670 feet in a second, lost about 125 feet in a second in passing through 50 feet of air; this it must have done in $\frac{1}{32}$ of a second, in which time it would have lost 1 foot if projected directly upwards; from which it appears, that the resistance was about 125 times the weight of the ball. He further observed, that till the velocity exceeded 1100 feet per second, the resistance increases pretty regularly; but that in greater velocities, the resistance

becomes suddenly triple of what it would have been according to the law it before increased it. This he thinks may be explained by the vacuum behind the ball; yet it must be observed, that the increase of the rarity of the air behind the ball, and of density before it, must take place gradually, till there is a vacuum behind. From the result of all Mr. ROBINS's observations, he collected this RULE.


Let $AB$ represent the velocity 1700 feet per second, and $AC$ any other velocity. Make $BD$ to $AD$ as the resistance given by the ordinary theory to slow motions, to the resistance actually observed for the velocity 1700; then will $CD$ be to $AD$ as the resistance according to the common rule to the velocity $AC$, to that which really corresponds to it.

To adopt this to experiment, it is to be observed, that a ball of the size of a 12lbs. iron shot, moving 25 feet in a second, is resisted by $\frac{1}{22}$ of a lb. Increase this in the ratio of $25^2 : 1700^2$, and we get 210 for the resistance by the common rule; but by comparing it's diameter 4,5 in. with 0,75 in. the diameter of a leaden ball which had a resistance of about 11lbs. with this velocity, we conclude that the 12lbs. shot would have had a resistance of at least 396lbs.; hence, $BD : AD :: 210 : 396$, and $AB : AD :: 186 : 396$; and $AB$ being 1700, $AD = 3631$.

Let $AD = a$, $AC = x$, $R =$ resistance to a 12lbs. iron shot moving 1 foot per second, $r =$ resistance in pounds for the velocity $x$; then $r = R \times \frac{ax^2}{a - x}$. Now Mr. ROBINS's experiments gave $R = \frac{1}{13750}$; hence,

$$r = \frac{0{,}2632\,x^2}{3613 - x} = \frac{x^2}{4\,(3613 - x)}$$ nearly, falling short of the truth about $\frac{1}{20}$th part. But if we go to greater velocities, the rule cannot be applied as an approximation; for if the velocity be supposed 3613, the resist-

ance becomes infinite, which can never be. It is of no use to give a ball a greater velocity than 1700 feet per second, because it would be reduced to that almost in an instant.

Mr. ROBINS observes, that when a 24lbs. shot is impelled by it's usual charge of powder, the opposition of the air is equivalent to at least 400lbs weight, which retards the bullet so powerfully, that when fixed at an elevation of 45°. it's range is not $\frac{1}{5}$th part of what it would be, if there were no resistance of the air. In lighter and smaller shots, the effect of resistance was still greater; for he made many experiments with a wooden bullet fired at an angle of 45°, where, instead of 15000 yards which would have been it's range in vacuo, the range was not more than 200 yards.

### PROP. XXV.

*When different planes move in directions perpendicular to their surfaces, in different fluids, and with different velocities, the resistances will be as the squares of their velocities × the densities of the fluids × the areas of the planes.*

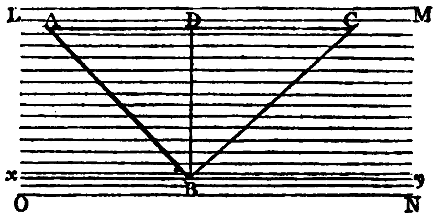
(64). For by increasing the density of the fluid, the number of particles struck in the same time will be greater in the same proportion, and consequently the resistance will, *cæteris paribus*, be greater in the same ratio. Also, by increasing the area of the plane, the greater will be the number of particles struck in the same ratio, and therefore the resistance will be greater in the same proportion. And (Art. 62.) when the velocities vary, *cæteris paribus*, the resistance varies as $V^2$. Hence, combining these ratios together, the resistance will be as $V^2$ × the densities of the fluids × areas of the planes.

### PROP. XXVI.

*If a plane move obliquely in a resisting medium, with an uniform velocity, and after the resolution of the*

*force with which the plane strikes the fluid, the whole of that part which acts* perpendicularly *to the plane take effect, the resistance perpendicular to the plane will vary as the square of the sine of the angle of inclination.*

(65.) Let $AB$ be the plane moving in the medium $LMNO$ in the direction $xBy$; draw $AC$ parallel to $xy$, meeting $BC$ perpendicular to $AB$ in $C$; and let $BD$ be perpendicular to $AC$. Now the quantity of fluid which $AB$ has to oppose by it's motion, being that which is contained between $AC$ and $xy$, is manifestly in pro-




portion to $BD$, or to the sine of $BAC$, because $AB$ : $BD$ :: rad. $= 1$ : sin. $BAD$ or $BAC$, and the first and third terms being constant, the second varies as the fourth. Also, as the plane acts against the fluid at the angle $CAB$, let $AC$ be taken to represent the whole force of the plane acting against the fluid, upon supposition that no part thereof was lost, which force would be uniform, the velocity of the plane being uniform; then, by the resolution of motion, the force acting perpendicularly to the plane will be in proportion to $BC$, or to the sine of $BAC$, for $AC$ : $BC$ :: rad. $= 1$ : sin. $BAC$, where the first and third terms being constant, the second varies as the fourth. Hence, the whole action of the plane against the fluid in the direction $BC$ (being in proportion to the whole quantity of fluid which opposes it's motion, and it's effect in the direction $BC$ conjointly,) will be as sin. $BAC \times$ sin. $BAC$, or as $\overline{\text{sin } BAC}|^2$. And as action and reaction are equal and contrary, the action of the fluid

against the plane in the direction $CB$, or the resistance of the fluid must vary in the same ratio.

## Prop. XXVII.

*The resistance of the same fluid to oppose the plane in the direction of it's motion, varies as* $\overline{\text{sin. BAC}}|^3$, *supposing, after the resolution of the reaction of the fluid in the direction* CA, *into* CB *and* BA, *the part* BA *to be entirely lost, and* CB *to take effect.*

(66.) As the whole effect of the resistance of the fluid upon the plane is that part which is perpendicular to it, let $CB$ represent that whole resistance, and resolve it into $CD$, $DB$, then will $CD$ represent the resistance which opposes the motion of the body; now rad.$=1$ : sin. $DBC$, or sin $BAC$, :: $CB : CD = CB \times \text{sin. } BAC$; but $CB$, as representing the whole resistance in the direction $CB$, varies as $\overline{\text{sin. } BAC}|^2$ (Art. 65.); hence, $CD$ varies as $\overline{\text{sin. } BAC}|^3$.

(67.) By experiments on plane bodies moving both in air and water, I find that they are not resisted according to the laws here deduced. Part of the difference may probably be owing to the two latter cases mentioned in Art. 61, but it principally arises from the force after resolution, not taking effect as here supposed, that part which is parallel to the plane not being all lost. But the further consideration of this subject falls not within the plan of this work, which is intended only to be an Elementary Treatise.

## Prop. XXVIII.

*The same supposition being made, the resistance of the plane, in a direction* perpendicular *to that of it's motion, varies as* $\overline{\text{sin. BAC}}|^2 \times \text{cos. BAC}$.

(68.) For by the last Art. $DB$ will represent that part of the whole resistance which acts perpendicu-

larly to the direction of the motion; hence, rad. = 1 : sin. $BCD$, or cos. $BAC$, :: $CB$ : $DB = CB \times$ cos. $BAC$; but $CB$, as representing the whole effective part of the force, varies as $\overline{\text{sin. } BAC}|^2$ (Art. 65.); therefore, $DB$ varies as $\overline{\text{sin. } BAC}|^2 \times$ cos. $BAC$.

(69.) If instead of supposing the plane to move in the fluid, we suppose the plane to be at rest and the fluid to move against it, the action of the fluid against the plane will be just the same as it's reaction when the plane moves. Hence, the last Article will show the effect which the wind has upon the sails of a wind mill, when at rest, to put them in motion, admitting our hypothesis respecting the efficacious part of the force, to be true.

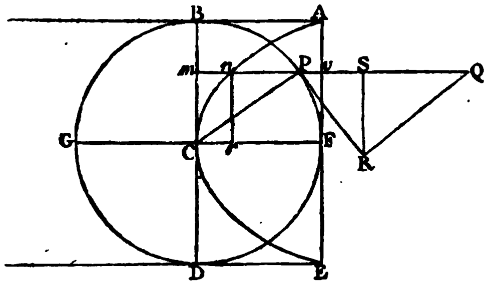
(70.) If, in the three last Propositions, the area of the plane, the velocity and density of the fluid be not given, then (for the reasons in Arts. 62. 64.) the resistance will vary in the above ratios, and as the area of the plane, square of the velocity, and density of the fluid conjointly.

## Prop. XXIX.

*The same supposition being made, let a cylinder move in the direction of it's axis, and a sphere of the same diameter, move in the same fluid with the same velocity; then will the resistance to the motion of the cylinder be double that of the globe.*

(71.) Let $AFE$ be a diameter of the end of a cylinder, parallel and equal to $BD$ a diameter of the sphere $BFDG$ whose centre is $C$, and $CF$, $DE$, $BA$ perpendicular to $AE$; draw $QP$ parallel to $CF$ the axis of the cylinder, and let it represent the force with which a particle of fluid would act perpendicularly at $v$, the end of the cylinder, in which case no part is lost. Now conceiving the same particle to act upon the globe at $P$, part of it's effect will be lost by the obliquity of the stroke; draw $PR$ a tangent to the

sphere in the plane *PCF*, and *QR* perpendicular to it; draw also *RS* perpendicular to *QP*, and produce




*QP* to *m*. Resolve the whole force *QP* into *RP*, *QR*, then we here suppose that the part *RP* is wholly lost, and the part *QR* only to be effective; resolve this into *QS*, *SR*, then *QS* is employed in opposing the motion of the globe, and *SR* (being perpendicular to it) can have no effect in that respect, and moreover it will be destroyed by an equal and opposite force, acting at a point, equidistant from *F*, on the other side. Hence, the force with which the point $v$ at the end of the cylinder is retarded : the force with which the corresponding point *P* on the globe is retarded :: $QP : QS$ :: (because $QP : QR :: QR : QS$) $QP^2 : QR^2$ :: (by sim. trian. *PRQ*, *PmC*) $PC^2 : Pm^2 :: PC : \frac{Pm^2}{PC}$ :: (taking $vn = \frac{Pm^2}{PC}$) $PC : vn :: vm : vn$; consequently the whole resistance on the cylinder : that on the globe :: the *sum of all the vm*'s : the *sum of all the vn*'s. Draw $nr$ parallel to $mC$. Now $vm : vn :: PC^2 : Pm^2$, therefore $vm$, or $CF$, : $mn$, or $Cr$, :: $PC^2$, or $AF^2$, : $PC^2 - Pm^2$, or $rn^2$, therefore $Cr$ varies as $rn^2$; hence, the locus *AnCE* of all the points $n$ is a parabola. Conceive now this whole figure to revolve about *FCG*, then will *AE* generate the end of the cylinder, and *BFD* will generate that half of the globe which is resisted;

also, the sum of all the $vm$'s will be the solid generated by the parallelogram $AEDB$, and the sum of all the $vn$'s will be the solid generated by the inscribed parabola $ACE$, which solids (*see the Fluxions*) are as $2 : 1$; hence, the resistance of the cylinder : the resistance of the globe :: $2 : 1$.

(72.) It appears by experiment, that this proposition is not true when bodies move either in air or water, the resistance of the globe, compared with that of the cylinder, being less than that which the theory gives it.

## Prop. XXX.

*The same supposition being admitted, if a globe whose diameter is* d, *move in a resisting medium whose density is* n, *with a velocity* v, *the resistance will vary as* $v^2d^2n$.

(73.) For the resistance of a globe is (Art. 71.) equal to half the resistance of the base of a cylinder of the same diameter, moving in the direction of it's axis with the same velocity; therefore the resistance of the globe varies as the resistance of the cylinder; but the end of the cylinder being a circle, whose area is as $d^2$, the resistance (Art. 64.) varies as $v^2d^2n$; therefore the resistance of the globe varies as $v^2d^2n$.

If the Reader wish to see any thing further on the motion of bodies in resisting mediums, he may consult the *Fluxions*.

## Prop. XXXI.

*As a body descends in a fluid, it continually adds more weight to the fluid until it has acquired it's greatest velocity, at which time, the weight added to the fluid from the resistance, is equal to the relative weight of the body.*

(74.) For as long as the velocity of the body increases, the action of the body upon the fluid will continue to increase; and when the body has acquired

it's greatest velocity, the resistance becomes equal to the weight of the body in the fluid, and the body then acts against the fluid with it's whole relative weight.

In the theory of resistances, it is supposed that the resistance against a plane perpendicular to the direction in which it move, is equal to the weight of a column of the fluid, whose base is equal to the area of the plane, and altitude equal to that through which a body must fall to acquire the velocity with which the plane moves. But from a number of experiments which we have made, it appears that the resistance is considerably greater than that; and this is confirmed by the experiments made by others. But there is another remarkable circumstance in the resistance of plane bodies, that is, that the resistances are not in proportion to the surfaces, but increase considerably faster. Surfaces of 9, 16, 36, 81 inches, moving with the same velocity, have been found to have resistances as 9, 17,5, 42,75, 104,75. BORDA's experiment on a plane of 81 inches, with the force of wind moving 1 foot per second, is about $\frac{1}{500}$ of a pound. According to the experiments we have made in air, it appears that a plane of 64 square inches moving with a velocity of 2 feet per second, meets with a resistance equal to 0,0462 troy ounces. Mr. ROUSE of Leicestershire had made a great many experiments on the resistances of wind, and they confirm the above opinion, that the resistance increases faster than the surfaces. But the principal deviation from theory appears from oblique impulses. Mr. ROBINS compared the resistance of a wedge whose angle was 90°, with the resistance on the base, and instead of finding it to be in the ratio of $\sqrt{2} : 1$ as by theory, he found it greater in the ratio of 55 : 68. And when he formed the body into a pyramid, of which the sides had the same surface and inclination as the sides of the wedge, the resistance of the base and sides were now as 55 : 39. Like deviations have been observed by BORDA, and by our own experiments.

The following table contains the result of experiments which are made by a plane of 3,73 inches, moving in water with a velocity of 0,66 feet in a second, under different angles of inclination; assuming the perpendicular resistance by theory to be the same as by experiment.

| Angle. | Experiment. | Theory. | Power. |
|---|---|---|---|
| $10^0$ | 0,0112 | 0,0012 | 1,73 |
| 20 | 0,0364 | 0,0093 | 1,73 |
| 30 | 0,0769 | 0,0290 | 1,54 |
| 40 | 0,1174 | 0,0616 | 1,54 |
| 50 | 0,1552 | 0,1043 | 1,51 |
| 60 | 0,1902 | 0,1476 | 1,38 |
| 70 | 0,2125 | 0,1926 | 1,42 |
| 80 | 0,2237 | 0,2217 | 2,41 |
| 90 | 0,2321 | 0,2321 | |

The first column shews the angle at which the plane struck the fluid; the second shews the resistance by experiment, in troy ounces; the third, by theory; the fourth, the power of the sine of the angle to which the resistance by experiment is proportional. The fourth column was thus computed. Let $s$ = sine of the angle, to radius unity, $r$ = resistance at that angle, and suppose $r$ to vary as $s^m$; then $1^m : s^m ::$ $0,2321 : r$ : hence, $m = \dfrac{\log. r - \log. 0,2321}{\log. s}$; and substituting for $r$ and $s$ their several corresponding values, we get the values of $m$ in the table. Now the theory varies as the cube of the sine; hence it appears that the actual resistance at these angles, is always greater than that given by theory. In theory, the resistance perpendicular to the plane = the weight of a column of fluid whose base is 3,73 inches and altitude 0,08124, the space through which a body must fall to acquire the velocity of 0,66 feet. Now that weight is 0,1598 troy ounces. Hence, the actual resistance : resistance by theory at an angle of $90^0$ :: 0,2321 : 0,1598, or nearly as 3 : 2.

By theory, the resistance of a globe : resistance of a cylinder of equal diameter :: 1 : 2; but by experiment, it is that of 1 : 2,23. Hence it appears, that the actual resistance of a globe : its resistance by theory :: 4 : 3, very nearly.

The results of our experiments do not agree with those made by D'ALEMBERT, CONDORCET, and BOSSET, whose results agree with theory, very nearly. The difference appears to lie in this, that in our experiments, the plane was immersed some depth in the water; in their experiments, the bodies floated on the surface, in which case the fluid can escape more readily, than when the body is immersed below the surface. The resistance of bodies descending in fluids manifestly comes under the case of our experiments.

The resistance of a body moving in a fluid, depends on the form of the back part of the body. We found by experiment, that the resistance on the flat side of a semi-globe : resistance of a cylinder of the same diameter moving with the same velocity :: 0,08339 : 0,07998.

Now instead of the plane moving in a fluid, let the plane be at rest, and the fluid strike it. Then the following table shows the force of water flowing horizontally through a pipe placed at the bottom of a vessel 45,1 inches high, against a plane, the area of the section of the pipe being 0,045 inches, and the velocity out of the pipe was found by experiment to have been that which a body would acquire in falling down 45,1 inches. The table shows the force at different angles by experiment and theory.

| Angle. | Experiment. oz. dwts. gr. | Theory. |
|---|---|---|
| $90^0$ | 1 . 17 . 12 | 1 . 17 . 12 |
| 80 | 1 . 17 . 0 | 1 . 16 . 22 |
| 70 | 1 . 15 . 12 | 1 . 15 . 6 |
| 60 | 1 . 12 . 12 | 1 . 12 . 11 |
| 50 | 1 . 18 . 10 | 1 . 18 . 17 |
| 40 | 1 . 4 . 10 | 1 . 4 . 2 |
| 30 | 0 . 18 . 18 | 0 . 18 . 18 |
| 20 | 0 . 12 . 12 | 0 . 12 . 19 |
| 10 | 0 . 6 . 4 | 0 . 6 . 12 |

The theory supposes the force to be as the sine of the angle, supposed at $90^0$ to be the same as by experiment. Hence it appears that the perpendicular force of the fluid varies as the sine of the angle, the difference of the theory and experiment being only such as may be supposed to have arisen from the inaccuracy to which the experiments must necessarily be subject.

By comparing the force of the fluid thus striking a plane at rest, with the resistance of the plane moving in the fluid with the same velocity, it appears that the former : the latter :: 6 : 5. This difference probably arises from the pressure of the fluid behind the body in the latter case. With different velocities the proportion will vary. For an account of those experiments, see the *Phil. Trans.* 1798.

Definition. If a plane body revolve in a resisting medium about an axis, that point into which if the whole plane were collected, the resistance would remain the same, we call the *Center of Resistance.*

To find the distance $d$ of that center from the axis, let $a$ = area of the plane, $\dot{a}$ = the fluxion of that area at the distance $x$ from the center of the axis. Now the effect of the resistance $\dot{a}$ to oppose the weight turning the axis, is as $x\dot{a}$; but the resistance is supposed to vary as the square of the volocity, or as $x^2$; hence the effect of the resistance is as $x^3\dot{a}$, and the whole effect is as the fluent of $x^3\dot{a}$. For the same

reason the resistance of the whole plane at the distance $d$ is $da^3$; hence, $d\dot{a}^3 =$ fluent of $x^3\dot{a}$, and $d = \sqrt[3]{\frac{\text{flu. } x^3\dot{a}}{a}}$.

Ex. If the plane be a parallelogram, two of whose sides are parallel to the arms, $m$ and $n$ the least and greatest distances of the other two sides from the axis, then $d = \sqrt[3]{\frac{n^4 - m^4}{4n - 4m}} = \sqrt[3]{\frac{\overline{n^2 + m^2} \times \overline{n + m}}{4}}$.

# Sect. IV.

## On the TIMES of EMPTYING VESSELS; and on SPOUTING FLUIDS.

### Prop. XXXII.

*If a fluid run through any tube, kept continually full, and the velocity of the fluid in every part of the same section be the same, the velocities in different sections will be inversely as the areas of the sections.*

(75.) For the same quantity of fluid runs through every section in the same time; now the quantity running through any section ($A$) with the velocity ($V$), in any given time, is manifestly in proportion to $A$ and $V$ conjointly, or to $A \times V$; and as this quantity is constant, $A \times V$ is constant, consequently $V$ varies inversely as $A$.

(76.) Cor. Hence, the velocity of water in a river increases as the breadth and depth decrease; the rule however cannot be applied with accuracy here, as the water at the bottom, from the unevenness, cannot move with the same velocity as at the top; and where the rise and fall at the bottom is very quick, there is probably a great deal of stagnant water.

### Prop. XXXIII.

*Let a vessel be kept filled with a fluid whilst it continues to flow out at an orifice; and let the quantity discharged in a given time, and the area of the orifice be given, to find the velocity at the orifice.*

(77.) Let $a$ = the area of the orifice, $m$ = the quantity discharged in the time $t''$, and $v$ = the velocity. Conceive all the fluid which runs out, to form a cylinder whose base is $a$, and length $l$; then $a \times l = m$; hence, $l = \frac{m}{a}$; therefore, in the time $t''$ the fluid, with the first velocity $v$, would have described the space $\frac{m}{a}$; hence, $t'' : 1'' :: \frac{m}{a} : v = \frac{m}{at}$ the velocity at the orifice.

## Prop. XXXIV.

*If a fluid run out from the bottom or side of a vessel, and the area of the orifice be very small when compared with the bottom, the velocity at the orifice is that which a body would acquire in falling through a space equal to half the altitude of the fluid above the orifice, very nearly.*

(78.) When a fluid issues from a vessel, the velocity through the orifice does not arise from a continual acceleration of descending particles by the force of gravity, as in the case of a body falling freely, but it is communicated by the whole pressure of the surrounding fluid; in consequence of which, the water rushing towards the orifice in all directions, causes a contraction in the stream; and at a distance from the orifice equal to it's diameter, Sir. I. Newton measured the diameter of the section of the stream (which section he called the *vena contracta*), and found it to be to the diameter of the orifice, as 21 : 25; hence, the area of the orifice : the area of the *vena contracta* (they being supposed to be similar) :: $25^2 : 21^2$, which is very nearly as $\sqrt{2} : 1$; and as (Art. 69.) the velocity is inversely as the area of the section, the velocity at the *vena contracta* : the velocity at the orifice :: $\sqrt{2} : 1$. Also, from the quantity of water running out in a given time, and the area of the *vena contracta*, Sir I. New-

TON found (Art. 71.) that the velocity at the *vena contracta* is that which a body acquires in falling down the altitude of the fluid above the orifice; hence, the velocity at the orifice (being less than that at the *vena contracta* in the ratio of $\sqrt{2} : 1$) is (*Mech.* Art. 248.) that which a body would acquire in falling down half the altitude.

It has been supposed, that analogous to this is the case of air rushing into a vacuum, the velocity being supposed equal to that which a body would acquire in falling through a space equal to the height of an homogeneous atmosphere, that is, about 1330 feet per second. But the two fluids are so different in their constitutions, that we can with no safety reason from one to the other. We know that when water flows out of a vessel, the velocity equal to that which a body would acquire in falling down the height of a fluid, is the velocity at the *vena contracta*; but in the case of the air, we do not know that there is any *vena contracta*, or if there be, we know nothing about its ratio to the area of the hole. No experiments have, nor probably can be instituted, to determine this point. It does not appear, therefore, that the conclusion deducing the velocity with which air rushes into a vacuum follows from the principle on which it rests.

(79.) The principle to be established, in order to determine the time of emptying a vessel through an orifice, is the relation between the velocity of the fluid at the orifice, and the altitude of the fluid above it. Most writers upon this subject have considered the column of fluid over the orifice as the expelling force, and from thence, some have found the velocity at the orifice to be that which a body would acquire in falling down the *whole* depth of the fluid, and others, that it is such as is acquired in falling through *half* the depth; and this, without regard to the magnitude of the orifice; whereas it is manifest from experiment, that the velocity at the orifice, the

depth of the fluid being the same, depends upon the proportion which the magnitude of the orifice bears to the magnitude of the bottom of the vessel. Conclusions thus contrary to matter of fact show, either that the principle assumed is not true, or that it is not applicable to the present case. The most celebrated theories upon this subject are those of D. BERNOULLI and M. D'ALEMBERT; the *former* deduced his conclusions from the principle of the *conservatio virium vivarum*, or, as he calls it, the *equalitas inter descensum actualem ascensumque potentialem*, where by the *descensus actualis* he means the actual descent of the center of gravity, and by the *ascensus potentialis* he means the ascent of the center of gravity, if the fluid which flows out could have it's motion directed upwards; and the *latter*, from the principle of the *equilibrium* of the fluid. This principle of M. D'ALEMBERT leads immediately to that assumed by D. BERNOULLI, and consequently they both obtain the same fluxional equation, the fluent of which expresses the relation between the velocity of the fluid at the orifice, and the perpendicular altitude of the fluid above it. How far the principles here assumed can be applied in our reasoning upon fluids, can only be determined by comparing the conclusions deduced from them with experiments.

(80.) The general fluxional equation above mentioned cannot be integrated, and therefore the relation between the velocity of the fluid at the orifice, and it's depth, cannot from thence be determined in all cases. If the magnitude of the orifice be indefinitely less than that of the surface of the fluid, the equation gives the velocity of the fluid equal to that which a body would acquire by falling *in vacuo* through a space equal to the depth of the fluid. But the velocity here determined is not that at the orifice, but at the *vena contracta*; for the fluid by flowing in all directions to the orifice contracts the stream, and the velocity being

inversely as the area of the section, the velocity continues to increase as long as the stream, by the expelling force of the fluid, continues to decrease, and when the stream ceases to be contracted by that force, at that section of the stream, or at the *vena contracta*, the velocity is found, by this theory, to be that which a body would acquire in falling through a space equal to the depth of the fluid. To determine therefore, by theory, the time in which a vessel empties itself, we must know the proportion between the area of the orifice and the area of the *vena contracta*; but no theory will give this. The times therefore of emptying vessels, even in the most simple cases, cannot be determined by theory alone.

## PROP. XXXV.

*If a vessel empty itself through a very small orifice, the velocity of the fluid at the orifice varies as the square root of the altitude* (a) *of the fluid above it.*

(81.) For the velocity at the orifice is that which is acquired (Art. 78.) in falling down $\frac{1}{2}a$, and consequently (*Mech.* Art. 241.) it varies as $\sqrt{\frac{1}{2}a}$, or as $\sqrt{a}$.

## PROP. XXXVI.

*If a vessel empty itself through an orifice at the bottom, and the area of the section, parallel to the bottom, be every where the same, the velocity of the surface of the fluid is uniformly retarded.*

(82.) For (Art. 75.) the velocity of the descending surface is to the velocity at the orifice, as the area of the orifice to the area of the surface, which is a constant ratio; hence, the velocity of the descending surface varies as the velocity at the orifice, or as $\sqrt{a}$ by the last Article; that is, the velocity of the descending surface varies as the square root of the space which it has to describe, which is exactly the case of a body

projected perpendicularly from the earth's surface, where (*Mech.* Art. 245.) the velocity is as the square root of the space to be described; and as the retarding force is constant in the latter case, it must also be constant in the former.

## Prop. XXXVII.

*If a cylindrical or prismatic vessel, having an orifice at the bottom, be kept constantly full, twice the quantity which the vessel contains, will run out in the time it would have emptied itself.*

(83.) For the surface of the fluid being uniformly retarded, and it's velocity becoming equal to nothing at the bottom, the space (*Mech.* Art. 236.) which the surface *would* describe with the first velocity continued uniform for the time in which the vessel would empty itself, is twice the space which the surface actually *does* describe in the time it empties itself; in that time therefore the quantity discharged in the former case is twice that in the latter, because the quantity discharged when the vessel is kept full, may be measured by what *would* be the descent of the surface, if it could descend with it's first velocity.

## Prop. XXXVIII.

*If a cylindrical or prismatic vessel whose altitude is* h, *empty itself through a very small orifice* (a) *at the bottom* (A), *and* $s = 16\frac{1}{12}$ *feet, the time* (t) *of emptying itself* $= \sqrt{\frac{2h}{s}} \times \frac{A}{a}$.

(84.) By the last Proposition, in the time $t$ the quantity discharged with the first velocity ($v$) is equal to $2A \times h$; hence, (Art. 77.) $v = \frac{2A \times h}{a \times t}$; therefore, $t = \frac{2A \times h}{a \times v}$; but (*Mech.* Art. 248, and *Hyd.* Art. 78.)

$v = \sqrt{4s} \times \frac{1}{2}h = \sqrt{2sh}$; consequently, $t = \frac{A}{a} \times \frac{2h}{\sqrt{2sh}} = \sqrt{\frac{2h}{s}} \times \frac{A}{a}$.

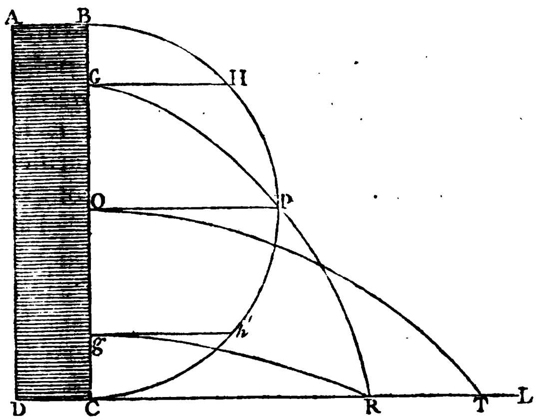
(85.) Cor. Hence, the time of emptying any depth $h$ varies as $\sqrt{h}$; if therefore equal intervals of time be taken from the end of the motion, the spaces described in these intervals are as 1, 3, 5, 7, &c. (*Mech.* Art. 247). Also, the time of emptying any other depth $k$ from the bottom is $\sqrt{\frac{2k}{s}} \times \frac{A}{a}$; consequently, the time of emptying any depth $h-k$ from the top $= \sqrt{\frac{2}{s}} \times \frac{A}{a} \times \overline{\sqrt{h} - \sqrt{k}}$.

For the time of emptying vessels in general, see the *Fluxions.*

## Prop. XXXIX.

*To find the distance to which fluids will spout from the side of a vessel placed upon an horizontal plane.*

(86.) Let $ABCD$ be a vessel filled with a fluid, and $BC$ be perpendicular to the horizontal plane $CL$, and




upon $BC$ let a semicircle be described, and $GH$ an

ordinate perpendicular to $BC$; then the distance $CR$ to which the fluid spouts through a very small orifice at $G$ is $=2\,GH$. By Art. 78. the velocity at the *vena contracta*, which is extremely near to the vessel, is that which a body would acquire in falling down $BG$; we are therefore to consider this as the velocity with which the fluid is projected, and not the velocity at the orifice. Now (*Mech.* Art. 313. and 320.) the curve $GR$ described by the fluid is a parabola, and $BG$ is one-fourth of the parameter belonging to the point $G$, which point is the vertex of the parabola, the fluid spouting out horizontally; hence, $GC$ is the abscissa, and $CR$ it's ordinate; and by the property of the parabola, $4BG\times GC=CR^2$, therefore $CR=2\sqrt{BG\times GC}$ $=2\,GH$, by the property of the circle.

(87.) Cor. If $Cg=BG$, then $gh=GH$, and the fluid spouts to the same distance from $g$ as from $G$. If $BC$ be bisected in $O$, then the distance $CT$, to which the fluid spouts, is equal to $2\,OP=CB$, and this is the greatest distance, $OP$ being the greatest ordinate.

(88.) If the fluid spout perpendicularly upwards, it ought (Art 78.) to rise to the altitude of the surface of the water in the vessel; but it falls a little short of this, partly from the friction at the orifice, partly from the resistance of the air, and partly from the falling back of the water. If the water spout upwards through a pipe, instead of simply a hole, it does not ascend so high, because there being no *vena contracta*, the velocity is not increased immediately after it leaves the pipe, as it is when it flows out of a simple orifice. Also, there is a certain measure of the hole, compared with the size of the vessel, through which the velocity is the greatest. For the retardation arising from the sides of the orifice varies as the circumference of the orifice, or as it's diameter, and the quantity of fluid passing through varies as the square of the diameter; therefore by diminishing the orifice, the retarding force does not decrease so fast as the quantity of fluid to oppose the retardation, decreases; consequently there is a diminution

of velocity from this cause, when you diminish the hole. On the contrary, when you diminish the hole, the effect of the pressure on a smaller quantity of fluid, appears to be in favour of increasing the velocity. These seem to be the reasons, why there is a certain size of the hole which gives the greatest velocity to the issuing fluid.

(89.) The very near agreement of this theory with experiments, proves, very satisfactorily, that the velocity of projection must be that which is acquired in falling down *BG*, the whole altitude of the fluid. And the agreement of the theory of emptying vessels with experiments, shows, very clearly, that the velocity *at* the orifice must be that which is acquired in falling through half the altitude of the fluid. Almost immediately, therefore, after the fluid gets out of the orifice, it's velocity is increased in the ratio of $1 : \sqrt{2}$.

According to Desaguliers, through a hole of 1 inch square, and 25 inches below the surface of the water, the water running out in 1 hour would be 5,2 tons; this he found from repeated experiments. Now as the quantity run-out is as the velocity and area of the hole conjointly, and the velocity is as the square root of the depth, if $a$ = area of the hole, $h$ = the altitude of the water above the hole, then $5 \times 1 : \sqrt{h} \times a :: 5,2 : 1,04 a\sqrt{h}$ the quantity run out in 1 hour, the water continuing at the same height above the hole.

For the motion of water through pipes, see our Experiments.

# SECT. V.

## ON THE ATTRACTION OF COHESION; AND ON CAPILLARY ATTRACTION.

### PROP. XL.

*If two globules of mercury, lying on a smooth plane, be brought to touch, an attraction then takes place, and they immediately rush together, and form one complete globule.*

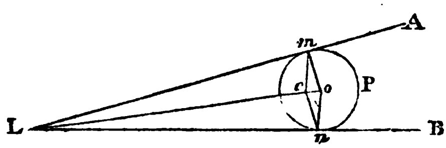
(90.) IF the globules be examined with a microscope, no effect is found to take place till they are actually in contact, and then they rush violently together; this therefore can only be accounted for by an attraction which begins at the instant they come into contact.

(91.) The same effect takes place between two globules of water, when they are laid upon a surface on which they can freely move.

### PROP. XLI.

*If two glass planes* AL, BL, (*of which* BL *is horizontal*) *be inclined at a very small angle, and just moistened with oil, and a drop* P *of oil be placed between them, it will move towards their concourse* L.

(92.) This arises from the attraction of cohesion be-




tween the drop and the glass planes; for if *o* be the

centre of the drop, and *om*, *on* represent the attraction of the drop to each plane (the whole attraction acting perpendicularly to the planes), then the compound motion *oc*, being directed to *L*, will give the drop a motion towards that point. The planes are first moistened, that the drop may move freely.

(93.) If the point *L* be elevated, until the motion of the drop ceases, the action *oc* then becomes equal to the accelerative force of *P* down the inclined plane *LB*; consequently the ratio of the accelerative force upon that inclined plane to the force of gravity, or (*Mech.* Prop. 61.) the height of the plane to the length, gives the ratio of the attraction *oc* to the weight of the drop.

## Prop. XLII.

*If two plane surfaces of metal, &c. be smeared with oil, grease, &c. and pressed together, they will cohere very strongly.*

(94.) This effect arises from two causes, the pressure of the surrounding air, in consequence of the air from between being expelled, and from the attraction of cohesion. That it arises partly from the former cause, is manifest from hence, that if two plates be thus put together and cohere pretty strongly in the air, when they are suspended in the receiver of an air-pump, after exhausting the air, the under plate will frequently fall; when it does not fall, it shews that there must be an attraction of cohesion, at least equal to the weight of that plate. If the air be expelled by different substances, as oil, turpentine, grease, &c. it is found that the attraction of cohesion is different. This therefore must arise, either from the air being expelled more perfectly by one than the other, or that the attraction is rendered stronger by one than by another.

It is this attraction of cohesion by which the constituent particles of a body, admitting them to be in contact, are kept together. When you break a body,

you overcome this attraction; and if you could join the parts together again exactly in the same manner, it would be as strong as before. On this principle we may explain the different degrees of hardness of bodies. Hard bodies may consist of constituent particles which touch in a great part of their surfaces, and thus their attraction may be very great. The constituent particles of soft bodies may touch in a few points, and thus their attraction will be weakened. Solids are supposed to be dissolved in menstruums, from the attraction of cohesion between the particles of the fluid and body being greater than the attraction between the constituent particles of the solid.

### Prop. XLIII.

*There is an attraction of cohesion between water and glass.*

(95.) For take a piece of very clean glass, and hold it in an horizontal position, and a drop of water will remain suspended from it's under side.

### Prop. XLIV.

*The constituent particles of water attract each other.*

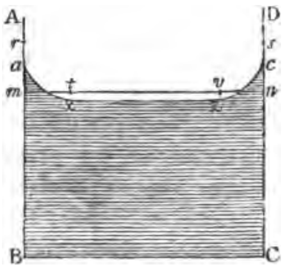
(96.) For a small quantity of water always forms itself into a globule, if no external circumstance tend to prevent it: this can arise only from the mutual attraction of it's parts. If the quantity be large, the upper surface will grow flat, from it's gravity overcoming the attraction of it's parts, but the sides will still be convex.

### Prop. XLV.

*There is an attraction between water and glass, at a distance.*

(97.) Pour water into the glass vessel $ABCD$, and if no attraction took place between the water and glass, the surface $mn$ would be horizontal. Take $mr = mt = ns = nv$, and suppose the glass to act upon the

water through these distances; then the glass *mr* will attract the surface *mt*, and *ns* will attract *nv*, and part of this attraction acting upwards, the gravity of the columns of water under *mt*, *vn* will be diminished by the attraction, and more diminished the




nearer to the sides; consequently their lengths must be increased in order to be in equilibrium with the other columns whose gravity is not diminished; hence, the water will rise in a curve *ax*, *cz*, from the points *x* and *z*, as far as the attraction extends, and the other part *xz* will be horizontal. Now when water is put into a glass vessel, the surface of the fluid puts on the form *axzc*; we conclude, therefore, that glass acts upon water at a distance. In like manner, if any piece of glass be immersed in water, the water will rise on each side of the glass.

(98.) Cor. 1. Hence, if two pieces of glass, parallel to each other, be immersed in the vessel, the water will rise against each. Let them be so near, that the two curves *ax*, *cz* may just meet, then will a certain quantity of the fluid between them be raised above the surface of the fluid in the vessel. Bring them nearer, and as the glass still exerts the same attraction, it must, upon this principle, raise the same quantity of water, and therefore the altitude will be inversely as the distance of the planes; for the same attraction being exerted, there must be the same quantity of fluid supported, consequently the altitude of the fluid will increase as the distance of the planes decreases.

This seems to be conclusive, admitting the water to be both raised and supported by the attraction of a small part of the glass contiguous to the upper surface of the fluid. But as (Art. 95.) there is an attraction of cohesion between glass and water, after the water is raised by the aforesaid attraction, it will then, in part, be supported by the attraction of cohesion. An additional quantity of water may therefore probably be *supported* from this cause.

(99.) Cor. 2. If the glass planes be *inclined* to each other, then it follows, from these principles, that as the distance between the glasses decreases, the altitude of the water will increase. Mr. Hauksbee informs us, that he very accurately measured the abscissas and ordinates of the curve formed by the upper surface of the water between the glass planes, and concluded it to be the common hyperbola, having the surface of the fluid, and concourse of the planes, for it's asymptotes. Now if we admit that the water is raised, and also supported, by the attraction of the glass lying just above the surface of the water, the curve ought to be the common hyperbola; for if we divide the water into laminæ of the same thickness, then there being the same attraction exerted upon each, the same quantity will be supported, and therefore the altitudes (or ordinates) must be inversely as the lengths of the laminæ, or as the distances (abscissas) of the laminæ from the concourse of the planes, which is the property of the hyperbola between the above mentioned asymptotes.

## Prop. XLVI.

*If very small glass tubes, called* capillary *tubes, be dipped into water, the water is found to stand in them, above the level of that in the vessel, at altitudes which are either accurately, or very nearly in the inverse ratio of their diameters.*

(100.) If we admit the fluid to be raised and supported only by an annular surface of the glass contiguous to the upper surface of the water, the ratio ought

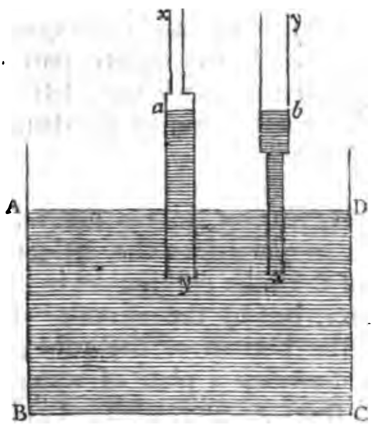
to be accurately so. For let $d =$ the diameter of the tube, $a =$ the altitude of the water in it; then the breadth of the attracting annulus being constant (it being the distance to which the attraction of the glass reaches), the area of this annulus, or the attracting surface, will be in proportion to the circumference of the tube, and consequently to the diameter $d$; also, the quantity of attraction must be in proportion to the quantity of water supported, or to $d^2 \times a$, the tube being cylindrical; hence, $d^2 \times a$ varies as $d$, therefore $d \times a$ is constant, and consequently $a$ varies inversely as $d$. Experiments show that this conclusion is accurately, or very nearly, true.

(101.) Cor. 1. If the tube be taken out of the fluid and laid in an horizontal situation, the fluid will recede from that end which was immersed. For at that end there is no attracting annulus beyond the fluid, whereas there is at the other end, and consequently the fluid will be drawn towards the empty part of the tube, until the length of the other end, left free from the fluid, be equal, or nearly so, to the distance to which the attraction reaches.

(102.) Dr. Hamilton, in his Essays, thinks, that the fluid is not supported by the attraction of the annular surface of the tube contiguous to the upper surface of the water, but by the annulus at the bottom, contiguous to the bottom of the tube; this he supposes will first draw up a plate of water immediately under it, and then a succession of plates, till the weight of the whole is equivalent to the attraction of that annulus. If this were the case, the quantity supported, and consequently the altitude of the fluid, would depend upon the orifice at the bottom; whereas, experiments show that the altitude at which the fluid is supported, depends upon the diameter of the tube at the upper surface of the fluid, without any regard to the form of the tube below it. Again, if in a capillary

tube, water will stand at the altitude of an inch above the surface of the fluid in the vessel, and you depress the tube till there be only an inch of it above the surface, the water will then not rise to the top of the tube; and if you depress the tube still lower, the water will not rise to the top. Thus there will always be an annular surface of the tube above the fluid, which is a strong argument in favour of the fluid being supported by the attraction of such a surface. If the fluid were raised by the attraction of the annulus at the bottom, when the length of the tube above the surface was less than an inch, the fluid ought to run over, and thus a perpetual motion would be formed.

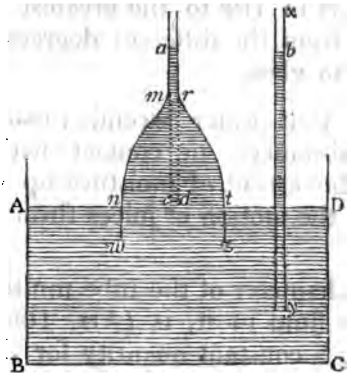
(103.) Dip the tube $xy$ (consisting of two cylindrical tubes joined together) into water, and suppose the fluid to rise to $a$ by the capillary attraction; invert




the tube, and the fluid will rise to $b$, the same altitude, where the diameter is the same. If we admit the principle in Art. 100, the annular surface being the same in both, the same quantity of fluid ought to be supported, whereas, when the small end is downwards, the quantity supported is less. But if we admit the fluid to be partly retained by the attraction

of cohesion, the quantity of the surface being less with the smaller end downwards, the whole support is less, and therefore the quantity supported ought to be less. This seems to be in favour of the support being partly by the attraction of cohesion.

(104.) If the water rise in the capillary tube $xy$ to $b$, and another vessel $az$ be put into the water, having the upper part capillary, and equal to that of $xy$, but the lower part of any size; then if the air be drawn out of this vessel by suction until the water enters into the capillary part, it will stand at the same altitude as in the tube $xy$, after the suction ceases and the air is




admitted into the capillary part. Here the cylindrical part $ad$ is supported by the same power as the water in the other tube, and the other part $mvn$, $rtd$ is supported by the pressure of the air upon the surface of the water in the vessel, in consequence of the air being drawn from within; and this is proved from hence, that if the whole be put under a receiver of an air-pump, and the air be exhausted from the surface of the vessel, the water will not be supported. Dr. JURIN says, that the water will be supported *in vacuo*; but in his time, the air-pumps would not exhaust sufficiently to determine this point; for the altitude to which the water rises being very small, it requires a considerable degree of exhaustion before the

water will fall. The pumps which are now made, show that the water will not be supported after a very great degree of exhaustion.

(105.) If a vessel of water be put under the receiver of an air pump, and the air be exhausted, and capillary tubes be then immersed in it, the water will rise to the same altitude as when the vessel was exposed to the air; the air, therefore, has no effect in causing the ascent of the water.

(106.) Different fluids will rise to different altitudes in the same tube. Spirituous liquors, which are lighter than water, rise to a less height than water, which, of all fluids appears to rise to the greatest height. This can arise only from the different degrees of attraction of these fluids to glass.

Hence it is, that water ascends in sugar, salt, and dry porous substances in contact with the water. Hence also, the ascent of moisture up the fine tubes of vegetables; the motion of juices through the glands in animals, &c.

(107.) The diameter of the tube multiplied into the altitude of the fluid in it, is (Art. 100,) accurately, or very nearly, a constant quantity for the same fluid, which, by experiment, is found to be ,053 of an inch when the fluid is water.

## Prop. XLVII.

(108.) *Let so much cork be attached to a man, as will just support him in water; to find the weight of the cork, that of the man being given.*

Let $W$ = weight of the man, $M$ his magnitude, $m$ = his specific gravity, $C$ = magnitude of the cork, $c$ = it's specific gravity, $w$ = that of water. Then $Mm$ = weight of the man, $Cc$ = that of the cork, and $\overline{M+C} \times w$ = weight of water equal in magnitude to $M+C$. Hence from Prop. 17. $Mm - Cc = \overline{M+C} \times w$; but $W = Mm$,

and $M = \frac{W}{m}$; hence, $Cc = W \times \frac{e}{m} \times \frac{m - w}{c + m}$ the weight of cork required.

Ex. Let $W = 150$ lbs. $c = 216$, $m = 1000$, $w = 900$; then $Cc = 3,126$ lbs. the weight of cork.

Ex. Let $A$ represent the magnitude of a man whose specific gravity is 900, and weight 150lbs. $B$ that of cork whose specific gravity is 240, and $C$ that of water whose specific gravity is 1000; then $a = 900$, $b = 240$, $s = 1000$, and the weight $Bb$ of cork $= 150 \times \frac{3}{85} =$ 5,3lb. the weight of cork to be attached to the man, that he may be of the same specific gravity as water. Any weight of cork therefore above that, would make him float.

## Prop. XLVIII.

*To explain the principle of the* Diving Bell.

(109.) The diving bell was not invented, but greatly improved by Dr. Halley; it's use is to convey in safety a person to the bottom of the sea. It was constructed in the form of a bell, that when it was let down, the water might rise to a less height within, than if it had been made of the same size all the way up. A stop cock was fixed at the top, which could be opened by a person sitting in the inside. The bell was suspended from the yard-arm of a ship, and let down by pullies. By the side of the bell, there was let down a barrel open at the bottom, but closed at the top, and into the top was fixed a leathern tube, and by a weight attached to the other end, that end descended below the bottom of the barrel, which contained fresh air for a supply to the bell, as that air became unfit for respiration. When the person within the bell felt the want of fresh air, he took the leathern tube and turned it's lower end up into the bell, and the fresh air from the barrel ascending in it,

rushed into the bell, and at the same time he opened the stop cock at the top and let out the foul air. By repeating this as often as the case required, the air within the bell was kept fit for respiration. This was Dr. HALLEY's invention. As the bell descends, the pressure of the water upward at the bottom of the bell increases, and condenses the air in the bell, and the water rises up in it; and the problem for our consideration, is, to what height will it rise?


(110.) Let $ABC$ be the bell, $FD$ the depth to which the bottom is immersed, $DE$ the height of the water in the bell. Put $FD = a$, $AD = b$, $AE = x$, $S =$ the capacity of the bell, $s =$ that of the part free from water. Now the height $DE$ to which the water rises, depends on the weight of the air; let therefore $m$ = altitude of the barometer; then $14m =$ the altitude of a column of water, which the air would support in a vacuum. Now the density of the air in it's natural state : density in the bell :: $s : S$, the density being inversely as the space occupied by the same quantity. But the density $S$ supports a column of water whose altitude is $14m$, and the density $s$ supports a column whose altitude is $14m + FE$, since, instead of the air pressing on the surface of the water, we may substitute a column of water whose height is $14m$. Hence, $s : S :: 14m : 14m + FE$, and $s : S - s :: 14m : FE = a - \overline{b - x} = a - b + x$, and $s \times \overline{a - b + x} = 14m \times \overline{S - s}$; and when the figure of the bell is given, we find $x$.

Ex. 1. Let the bell be a *cylinder* or *prism*, with the end downwards; then $s : S :: x : b$, and $s : S - s :: x : b - x$, hence, $x \times \overline{a - b + x} = 14m \times \overline{b - x}$, and

$$x = \sqrt{\frac{\overline{a - b + 14m}|^2}{4} + 14mb} - \frac{a - b + 14m}{2} = AE;$$

hence $DE$ is known. If $a = 50$, $b = 5$, then $DE =$ 2,75 feet.

Ex. 2. Let the bell be a *cone*; then $s : S :: x^3 : b^3$, and $s : \overline{S-s} :: x^3 : b^3 - x^3$; hence, $x^3 \times \overline{a-b+x} = 14m \times \overline{b^3-x^3}$, from which $x$ may be found, and then $DE$.

Ex. 3. Let the bell be a *paraboloid*; then $s : S :: x^2 : b^2$, and $s : \overline{S-s} :: x^2 : b^2 - x^2$; hence, $x^2 \times \overline{a-b+x} = 14m \times \overline{b^2-x^2}$, from which $x$ may be found, and then $DE$.

(111.) At the top of the bell, Dr. HALLEY fixed a meniscus with the concave part downwards, which let in sufficient light to read a small print by in calm weather. When the sea was rough, the Dr. was obliged to have a candle; and a candle of 6 in the pound is found to consume as much air as a man. When the cock was opened at the top of the bell, the water did not enter for this reason. The column of water pressing down to come in at the top, is measured by the depth of the top; but the column of water driving the air up the bell to force it's way out at the top, is measured by the depth of the bottom. The Dr. dispatched a man from the bell, by putting a glass cap over his head made water-tight, a small pipe going from the bell into the cap to supply him with air. The man was obliged to have weights on his feet, to keep him firm on the ground. Dr. HALLEY, who made the experiment, mentioned a small inconveniency he felt in the bell. At first he had a small pain in the ears, as if the end of a tobacco-pipe was thrust in; but after a little time, there was a small puff of air with a little noise, and then he was easy. One of the men with him, to prevent this, stopped his ears with chewed paper, which was forced in so far, that a surgeon got it out with difficulty. The correspondence between the Dr. and those in the ship was kept up by writing with a nail on a plate of lead, fixed to the barrel which conveyed down the air. The bell should be let down gently, or the pressure of the air in it will increase too quickly, and materially affect the body. For the same

reason, it should not be drawn up too fast. When the pressure of the air is the least, a man of middle size sustains a pressure of 30000 lbs., and every 5 fathom of depth of water, adds a weight of about 30000 lbs more.

(112.) Mr. SMEATON changed the form of the bell to that of a parallelopepidon, or as he called it, a chest. This was more convenient for the workmen, a seat being fixed at each end for them to sit on. The chest was supplied with fresh air by a forcing pump through a leathern tube fixed to the top of the chest, and connected with the pump which was in a boot. Several glasses were fixed to the top of the chest.

## PROP. XLIX.

*There is a small attraction of cohesion between mercury and glass.*

(113.) For a very small globule of mercury will adhere to the under side of a clean piece of glass.

## PROP. L.

*There is a strong attraction of the constituent particles of mercury towards each other.*

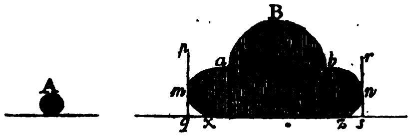
(114.) For a small quantity of mercury laid upon a piece of glass, will, as to sense, form itself into a perfect sphere. If two of these spheres be brought into contact, at the instant they touch they will rush together with a surprizing quickness, and form a single sphere. This can only be explained upon the principle of the mutual attraction of all the parts. As you increase the quantity of mercury, it will begin to deviate from a perfect sphere, and grow flatter on the upper side, arising from the gravity of the mercury becoming sensible, when compared with the mutual attraction of it's constituent particles; and when the quantity becomes considerable, the upper surface will not sen-

sibly differ from a perfect plane, but the sides will retain their convexity.

## Prop. LI.

*If mercury be put into a glass vessel, it will stand lowest at the sides, and rise in a curve till the surface becomes, as to sense, a plane.*

(115.) This arises from the mercury attracting itself by a greater force than it is attracted to the glass, and may be thus explained. A very small quantity *A* of mercury laid upon glass, will, as to sense, form itself into a perfect sphere. If we take a large quantity *B*, it




will not preserve it's spherical form, the force of gravity destroying that figure by counteracting the mutual attraction of the particles, and the mercury will put on the form *xmabnz*; and if two pieces of glass *pq*, *rs* be made to touch the mercury at *m* and *n*, the form will not be sensibly altered. If therefore we take a glass vessel *pqsr* and put mercury into it, the upper surface will still be in the form *mabn*; for it can manifestly make no difference, whether we put the glass to the mercury, or the mercury to the glass; except that in the latter case, the spaces *mqx*, *nsr*, will be filled with mercury, which can have no sensible effect upon the upper surface.

Cor. 1. Hence, if a piece of glass be dipped into mercury, the mercury will be depressed on each side of the glass, in the same manner.

Cor. 2. If the two pieces of glass, *pq*, *rs* be brought so near, that the depressed parts of the mercury may meet, the mercury will be depressed to a distance

which, by experiment, is found to be inversely as th distance of the glasses, accurately, or nearly so.

COR. 3. If small capillary tubes be put into a vess of mercury, the fluid in the tubes will be depressed distances below the surface of the fluid in the vess which are found, by mensuration, to be inversely their diameters, accurately, or nearly so.

COR. 4. If two glass planes, inclined at a sma angle, be put into a vessel of mercury, the mercu between them will be depressed below the surface the mercury without the planes, and that depression found, (as nearly as it can be determined by mensura tion) to be inversely as the distance from the concours of the planes, and therefore the curve is an hyperbol having the concourse of the planes for one asymptot and the surface of the mercury against the planes with out, for the other asymptote.

# SECT. VI.

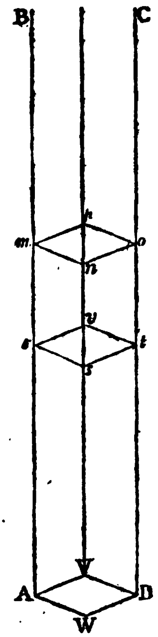
## ON ELASTIC FLUIDS.

### PROP. LII.

*If the particles of an elastic fluid repel each other with forces varying inversely as the* n^th *power of their distances, and* d *represent the density of any part, and* c *the compressive force upon it, then* c *varies as* $d^{\frac{n+2}{3}}$.

(116.) LET *ABCD* be a square column of the fluid, *mnop, rstv* two sections parallel to the base (and consequently each a square equal to the base) whose distance *mr* is equal to *mn*, one side of the square, then will the fluid contained between these sections be a cube. Let *d* be the density of the fluid in this cube, supposed to be indefinitely small, *c* the compressive force on it, and *r* the distance of the particles; then one side of the cube being indefinitely small, *d, c* and *r* may be considered as the same for the whole of this cube. Now the number of particles in *mn* is as $\frac{1}{r}$, consequently the number in the whole section *mnop* is as $\frac{1}{r^2}$. Also, the repulsive force of all the particles in *mnop*, being as the number of particles and force of

each conjointly, is as $\frac{1}{r^2} \times \frac{1}{r^n} = \frac{1}{r^{n+2}}$; and as the repulsive force of each particle acts in every direction, this




repulsive force acting upwards must be equal to the compressive force which it sustains, or $c$ will vary as $\frac{1}{r^{n+2}}$; they will not necessarily be equal, because $\frac{1}{r^{n+2}}$ does not represent the *quantity* of the repulsive force, only what it is proportional to. Also, the number of particles in the cube is as $\frac{1}{r^3}$, and therefore (Art. 5.) $d$ varies as $\frac{1}{r^3}$; hence, $d^{\frac{1}{3}}$ varies as $\frac{1}{r}$, and $d^{\frac{n+2}{3}}$ varies as $\frac{1}{r^{n+2}}$; but $c$ varies as $\frac{1}{r^{n+2}}$; consequently $c$ varies as $d^{\frac{n+2}{3}}$.

(117.) It appears by experiment, that the compressive force of the atmospheric air, varies as the density or $c$ varies as $d$; therefore in this case $\frac{n+2}{3}=1$, and hence $n=1$; consequently the particles of air repel each other with forces which vary inversely as their distances. Also, as the compressive force of air is equal to it's elastic force, these balancing each other, the elastic force must vary as the density.

(118.) It is manifest that there can be no fluid whose density varies in any inverse ratio of the compressive force, that is, you can never, by increasing the compressive force, diminish the density, as any increase of the compressive force must compress the fluid into a less space, and therefore increase the density, unless the particles of the fluid were absolutely in contact, in which case the density would remain the same under any pressure, which is probably not the case with any fluid. Hence $n+2$ must be always positive, that is, $n$ must be some whole positive number, or a negative number less than 2, in order to constitute a fluid consisting of particles which repel each other. If we admit water to be compressible in a very small degree, the particles must be kept at a distance by some repulsive force, and $d$ is nearly constant; but $d$ varies as $c^{\frac{3}{n+2}}$ and that $c^{\frac{3}{n+2}}$ may be nearly invariable, $n$ must be a very great number; hence, upon this supposition, the repulsive force of the particles of water varies inversely as a very high power of their distances.

## Prop. LIII.

*Air has weight.*

(119.) If a vessel be exhausted of air, and balanced at one end of a beam, upon admitting the air, the vessel preponderates. This clearly proves that air has weight, and therefore it must press upon all bodies;

and from the weight necessary to balance the vessel after the air is admitted, compared with the weight of the same vessel of water, we get the specific gravity of air to that of water (Art. 5.), which is as 1 to about 885 in the mean state of the air, or when the barometer stands at $29\frac{1}{2}$ inches, according to Mr. HAUKSBEE. Others have made the specific gravities as 1 to 850, when the baromter stands at 30 inches.

## PROP. LIV.

*The density of the air is in proportion to the force which compresses it, under a certain limitation.*

(120.) Let $abc$ be a glass cylindrical tube, hermetically sealed at $c$, and let the bottom be covered with mercury, whilst the air in $wc$ is in it's natural state. Pour in mercury at $a$ and it will force the mercury to rise in $wc$, and continue to pour in, till the mercury stands at $y$ as high above the point $x$ to which it has now risen in $wc$, as the altitude of the mercury in the common barometer; then that column of mercury (Art. 130.) is equivalent to the weight of the column


of air incumbent upon it; hence, the pressure against the air in $cx$ is now twice as much as it was against the

air in $cw$, and $cx$ is observed to be $= \frac{1}{2} cw$; hence, the air being compressed into half the space, the density is doubled. In like manner, if another column of mercury of the same altitude be added, $cx$ is found to be $= \frac{1}{3} cw$; thus the compressing force is made three times greater, and the density is three times greater. In this manner, the compressing force is found in any other case to be in proportion to the density. The same is observed to be true in all kinds of factitious airs, upon which experiments have been made.

But when $cx = \frac{1}{4} cw$, or less than that quantity, the density does not vary as the compressive force, but in a less ratio. For if the compressive force become infinite, $cx$ does not vanish; the least value being when the particles come into contact.

(121.) By increasing the compressing force of the air, the particles are brought nearer together, but are kept from coming into contact by their repulsive force; these forces must therefore be equal, when the fluid is at rest. The repulsive force is what we usually call the air's elasticity; hence, the elasticity of the air being in proportion to it's compressive force, must be also in proportion to it's density.

## *LEMMA.*

(122.) If $a : b :: b : c :: c : d ::$ &c. then by Euclid, *B*. 5. p. 12. $b : c :: b + c + d +$ &c. : $c + d + e$ + &c.; hence, $a : b :: b + c + d +$ &c. : $c + d + e +$ &c.; for the same reason, $b : c :: c + d + e +$ &c. : $d + e + f$ + &c. and so on to the end of the series. Hence, *vice versâ*, if $a : b :: b + c + d +$ &c. : $c + d + e +$ &c. and $b : c :: c + d + e +$ &c. : $d + e + f +$ &c. and so on, then will $a : b :: b : c :: c : d :: d :$ &c.

## Prop. LV.

*If the force of gravity be considered as constant, and altitudes from the earth's surface be taken in arith-*

*metic progression, the corresponding densities of the air will decrease in geometric progression.*

(123.) Conceive the whole atmosphere to be divided into an indefinite number of laminæ of *equal* thickness, parallel to the earth's surface; and let $a$, $b$, $c$, &c. represent the respective densities of these laminæ, beginning at the surface of the earth; then the compressive force on the laminæ $a$, $b$, $c$, &c. will be proportional to the weight incumbent upon each, that is, the sum of the weights of all the laminæ above; but the weight of each lamina is as the density × it's thickness, or, as the thickness is the same, as it's density; hence, the compressive forces on $a$, $b$, $c$, &c. will be as the sums of all the quantities which represent the densities above them, or as $b+c+d+$&c. $c+d+e+$&c. $d+e+f+$&c. &c. &c. But (Prop. LII.) the compressive force of the air is as it's density; hence, $a : b :: b+c+d+$&c. $: c+d+e+$&c. and $b : c :: c+d+e+$&c. $: d+e+f+$&c. and so on; hence, by the above Lemma, $a : b :: b : c :: c : d :: d : e ::$ &c. Now the laminæ being of the same thickness, the last proportion shows, that as you ascend by equal spaces, or in arithmetical progression, the densities decrease in geometrical progression.

The density is inversely as the rarity; therefore as you ascend by equal spaces, the rarities increase in geometrical progression.

## Prop. LVI.

*Given the density of the air, to find the corresponding altitude; and the converse.*

(124.) By the nature of logarithms, if the natural numbers be in geometrical progression, their logarithms are in arithmetical progression; hence, as the altitudes increase in arithmetical progression, whilst the corresponding rarities of the air increase in geometrical progression (Art. 123), it follows, that the altitudes increase as the logarithms of the rarities increase.

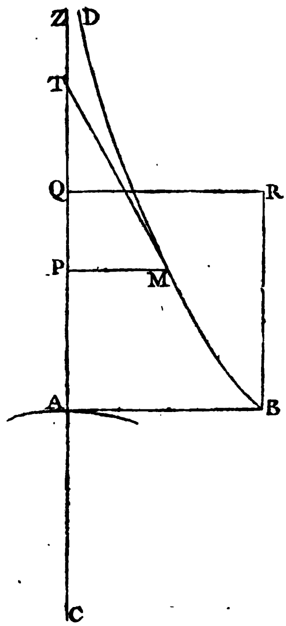
Hence, if at the altitudes $x$ and $y$, the rarities be $m$ and $n$, the rarity at the surface being unity, we have $x : y ::$ log. $m$ : log. $n$. Now Mr. COTES (*Hyd.* p. 103.) collected from experiment, that at the altitude of 7 miles, the rarity is 4 times greater than at the surface; hence, if $y=7$, $n=4$, then $x : 7 ::$ log. $m$ : log. 4, therefore $x=7\times\frac{\log. m}{\log. 4} = 11{,}626\times\log. m$; if therefore the rarity $m$ be given, we know $x$. Also, $\log. m=\frac{x}{7}\times\log. 4$, therefore (*Flux.* Art. 109.) $m=\overline{4}|^{\frac{x}{7}}$; hence, if the altitude be given, the rarity $m$ will be known. This rule is not accurate, because it supposes the compressive force of the air to be as it's density, which is not true, unless the temperature be the same.

If it should appear that the altitude at which the density is 4 times less than at the surface, be not 7 miles, then 11,626 must be altered in the ratio of 7 to that altitude.

*Otherwise solved.*

Let (Fig. next, see that Art.) $AP=x$, $PM=y$, $AB=1$; then (Flux. Art. 49. Ex. 4.) $\dot{x} : -\dot{y} :: y : PT=AQ$, and $\dot{x}=AQ\times\frac{-\dot{y}}{y}$; hence, $x=AQ\times 0{,}4342945\times-\log. y=AQ\times0{,}4342945\times\log.\frac{1}{y}$. By Art. 130. $AQ=4342{,}9$ fathoms; hence, $x=10000\times\log.\frac{1}{y}$, very nearly, expressing, in fathoms, the altitude in terms of the density. This is probably a little more correct in numbers, than the other solution. Also, if $c=$ the number whose hyp. log. $=1$, then (Flux. Art. 111.) $c^{\frac{x}{10000}}=\frac{1}{y}$, and $y=\frac{1}{c^{\frac{x}{10000}}}$, the density in terms of the altitude.

(125.) Let *CA* be the radius of the earth, which produce to *Z*; draw *AB* perpendicular to *CA*, and let




it represent the density of the air at the surface, and *PM* represent the density at any altitude *AP*, and let *BMD* be a curve passing through the extremities of all the ordinates *PM*. Then (Art. 123.) as the altitude *AP* increases in arithmetic progression, the density *PM* will decrease in geometric progression; hence, *PM* is the logarithmic curve, whose subtangent *PT* is the modulus of the system. Let *AQ* be the altitude of an homogeneous * atmosphere whose density is *AB*, and complete the parallelogram *BAQR*. Now consider the whole atmosphere *AZ*, and the homogeneous atmosphere *AQ*, to be divided into an indefinite number

* An homogeneous atmosphere is an atmosphere supposed to be of the same weight as that which surrounds the earth, and whose density is uniform and equal to the density of the air at the earth's surface.

of laminæ of equal thickness; then (Art. 123.) the whole pressure on $AB$ in both cases may be measured by the sum of all the densities of these laminæ, and the density being as the laminæ $PM$ and $AB$ respectively, the pressures will be as the sum of all the $PM$'s, and the sum of all the $AB$'s, or as (*Flux.* Art. 49, Ex. 4.). $AB \times PT$, and $AB \times AQ$; but these pressures are equal; hence, $AB \times PT = AB \times AQ$, consequently $AQ = PT$; the modulus of this system of logarithms is therefore the altitude of an homogeneous atmosphere.

For the general investigation of the density of the air, when the force of gravity is supposed to vary as any power of the distance from the earth's centre, see the Treatise on *Fluxions*.

## Prop. LVII.

*The altitude of an homogeneous atmosphere at any point* P, *is the same as that at the earth's surface, the temperature being the same.*

(126.) For let $D$ = the density of the air at $P$, $C$ = the compressive force of the air at that point, or the weight $W$ of the superincumbent air, $H$ = the altitude of an homogeneous atmosphere at $P$, or the height of the atmosphere above that point, upon supposition that it was reduced to the uniform density $D$; then $W \propto D \times H$, therefore $H \propto \frac{W}{D} \propto \frac{C}{D}$; but $C \propto D$ (Art. 120.); hence, $H$ is constant.

## Sect. VII.

### On the BAROMETER.

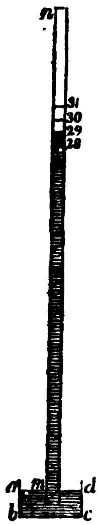
The ancients were ignorant of the weight and pressure of the air, for they attributed the cause of the ascent of water in pumps, syphons, &c. to nature's *abhorrence of a vacuum.* Even Galileo himself, who first observed that water would not rise above 33 or 34 feet in a pump (though the pump was 40 feet long and the bucket worked at that height above the water) only thence concluded, that nature abhorred a vacuum but to a certain degree. But Toricelli, who first made a Barometer, observed that at different times, the mercury would stand at different altitudes, and thence concluded, not only that the mercury was supported by the pressure of the air, but that the pressure was subject to a variation. Hence he called this instrument a *Barometer* or *Baroscope.* He further observed, that in, or a little before, fair weather, the mercury would rise, and in stormy and rainy weather it would fall. Hence, this instrument is called a *Weather-glass.*

### Prop. LVIII.

*To make a Barometer.*

(127.) If a glass tube *mn* above 31 inches long, hermetically sealed at one end, be filled with mercury,

and, putting the finger on the open end, the tube be inverted, and then immersed in a bason *abcd* of




the same fluid, the altitude at which the mercury will stand in the tube above the surface of the mercury in the bason, is between 28 and 31 inches. A tube thus filled is called a *Barometer*.

This is the most perfect barometer; for although there have been contrivances to lengthen the scale above 3 inches, yet there are objections arising from the construction, which more than counterbalance the advantage. Sometimes when the tube comes to the height of 28 inches, it is bent in an angle, so as to make the scale longer; for the mercury will always rise to the same perpendicular height. But the mercury here does not rise so freely as when the tube was continued upright. This is called a *diagonal* barometer. Instead of a bason, the lower end is sometimes bent and turned upwards, terminating in a large bulb containing the mercury, the bulb having a small hole to admit the air. Sometimes the tube is bent and terminating in a large cylindrical form, and on the mercury is laid a

ball of iron from which goes a string passing round an axis, and having a weight attached to the other end; to this axis there is fixed an index; and as the mercury rises and falls, it carries the iron ball with it, and that turns the axis with the index, the extremity of which moves against a graduated circle. Thus you extend the length of the scale; but the friction of the axis obstructs the free motion of the index; the string is also affected by the variation of the humidity of the air. This is called a *wheel* barometer. The upright barometer is therefore the most perfect.

If in the dark you shake the mercury in the tube, so as to make it rise and fall, if you see no light on it's surface, it is well filled; but if light be perceived, there is a little air within. The air is best expelled by boiling the mercury in the tube, by putting the closed end into a sand bath, and gradually heating it till the mercury boils. The section of the bason should be large compared with that of the tube, so that the surface of the mercury in the bason do not sensibly rise or fall, on the fall or rise of the mercury in the tube.

## Prop. LIX.

*The mercury is suspended in the tube of a barometer, by the pressure of the air upon the surface of the mercury in the bason.*

(128.) For if a barometer *mn* be put under the receiver of an air-pump, and the air be exhausted, as you continue to exhaust the air, and consequently to diminish it's pressure upon the surface of the mercury in the bason, the mercury in the tube will continue to descend; and when no sensible quantity of air is left, the altitude of the mercury will not be sensibly above that in the bason; and upon admitting the air again into the receiver, the mercury will rise in the tube to it's former height.

As the density of the air, and consequently it's compressing force, is subject to a variation, the altitude of

the mercury must be subject also to a corresponding variation; it is always however contained between the limits of 28 and 31 inches. A tube thus filled is therefore graduated from 28 to 31 inches. When the mercury *rises*, it's surface becomes *convex*; and when it *falls*, *concave*. This arises from the attraction of the mercury to the glass; shake therefore the tube gently, to make the surface level, before you observe the height.

(129.) GALILEO was the first person who discovered the pressure of the air. He found by experiment, that water could be raised, by the common pump, to a certain height, and no higher; whereas, had nature abhorred a vacuum, according to the opinion of some of the philosophers at that time, the water might have been raised to any height. He conjectured therefore, that it was owing to the air's gravitation; the truth of which was afterwards confirmed by his pupil TORRICELLIUS, who considered, that if the pressure of the air could support a column of water 35 feet high, which is about the mean height to which a pump can raise water, it could suspend a column of mercury, whose density is about 14 times as great, only about one $14^{th}$ part of 35 feet high, or about 30 inches; he accordingly tried the experiment, and found that the mercury stood at the altitude which he expected. Thus he clearly proved the gravitation of the air; and hence this is called the *Torricellian* experiment; and the vacuum which is left above, when the mercury descends from the top of the tube, after immersing it in the bason, is called the *Torricellian* vacuum. When the tube is filled with great care, this vacuum is supposed to be the most perfect that can be made.

## PROP. LX.

*To find the height of an homogeneous atmosphere.*

(130.) The mercury in the tube of a barometer is

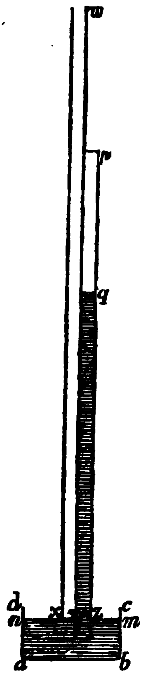
(Art. 128.) sustained by the pressure of the air; and (Art. 32.) when two fluids communicate, the altitudes at which they stand are inversely as their specific gravities. Let us take the specific gravity of air to that of water as 1 : 850, the barometer standing at 30 inches. Now if we take the specific gravity of water to that of mercury as 1 : 14, we have the specific gravity of air to that of mercury as 1 : 11900; hence, (Art. 32.) 1 : 11900 :: 30 in. : $11900 \times 30 = 357000$ in.$=5,63$ miles, the height of an homogeneous atmosphere, or an atmosphere of the same weight as the present atmosphere, and whose specific gravity is every-where the same as that of the air at the earth's surface. If we take the specific gravities of air and water as 1 : 885, when the barometer stands at $29\frac{1}{2}$ inches, we shall have the altitude of an homogeneous atmosphere 5,77 miles. The specific gravity of mercury has been here supposed 14, that of water being 1; but when the mercury is very pure, it's specific gravity has been found to be only 13,6. To determine with accuracy the height of an homogeneous atmosphere by this method, the specific gravity of the mercury in the barometer, at the time of observation, should be determined, as it is subject to a small variation from the different temperatures of the air. By some very accurate experiments at the temperature of $32^0$ of Fahrenheit's thermometer, when the barometer stood at 30 inches, the height of an homogeneous atmosphere was found to be 4342,9 fathoms.

## Prop. LXI.

*The weight of the mercury in the barometer (the tube being cylindrical) above the level of that in the bason, is equal to the weight of a cylinder of air of the same base, reaching to the top of the atmosphere.*

(131.) Let $qz$ be the altitude of the mercury in the tube $pv$; take a cylindrical column $xvw$ of the air,

whose base $xv$ is equal to $vz$ that of the mercury. Now the section $nxvm$ of mercury being at rest, every point thereof must be equally pressed, and therefore




equal parts must be equally pressed; but the pressures on $xv$, $vz$ arise from the incumbent columns $xw$ of air and $vq$ of mercury, and these being perpendicular cylindrical columns, the pressures are equal to their weights (Art. 15.); consequently the weights of these columns are equal.

(132.) Some have found it difficult to conceive, why the weight of the mercury in the tube should not be equal to the weight of the air pressing upon the *whole* surface of the mercury in the bason. This difficulty has arisen from their not making a proper distinction between pressure and weight: the column of mercury gives a pressure upwards to the surface of the mercury

in the bason, equal to the weight of the whole incumbent air; but as fluids press equally in all directions, this pressure which the mercury gives is as much greater than it's weight, as the surface of mercury in the bason is greater than the orifice of the tube. It is a fact similar to the hydrostatical paradox, where a smaller weight sustains a greater.

## Prop. LXII.

*When the mercury in the barometer stands at* 30 *inches, the pressure of the air upon every square inch is about* 15*lbs. avoirdupoise.*

(133.) For by Art. 131, a column of air, of mean density, whose base is a square inch, presses as much as a column of mercury of the same base 30 inches high, the weight of which is about 15lbs. avoirdupoise.

(134.) Cor. Hence, if we take the surface of a middle-sized man to be $14\frac{1}{2}$ square feet, when the air is lightest it's pressure on him is 13,2 tons, and when heaviest, it is 14,3; the difference of which is 2464lbs. This difference of pressures must greatly affect us in regard to our animal functions, and consequently in respect to our health, more especially when the change takes place in a short time. The pressure of the air upon the whole surface of the earth is about 77670297973563429 tons.

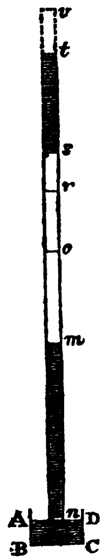
The reason why air is thought to be light in fair weather, when it is really most heavy, is, because when the barometer is nearly 31 inches high, an additional weight of about 3000lbs. of air, above the weight when the mercury stands lowest, acts like a bandage all over the body, which quickens the circulation of the blood, and we find ourselves more lively and alert.

## Prop. LXIII.

*If the tube* ns *of a barometer be perfectly cylindrical, and be in part only filled with mercury, and then*

*it's open end be immersed in a bason of the same fluid, the mercury will sink below the point, called the standard altitude, or the point at which it would have stood if no air had been left in; and the standard altitude will be to the depression below that altitude, as the space occupied by the air after the immersion to the space occupied before.*

(135.) Let *rs* be equal to the space occupied by the air before the tube is immersed, or when the air is in it's natural state; after the immersion of the tube into




the bason *ABCD* let the mercury sink to *m*; then the air which, in it's natural state, occupied the space *rs*, now occupies the space *ms*; and the space occupied by the same quantity being inversely as the density, or (Art. 121.) inversely as the elasticity, we have the elasticity in *rs* : the elasticity in *sm* :: *sm* : *rs*. Let *no* be the height at which the mercury would have stood if no air had been left in the tube, or the height of the mercury in the barometer. Then (Art. 128.) the compressive force of the air, and (Art. 121.) consequently it's elasticity, when it occupies the space *rs*,

would support a column of mercury $no$, because the air, when it occupied that space, was in it's natural state; and the elasticity of the air when it occupies the space $sm$ would support a column of mercury $mo$, because it depresses the mercury from $o$ to $m$; hence, the elasticity of the air in $rs$ : the elasticity in $sm$ :: $on$ : $om$; consequently $on : om :: sm : sr$.

(136.) This proposition may be applied to the solution of two problems; for we may either give the quantity of air left in before immersion, to find the altitude of the mercury after immersion, or we may give the altitude of the mercury after immersion, to find the quantity of air left in before. As the standard altitude $no$ (Art. 128.) is subject to a variation, it has been usual in this case to assume it 30 inches; but when accuracy is required, it must be taken equal to the height of the mercury in the barometer at the time.

*Ex.* 1. Let the length $ns$ of the tube be 35 inches, and the depression $om$ below the standard altitude $no$ (=30 in.) be 10 inches, to find the quantity of air left in before inversion.

As $ns=35$, and $no=30$, we have $so=5$; also, $om=10$; hence, $sm=15$; therefore, $30 : 10 :: 15 : rs=\frac{10\times 15}{30}=5$ inches.

*Ex.* 2. Let 5 inches of air be left in the same tube before inversion, to find the altitude of the mercury after.

In this case the point $m$ being unknown, the second and third terms of the proportion are unknown; put therefore $x=om$, then $x+5=sm$; hence, $30 : x :: x+5 : 5$, therefore $x^2+5x=150$, consequently $x=10$ or $-15$. The answer $+10$ shows that the mercury will stand at 10 inches below $o$; and the answer $-15$ shows, that if the tube were continued to $v$, and $ot$ taken equal to 15 inches, and the space $st$ were filled with mercury, the space $tv$ above being a vacuum, that this column $st$ of mercury would also be supported

by the elasticity of the air in $sm$. In fact, $st=om$, and therefore the elasticity of the air which depresses a column $om$, must necessarily sustain an equal column $st$.

## Prop. LXIV.

*If* a *be the altitude of the mercury in a barometer at the bottom of an hill, and* b *the altitude at the top; to find it's height.*

By Prop. $x=10000\times\log.\frac{1}{y}$; but $a:b::1\ (AB):y$, and $\frac{1}{y}=\frac{a}{b}$; hence, $x=1000\times\log.\frac{a}{b}=10000\times(\log.a-\log.b)$ very nearly in fathoms. If therefore when we take $\log.a-\log.b$ we remove the decimal point 4 places to the right, we get $x$ in fathoms.

### Example.

| | | |
|---|---|---|
| Alt. mer. at $A$ = 29,8 | .... log. | 1,4742163 |
| $P$ = 29,1 | .... log. | 1,4638930 |
| $AP$ in fathoms = | ............ | 103,233 |

Further, $\log.b=\log.a-\frac{x}{10000}$; and giving $a$, $x$, we get $b$ the density of $P$ compared with that of $A$.

The difference of the temperatures of the air and mercury at the top and bottom, is not here considered. A connection therefore must be applied; and that is done by two thermometers having their bulbs immersed in mercury below and above, and two in the air, by the following

### Rule.

(137.) Let $D=$ the difference of the barometic altitudes in tenths of an inch; $a=$ the difference between

$32^0$ and the mean temperature of the air; $m =$ the mean of the barometic heights of the mercury; $d =$ the difference of the mercurial temperatures; $E =$ the correct elevation; then,

$$E = \frac{30\ (87 \pm 0{,}21\,a)}{m} D \pm 2{,}83d.$$

In the first part $+$ or $-$ is used, according as $a$ is greater or less than 32; and in the second part $+$ or $-$ is used, according as the upper barometer is the warmer or colder.

Ex. Let the mercury in the barometer at the lower station be at 29,4 inches, it's temperature 50°; and the temperature of the air 45°; and let the height of the mercury of the upper station be 25,19 inches, it's temperature 46°, and that of the air 39°; to find the altitude of the mountain.

Here, $D = 294 - 251{,}9 = 42{,}1$, $a = 10$, $m = 27{,}29$, $d = 4$; hence, $E = \frac{30 \times 89{,}1 \times 42{,}1}{27{,}29} - 2{,}83 \times 4 =$ 4111,92 feet.

The principles on which the above Rule is deduced, are these:

1. To produce any fall of the mercury by ascending, the altitude is inversely as the density of the air, or as the height of the mercury.

2. When the mercury stands at 30 inches, and the air and mercury are of the temperature 32°, we must rise up 87 feet to produce a depression of $\frac{1}{10}$ of an inch.

3. If the air be of a different temperature, this 87 feet must be increased or diminished by 0,21 of a foot for every degree of difference of temperature from 32°.

4. Every degree of difference of temperature of the mercury at the two stations, makes a change of 2,83 feet in the elevation.

Hence it is easy to collect the above Rule.

The altitudes of some of the most remarkable Mountains, &c. above the height of the Sea.

| | | Feet. |
|---|---|---|
| Mount Puy de Domine in Auvergne | | 5088 |
| Mount Blanc | Alps | 15662 |
| Monte Rosa | Alps | 15084 |
| Arguille d'Argenture | Alps | 13402 |
| Monastery of St. Bernard | Alps | 7944 |
| Mount Cenis | | 9212 |
| Pic de los Reyes | Pyrennees | 7620 |
| Pic du Medi | Pyrennees | 9300 |
| Pic d'Ossano | Pyrennees | 11700 |
| Canegon | Pyrennees | 8544 |
| Lake of Geneva | | 1232 |
| Mount Ætna | | 10954 |
| Mount Vesuvius | | 3938 |
| Mount Heola in Iceland | | 4887 |
| Snowdon | | 3555 |
| Ben Moir | | 3723 |
| Ben Laur | | 3858 |
| Ben Gloe | | 3472 |
| Shihallion | | 3461 |
| Ben Lomond | | 3180 |
| Tinto | | 2342 |
| Table Hill, Cape of Good Hope | | 3454 |
| Gondar City in Abyssinia | | 8440 |
| Source of the Nile | | 8082 |
| Peak of Teneriffe | | 14026 |
| Chimboraçon | | 19595 |
| Cayambourow | | 19391 |
| Antisana | | 19290 |
| Pichinha | | 15670 |
| City of Quito | | 2977 |
| Caspian Sea below the Ocean | | 306 |

## Dr. HALLEY'S *Account of the Rising and Falling of the Mercury in a Barometer, upon the Change of Weather.*

(138.) To account for the different heights of the mercury at several times, it will not be unnecessary to

enumerate some of the principal observations made upon the barometer.

1st. The first is, that in calm weather, when the air is inclined to rain, the mercury is commonly low.

2dly. That in serene, good, settled weather, the mercury is generally high.

3dly. That upon very great winds, though they be not accompanied with rain, the mercury sinks lowest of all, with relation to the point of the compass the wind blows upon.

4thly. That *cœteris paribus*, the greatest heights of the mercury are found upon easterly and north-easterly winds.

5thly. That in calm, frosty weather, the mercury generally stands high.

6thly. That after very great storms of wind, when the quicksilver has been low, it generally rises again very fast.

7thly. That the more northerly places have greater alterations of the barometer than the more southerly.

8thly. That within the tropics, and near them, those accounts we have had from others, and my own observations at St. Helena, make very little or no variation of the height of mercury in all weathers.

Hence I conceive, that the principal cause of the rise and fall of the mercury, is from the variable winds which are found in the temperate zones, and whose great inconstancy here in England is most notorious.

A second cause is the uncertain exhalation and precipitation of the vapours lodging in the air, whereby it comes to be at one time much more crouded than at another, and consequently heavier; but this latter in a great measure depends upon the former. Now from these principles I shall endeavour to explicate the several phænomena of the barometer, taking them in the same order I laid them down.

1st. The mercury's being low inclines it to rain,

because the air being light, the vapours are no longer supported thereby, being become specifically heavier than the medium wherein they floated; so that they descend towards the earth, and in their fall meeting with other aqueous particles, they incorporate together and form little drops of rain. But the mercury's being at one time lower than at another, is the effect of two contrary winds blowing from the place where the barometer stands; whereby the air of that place is carried both ways from it, and consequently the incumbent cylinder of air is diminished, and accordingly the mercury sinks. As for instance, if in the German ocean it should blow a gale of westerly wind, and at the same time an easterly wind in the Irish sea, or if in France it should blow a northerly wind, and in Scotland a southerly, it must be granted me that, that part of the atmosphere impendent over England would thereby be exhausted and attenuated, and the mercury would subside, and the vapours which before floated in those parts of the air of equal gravity with themselves, would sink to the earth.

2dly. The greater height of the barometer is occasioned by two contrary winds blowing towards the place of observation, whereby the air of other places is brought thither and accumulated; so that the incumbent cylinder of air being increased both in height and weight, the mercury pressed thereby must needs rise and stand high, as long as the winds continue so to blow; and then the air being specifically heavier, the vapours are better kept suspended, so that they have no inclination to precipitate and fall down in drops; which is the reason of the serene good weather, which attends the greater heights of the mercury.

3dly. The mercury sinks the lowest of all by the very rapid motion of the air in storms of wind. For the tract or region of the earth's surface, wherein these winds rage, not extending all round the globe, that

stagnant air which is left behind, as likewise that on the sides, cannot come in so fast as to supply the evacuation made by so swift a current; so that the air must necessarily be attenuated when and where the said winds continue to blow, and that more or less according to their violence; add to which, that the horizontal motion of the air being so quick as it is, may in all probability take off some part of the perpendicular pressure thereof; and the great agitation of it's particles is the reason why the vapours are dissipated, and do not condense into drops so as to form rain, otherwise the natural consequence of the air's rarefaction.

4thly. The mercury stands the highest upon an easterly or north-easterly wind, because in the great Atlantic ocean, on this side the 35th degree of north latitude, the westerly and south-westerly winds blow almost always Trade, so that whenever here the wind comes up at east and north-east, it is sure to be checked by a contrary gale as soon as it reaches the ocean; therefore according to what is made out in our second remark, the air must needs be heaped over this island, and consequently the mercury must stand high, as often as these winds blow. This holds true in this country, but is not a general rule for others where the winds are under different circumstances; and I have sometimes seen the mercury here as low as 29 inches upon an easterly wind, but then it blew exceeding hard, and so comes to be accounted for by what was observed upon the third remark.

5thly. In calm frosty weather the mercury generally stands high, because (as I conceive) it seldom freezes but when the winds come out of the northern and north-eastern quarters, or at least unless those winds blow at no great distance off; for the northern parts of Germany, Denmark, Sweden, Norway, and all that tract from whence north-eastern winds come, are subject to almost continual frost all the winter; and thereby the lower air is very much condensed, and in that state is brought hitherwards by those winds, and being

accumulated by the opposition of the westerly wind blowing in the ocean, the mercury must needs be prest to a more than ordinary height; and as a concurring cause, the shrinking of the lower parts of the air into lesser room by cold, must needs cause a descent of the upper parts of the atmosphere to reduce the cavity made by this contraction to an *æquilibrium*.

6thly. After great storms of wind, when the mercury has been very low, it generally rises again very fast. I once observed it to rise 1½ inch in less than 6 hours, after a long continued storm of south-west wind. The reason is, because the air being very much rarefied, by the great evacuations which such continued storms make thereof, the neighbouring air runs in the more swiftly to bring it to an *æquilibrium*; as we see water runs the faster for having a great declivity.

7thly. The variations are greater in the more northerly places, as at Stockholm greater than at Paris, (compared by Mr. Pascall*), because the more northerly parts have usually greater storms of wind than the more southerly, whereby the mercury should sink lower in that extreme; and then the northerly winds bringing the condensed and ponderous air from the neighbourhood of the pole, and that again being checked by a southerly wind at no great distance, and so heaped, must of necessity make the mercury in such case stand higher in the other extreme.

8thly. Lastly, this remark, that there is little or no variation near the equinoctial, as at Barbadoes and St. Helena, does above all others confirm the hypothesis of the variable winds being the cause of these variations of the height of the mercury; for in the places above named, there is always an easy gale of wind blowing nearly upon the same point, viz. E.N.E. at Barbadoes, and E.S.E at St. Helena, so that there being no contrary currents of the air to exhaust or

* Equilibre des Liqueurs.

accumulate it, the atmosphere continues much in the same state: however, upon hurricanes (the most violent of storms) the mercury has been observed very low, but this is but once in two or three years, and it soon recovers it's settled state of about $29\frac{1}{2}$ inches.

The principal objection against this doctrine is, that I suppose the air sometimes to move from those parts where it is already evacuated below the *æquilibrium*, and sometimes again towards those parts where it is condensed and crouded above the mean state, which may be thought contradictory to the laws of statics, and the rules of the *æquilibrium* of fluids. But those that shall consider how, when once an impetus is given to a fluid body, it is capable of mounting above it's level, and checking others that have a contrary tendency to descend by their own gravity, will no longer regard this as a material obstacle; but will rather conclude, that the great analogy there is between the rising and falling of the water upon the flux and reflux of the sea, and this of accumulating and extenuating the air, is a great argument for the truth of this hypothesis. For as the sea, over against the coast of Essex, rises and swells by the meeting of the two contrary tides of flood, whereof the one comes from the S.W. along the channel of England, and the other from the north, and, on the contrary, sinks below it's level upon the retreat of the water both ways, in the tide of ebb; so it is very probable, that the air may ebb and flow after the same manner; but by reason of the diversity of causes whereby the air may be set in motion, the times of these fluxes and refluxes thereof are purely casual, and not reducible to any rule, as are the motions of the sea, depending wholly upon the regular course of the moon.

# Sect. VIII.

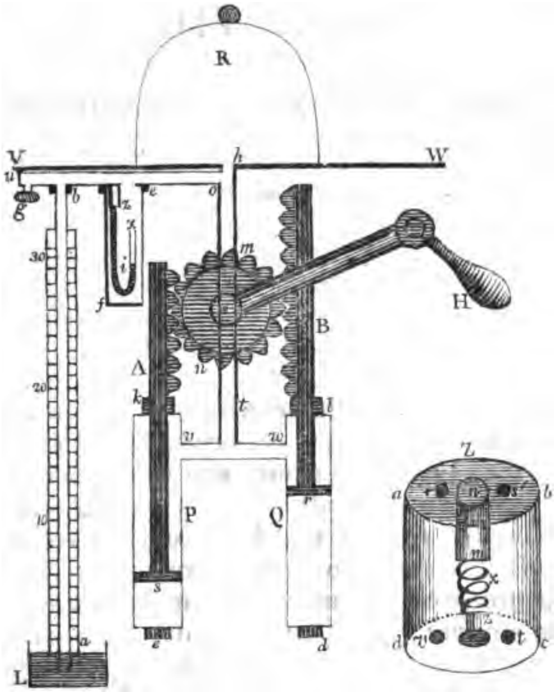
## On the AIR-PUMP, and CONDENSER.

### Prop. LXV.

*To construct an Air-Pump.*

(139.) The Air-Pump which I use in my experimental Lectures is thus constructed. *WV* is a brass plate ground perfectly plane, and strengthened on the under side with ribs; at *h* there is a small orifice, over which stands a glass vessel *R*, called a receiver, the edge of which is also ground truly plane, so that if a little grease be put upon the edge before it is placed on the receiver, it will be air-tight; in general, however, a piece of leather well prepared with grease is laid upon the plate for the receiver to stand upon; but you may make a more perfect exhaustion by the other method, on account of the air which the leather will give out; in this leather there is a hole made corresponding to *h* in the plate. From *h* a brass pipe *ht* descends, and turning each way at the bottom enters the two barrels, *P*, *Q*, at *v* and *w*. At the bottom of each barrel there is a small hole against which there are two pieces *c*, *d*, screwed on, containing the valves, each of which is represented by the figure *Z*, which is a solid piece *abcd* of brass, through the middle of which there is a cylindrical hole, partly filled with a solid brass cylinder, against the bottom *m* of which a spiral spring *x* acts, which rests below against a screw *z*, by means of which the spring may be rendered

stronger or weaker; through the brass there are also two holes, one from *r* to *v*, and the other from *s* to *t*;




and over the top *ab* there is tied a piece of oiled silk, having two holes corresponding to *r* and *s*; and when this piece *Z* is screwed on to the bottom of the barrel, the end *n* of the cylinder *nm* is pressed against the hole in the barrel, by means of the spring *x*. The barrels are truly cylindrical, having each a sucker *r*, *s*, (without a valve) surrounded with leather, and fitted so close to the barrel as to be air tight; these suckers are fixed to two brass rods *A*, *B*, having cogs above; *mn* is a small wheel with cogs acting on those of the rods, and moving by an handle *H*, which being turned backwards and forwards, the rods *A*, *B*, and consequently the suckers *s*, *r*, ascend and descend alternately. From the top of the pipe *ht* there

proceeds another pipe *ou*, into an orifice of which there is a fixed glass tube *ab*, having it's lower end immersed in a bason *L* of quicksilver; this tube is called the *gage*, at the back of which there is fixed a frame of wood, which is graduated from the mercury in the bason up to 31 inches. At *g* there is a screw, by unscrewing which, you can admit the air into the pipe *ou* when it is exhausted. The rods *A*, *B*, pass each through a collar of leathers at *k* and *l*, which are air-tight. The supporters to the whole of this are here omitted, as they would have rendered the figure confused, and have been of no use for the understanding of the instrument. This being the construction, the exhaustion takes place in the following manner:

(140.) Turn the handle, and bring the sucker *r* down to the bottom of the barrel then the sucker *s* will be carried just above the orifice *v*; and by turning the handle in the contrary direction, *s* will be depressed to the bottom, and *r* will rise just above the orifice *w*. Now upon the descent of *s*, it must manifestly force all the air in the barrel *P* before it, the sides being air-tight; the air therefore will depress the cylinder *nm* (Fig. *Z*) and escape through the holes *rv*, *st*; after which the screw *x* will force *mn* up against the orifice at the bottom of the barrel and prevent any air from returning into it. Then elevate *s* and depress *r*, and *r*, in like manner, will force out all the air before it. Now as *s* ascends, it leaves a vacuum between *s* and the bottom; but when *s* has gotten above *v*, the air will rush from the pipe *t*, which communicates with the receiver *R* and gage *ab*, into this vacuum, the consequence of which is, that the air in the receiver and gage becomes rarefied by being expanded into a greater space; and as this must take place every time each sucker descends, or at each turn of the handle, there must be a continued exhaustion, and consequently a continued rarefaction of the air in the receiver and gage. But besides this gage, there is another included in a

glass cylinder *ef* which has also a communication with the pipe *ou*; in this there is a bent glass tube *zix*, hermetically sealed at the upper end *z*, and filled with mercury to *i*, as represented by the shaded part. Then when the air is exhausted to a considerahle degree, the pressure of the air upon the mercury at *i* will not be able to sustain the mercury in the other leg, and therefore it will descend, and the two surfaces will approach to the same level, and if you could make a perfect exhaustion, they would stand in the same horizontal line; the difference of the altitudes therefore (measured upon a scale which lies against the tube) shows how much there wants of a perfect vacuum. If to the height of the mercury in the other gage, you add the difference of the altitudes in this gage, it gives the altitude in the gage *ab*, if you could make a perfect vacuum, or it gives the altitude at which the barometer stands at that time. By this method you may try whether a barometer be properly filled and graduated.

## *LEMMA.*

(141.) Let a quantity $a$ be diminished till it becomes successively $b, c, d$, &c. and let the decrements $a-b$, $b-c$, $c-d$, &c. be always in proportion to the quantities themselves, $a, b, c, d$, &c. then will both these quantities and their decrements be in geometrical progression.

For by supposition, $a : a-b :: b : b-c :: c : c-d ::$ &c.; hence, *dividendo*, $a : b :: b : c :: c : d ::$ &c. Also, *alternando*, $a : b :: a-b : b-c$, $b : c :: b-c : c-d$, &c.; hence, $a-b : b-c :: b-c : c-d ::$ &c.

## Prop. LXVI.

*If* b *represent the capacity of one of the barrels, and* r *that of the receiver, together with the pipes and gages connected with it; then the quantity of air extracted after every turn : the quantity before that turn* $:: \mathrm{b} : 2\mathrm{b}+\mathrm{r}$; *and the quantity left in : the quantity before* $:: \mathrm{b}+\mathrm{r} : 2\mathrm{b}+\mathrm{r}$.

(142.) For conceive the sucker $r$ to be down and $s$ to be up, and the receiver, pipes, gages, and barrels, which all now communicate, to be filled with air; then as the whole capacity of these is $2b+r$, the quantity of air may be represented by $2b+r$, from which, by the descent of $s$, the quantity $b$ will be driven out; and this must evidently be the case at every turn. And as the quantity $b$ is taken away from $2b+r$, there must remain the quantity $b+r$.

Cor. Hence, the quantity taken away at every turn being always in the same ratio to the whole quantity before the turn, the air can never be all exhausted.

## Prop. LXVII.

*The density of the air in the receiver at first : the density after* t *turns* :: $\overline{2b+r}|^t : \overline{b+r}|^t$.

(143.) For the density is (Art. 4.) as the quantity of air contained in the same space. Now the quantity before any turn : the quantity after :: $2b+r : b+r$ by Art. 142. and therefore the density at every turn is diminished in the same ratio; hence, by the composition of ratios, after $t$ turns, the density is diminished in the ratio of $\overline{2b+r}|^t : \overline{b+r}|^t$.

Hence, the density is diminished in geometrical progression.

## Prop. LXVIII.

*When the density of the air is diminished in the ratio of* n : 1, *the number of turns* $t = \dfrac{\log. n}{\log. \overline{2b+r} - \log. \overline{b+r}}$.

(144.) For (Art. 143.) $n : 1 :: \overline{2b+r}|^t : \overline{b+r}|^t$; hence, $n = \overline{\dfrac{2b+r}{b+r}}\Big|^t$, consequently (*Fluxions*, Art. 109.) $\log. n = t \times \log. \dfrac{2b+r}{b+r} = t \times \overline{\log. \overline{2b+r} - \log. \overline{b+r}}$; hence, $t = \dfrac{\log. n.}{\log. \overline{2b+r} - \log. \overline{b+r}}$.

### Prop. LXIX.

*As the air is exhausted, the mercury will rise in the gage; and the* defects *of the mercury in the gage from the standard altitude, after each successive turn, form a geometric series, the ratio of whose terms is* $2b+r:b+r$.

(145.) For as the density of the air within the gage, and consequently (Art. 120.) it's compressing force on the mercury, is diminished at every turn, the compressing force of the air upon the mercury in the bason, which remains the same, must cause the mercury to rise in the gage. If all the air were exhausted, the mercury would rise as high as in the common barometer, or to what is called the standard altitude. Now the compressing force of the quantity of air left in, prevents the mercury from rising to the standard altitude, and therefore it's compressing force must be equivalent to a column of mercury equal to the defect; therefore the defect, being as the compressing force, must be (Art. 120.) in proportion to the density, which, at every turn, diminishes in the ratio of $2b+r : b+r$, by Art. 143.

### Prop. LXX.

*The ascents of the mercury in the gage, at each successive turn, form a geometric series, the ratio of whose term is* $2b+r : b+r$.

(146.) The defects of the mercury from the standard altitude diminish in the ratio of $2b+r:b+r$; and the differences of these defects are the successive ascents of the mercury; but, by the Lemma, if a set of quantities decrease in geometrical progression, their differences will also decrease in the same geometrical progression; hence, the ascents of the mercury successively decrease in the ratio of $2b+r : b+r$.

(147.) The various properties of the air are very readily shown by the air-pump; as in the following experiments.

*Ex.* 1. Air is necessary for the *production* of sound. For if a bell be put under the receiver of an air-pump, and the air be exhausted, the bell, when struck, cannot be heard; and if the air be gradually let in, the sound will gradually increase.

*Ex.* 2. Air is necessary for the *propagation* of sound. For if a receiver be put over a bell, and then another receiver over that, and the air be exhausted from between them, no sound is heard; the sound therefore is not propagated through the vacuum.

*Ex.* 3. Air is necessary for the existence of fire.

For if a candle be put under the receiver and the air be exhausted, it immediately goes out.

*Ex.* 4. Air is necessary for the existence of animal life.

For most animals put under the receiver, die almost immediately upon exhausting the air, and probably all would, could we make a perfect vacuum.

*Ex.* 5. The effect of the pressure of the air is rendered visible, by taking away the air from one side of a body, whilst it continues on the other.

For if a bladder be tied over the top of a glass receiver, and the air be exhausted from within, at every exhaustion, the pressure of the air upon the bladder will continue to depress it, until it bursts with a very great explosion.

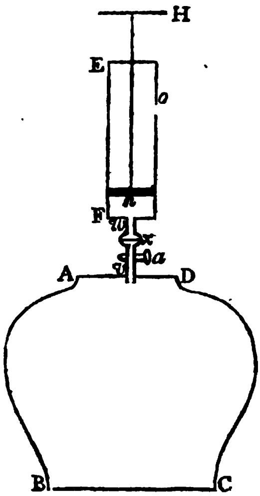
These are a few of the properties of the air which are shown by this instrument; but the experiments are too many to be all here enumerated.

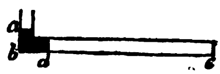
## Prop. LXXI.

*To construct a Condenser.*

(148.) A Condenser is thus constructed: *ABCD* is a strong vessel called a receiver, made either of glass or metal; if of glass, upon the top there is laid a brass plate with a stop-cock *a*, having under it a prepared piece of leather to make it air-tight, and also a like plate at the bottom. Into the cock at *x* there

is screwed a syringe *EF*, having a sucker *n*, which is moved by a handle at *H*; at *w* there is a valve which opens downwards, and at *o* there is an orifice. Now let the sucker be drawn up above the orifice *o*, and both the barrel of the syringe and the receiver



be filled with air in it's natural state. Then upon forcing down the sucker, the air opens the valve at *w*, and a barrel of common air is forced into the receiver. Upon raising again the sucker *n*, a vacuum is left under it, the valve preventing the air from returning; and when the sucker gets above *o*, the air will immediately rush in and fill the barrel; thus upon every descent of the sucker, you force into the receiver a barrel of common air, and consequently you condense the air in the receiver.

After the receiver is charged, the stop-cock at *a* may be turned to prevent the return of the air, and the syringe may be taken off, and any other apparatus may be screwed on for experiments with the condensed air in the receiver.

If the reservoir be partly filled with water, and the pipe with the stop-cock descend into it; then if there be screwed on to its top, a pipe with small holes, upon opening the stop-cock, the superior pressure of the air within the condenser, will force the water up through the said holes, and form a fountain.

## Prop. LXXII.

*If* b *represent the capacity of the barrel of the syringe, and* r *that of the receiver, then after* t *descents of the sucker, the density of the air in the receiver will be to the density at first, in the ratio of* r + tb : r.

(149.) For the quantity of air at first may be represented by $r$, and after $t$ descents of the sucker, a quantity represented by $tb$ will be forced into the receiver, and therefore the whole quantity in it will be $r+tb$; hence, (Art. 4.) the density after $t$ descents : the density at first :: $r+tb : r$.

Cor. Hence, the densities after any number of successive descents, are in arithmetic progression.

(150.) If $abc$ be a glass tube with the end at $a$ open, and the other end hermetically sealed, and a small quantity of mercury put in so as to leave the air in $dc$ in it's natural state; then if this be put into the receiver with the part $bc$ horizontal, and the air be condensed, the condensed air pressing on the mercury will force it towards $c$, and the air in $dc$ will continue of the same density as that in the receiver. Now as the density is inversely as the space occupied by the same quantity (Art. 37.), the density in $dc$, and consequently in the receiver, is inversely as $dc$; when therefore $dc$ is diminished until it be $n$ times less than it was at first, the density will be increased $n$ times. Hence, as the density, after any number of successive turns, increases in arithmetic progression,

the reciprocals of the spaces will be in arithmetic progression, and therefore the spaces themselves will decrease in musical progression. This instrument is called a *gage*.

(151.) A bell in condensed air sounds louder than in air in it's natural state. Fire-engines, air-guns, artificial fountains, some kinds of forcing pumps, &c. act by condensed air.

## SECT. IX.

### ON PUMPS AND SYPHONS.

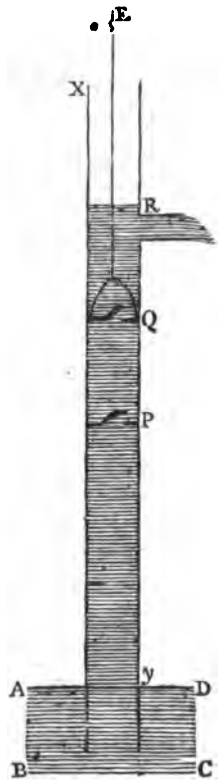
#### PROP. LXXIII.

*To construct a common Pump.*

(152.) THE Pump was invented by CTESIBIUS, a Mathematician of *Alexandria* about 120 years before Christ.

The common Pump is thus constructed. $Xy$ is a hollow cylinder having it's lower end in water; $P$ is a fixed sucker; $Q$ a sucker moveable by means of a handle fixed to the rod $E$, and each sucker has a valve opening upwards. Now let us suppose $Q$ to descend as low as it can, and each valve to be shut, and that the pump has at present no water in it; then when $Q$ ascends, the air between $P$ and $Q$ will follow it, and consequently it will become rarefied, therefore the air under $P$ being now denser than the air above, it will open the valve at $P$ and rush into $PQ$, and the whole air within being thus rarefied, it will not open the valve at $Q$, which is pressed down with air that is not rarefied. The air therefore in the pump being rarefied, the pressure of the air upon the surface of the water without the pump will force the water a little way up the pump. Now when $Q$ descends, it will press down the air under it, and that air will shut the valve at $P$ by pressing *upon* it, but it will open the

valve at Q by pressing *under* it, and thus some of the air will escape. Then when Q ascends again, the pressure of the air upon it's valve will shut it, and the same operation will be repeated. Thus at each ascent of Q the water will rise, till at length it comes up to Q,




and then upon the descent of Q it will open it's valve and get above the sucker, and the sucker then being drawn up, it will carry the water up and throw it out of the spout *R*.

If *h* be the height to which the atmosphere will raise the water in the pump, Q has to raise only the column of water above that altitude *h*; it therefore

matters not how near $P$ is placed to the water in the hole, since the pump will act equally easy. This is called a *sucking* pump.

## PROP. LXXIV.

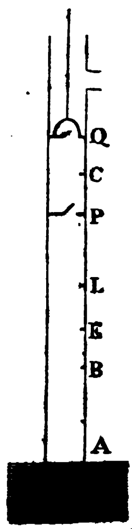
*To construct a Forçing Pump.*

(153.) Here the sucker $Q$ has no valve, and the air between $P$ and $Q$ is, by depressing the sucker $Q$, expelled through a valve opening outwards at $R$, instead of being expelled through $Q$, as in the other pump.

## PROP. LXXV.

*To find what height the water in the pump is raised at every stroke.*

Let the sucker $Q$ be at $C$ when there is no water in the pump; and on the ascent of the sucker to $Q$, let the water rise to $B$. Put $AC=n$, $AQ=m$, $m-n=p$, $a=$ the altitude of a column of water which the air would support, $\frac{m+a}{2}=b$, $AB=x$, then $BQ=m-x$.




On the ascent of the sucker $Q$ from $C$ to $Q$, the water rises from $A$ to $B$, and the air which at first occupied the space $AC$, now occupies the space $BQ$, and the elastic force of the air is inversely as the space occupied; also, the elasticity of the air in $AC$ is equivalent to a pressure of water whose altitude is $a$; hence, $m-x:n::a:\frac{na}{m-x}$, the altitude of a column of water which would be supported by the elasticity of the air in $BQ$; but that elasticity with the column $AB$ of water, is equivalent to a column of water whose altitude is $a$; therefore $\frac{na}{m-x}+x=a$, and $x=b\pm\sqrt{b^2-pa}$. But it is the least root we must here use, for the water cannot

rise to both altitudes by the same stroke, it must therefore stand at the least, to which it must first come, and at which, when it does come, there is an equilibrium. If the fluid could be placed at the height denoted by the other root, there would also be an equilibrium.

The fluid being got to $B$, and the sucker at $C$, to find the altitude $BE$ to which it rises at the second stroke, put $BC=n'$, $BQ=m'$, $AB=r$, $BE=x'$; then in the above equation put $n'$, $m'$, $r+x'$, for $n$, $m$, $x$, and we get $\frac{n'a}{m'-r-x'}+r+x'=a$, and $x$ gives $BE$; and so on for any number of strokes.

In the descent of the sucker $Q$ to $C$, the valve $P$ is shut, and when the air between $Q$ and $P$ is compressed so as to become of greater elasticity than the external air, the valve of $Q$ will be opened, and the water at each stroke will ascend as above described. But when in the descent of $Q$, the elasticity of the air in the pump is not sufficient to open the valve of $Q$, the air within the pump will suffer no further rarefaction, and the water will rise no higher. Let $L$ be that height, to find which, put $AL=x$, then $LC=n-x$, $LQ=m-x$; put therefore $n-x$ for $n$ in our proposition, and we have $\frac{n-x}{m-x}\times a+x=a$, and $x=\frac{1}{2}m\pm\sqrt{\frac{1}{4}m^2-pa}$, where the least root is to be taken, for the reason before stated. The other root would give an equilibrium, but the water could not be raised so high by the pump. At any height *between* these two roots, the water would be too *heavy*, and *without* them, too *light*.

If $pa$ be greater than $\frac{1}{4}m^2$, $x$ is impossible, and this case can have no existence. If $m$ be given, the greatest altitude to which the water can rise, is when $\frac{1}{4}m^2=pa$, and then $x=\frac{1}{2}m$; and $p=\frac{m^2}{4a}$ the length of the

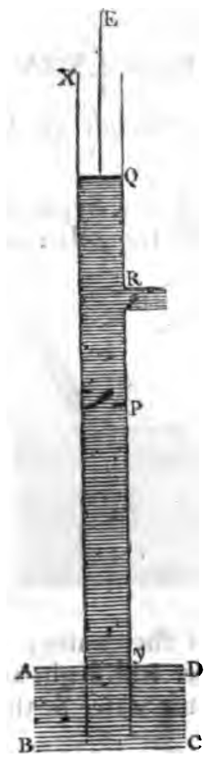
stroke. That the pump therefore may work, $p$ must be greater than $\frac{m^2}{4a}$.

If $m = 16$ feet, $a = 32$, $p = \frac{m^2}{4a} = 2$; the length of the stroke must therefore be greater than 2 feet, that the pump may work.

In our original equation $\frac{na}{m-x} + x = a$; any three of the four quantities being given, we may find the fourth.

*Ex.* Given $n$, $a$, $x$ to find $m$, we get $m = \frac{na}{a-x} + x$.

Hence $m - n = \frac{na}{a-x} + x - n$ the length of the stroke.



If $n=36$, $a=32$, $x=2$, $m-n=4,4$ the length of the stroke.

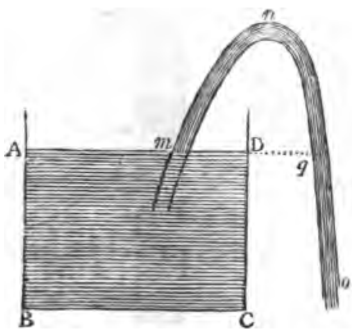
If the orifice at $R$ be very small, the water will be forced out with a great velocity. On this principle may be constructed the barrels of a fire engine. Then when the water follows $Q$ as $Q$ ascends, upon it's descent it shuts the valve at $P$ by pressing upon it, and opens that at $R$ and forces out the water.

(154.) In this pump, $Q$ must, at it's highest point, be within 32 feet of the water in the reservoir $ABCD$, because in the rarest state of the atmosphere the pressure of the air will not raise the water in a vacuum above that altitude. In the other pump, $P$ must be within a little less than the same distance, in order that the water may always rise above it.

## Prop. LXXVI.

*To explain the principle of the motion of water through a Syphon.*

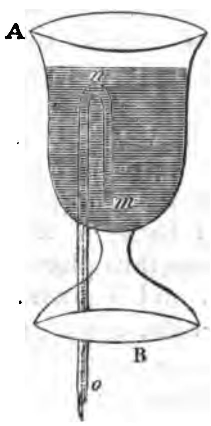
(155.) If one end of a Syphon $mno$ be put into a vessel of water, and the other end without be lower




than the surface of the water; then if the air be drawn out, the water will begin and continue to run, until the surface of the water in the vessel is on a level with the end $o$.

(156.) For when the air is drawn out of the Syphon, the water will rise in it to *n* by the pressure of the air upon the surface of the water in the vessel, and then it will descend to *o* by it's gravity. Now the pressure of the air at *o* to force the water in the direction *onm*, is equal to the pressure of the air on the surface of the fluid in the vessel to force the water in the direction *mno*, at least, extremely nearly so, on account of the very small difference of the altitudes of the air above *m* and *o*; but the former pressure is opposed by the pressure of the column *no*, and the latter pressure is opposed by the pressure of the column *mn*; the latter pressure of the air therefore being less opposed than the former pressure, the fluid must move in the direction of the latter pressure, or in the direction *mno*; and the fluid will continue to run till the pressures of *on*, *mn*, become equal, or till *o* and *m* are in the same horizontal line, for then their perpendicular heights being equal, their pressures will be equal by Art. 33. Thus it appears that suction is owing to the pressure of the air. If the tube be capillary, *qo* must be longer, than the column which is supported by capillary attraction, or the fluid will not run out.

Cor. If a syphon be put into a cup *ABC*, and the




longer leg pass through the bottom, and water be poured

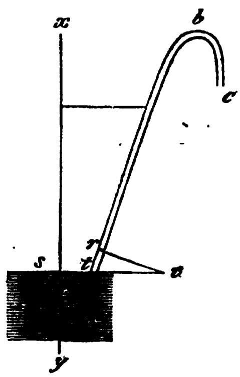
into the cup till it gets above the top *n*, it then begins to run, and all the water in the cup will run out. For the water ascends in the leg in *n* as the cup fills, and when it gets to *n*, the syphon begins to act as before described. Sometimes, instead of the leg *mn*, a glass tube with it's top closed is put over the other leg, and the space between this tube and the leg, acts as the leg *mn* before acted, and is considered as one leg of the syphon. This is called TANTULUS's cup.

If there be a regular supply of water, but not sufficient to keep it up, above the top *n*, so that the surface continually descend, the cup will be emptied, and the water will cease to flow out till the supply raises the water above *n*, when it will empty itself again through the syphon; thus there will be an intermission.

## PROP. LXXVII.

*To describe the* HESSIAN *Pump.*

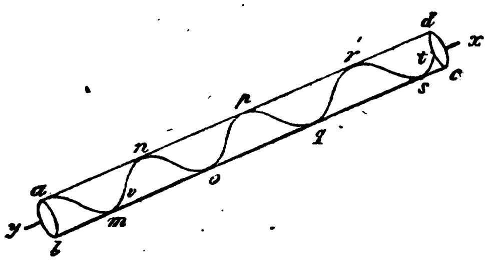
Let *abc* be a hollow tube open at both ends, revolving about a perpendicular axis *xy*, having its lower end *a* immersed in water. Let *stv* be perpendicular to *xy*, and *vr* to *tr*; and let *tv* represent the centrifugal force of the point *t*, and resolve this into two forces *tr*, *rv*; then the force *tr* will tend to raise the water up the tube and carry it out at *c*. The velocity of the tube therefore about *xy* must be such, that the centrifugal force may be sufficient to overcome the tendency of the water to fall back in the tube. Several tubes are thus placed about the axis *xy*, and a round trough is fixed under the end *c*, to receive the water.



## PROP. LXXVIII.

*To describe* ARCHIMEDES'S *Spiral Tube for raising Water.*

Let $xy$ be the axis of an inclined cylinder $abcd$, $amnopqrst$ a spiral tube running round it, having its ends open; and let such be the position of the axis $xy$,




that the point $m$ on the under side may be lower than the point $a$; and so on through the spiral; then if a ball be put into the spiral at $a$, it will descend to $m$. Let $v$ be very near $m$, and turn the cylinder about the axis, so that $v$ may come to the under side, and then the point $m$ will ascend; the body therefore will fall down to $v$, that being now the lowest point; the ball therefore has advanced in the spiral towards $t$; and by continuing to turn the cylinder, it will continue to advance towards $t$, and at length fall out at the end $t$. The operation by which the ball is made to ascend, is like that by which a body is made to ascend an inclined plane by forcing the plane under it. If the end $ab$ be immersed in water, the water will descend to $m$, and by turning the cylinder, will

ascend through the spiral and flow out at *t*, for the same reason the ball did.

If the end *ab* were put into a running stream and a water-wheel put on the axis at *y*, so that the cylinder might be turned by the stream, the water would continually run out at *t*, and be received for any purposes for which it might be wanted. Or water may be thrown out of a pond, by turning the cylinder with the hand.

# SECT. X.

## ON THE THERMOMETER, HYGROMETER, AND PYROMETER.

(157.) A THERMOMETER is an instrument constructed to measure different degrees of heat. It is a glass tube with a bulb at the bottom, having the bulb and part of the tube filled with a fluid; the tube is hermetically sealed at the top, and the part not occupied by the fluid is a vacuum. Against the tube there is a scale to measure the expansion of the fluid under different temperatures.

### PROP. LXXIX.

*To find what fluids are proper for Thermometers.*

(158.) Fluids expand by being heated, and contract again as they grow cold. Those fluids, therefore, which are not subject to be frozen, and whose expansion is sensible, and in proportion to the heat applied, are proper for thermometers. Now the expansion of mercury, linseed oil, and spirits of wine, is, as to sense, proportional to the heat applied. This BROOK TAYLOR found by the following experiment. Having constructed a thermometer with linseed oil, he put it into cold water, and then into water heated to any

degree, and noticed the altitudes at which the fluid in the thermometer stood in each case. He then put equal quantities of these waters together, which gave a mean heat; and by putting the thermometer into this mixture, he found that it stood at a mean altitude between the two former altitudes. And this appeared to be true, of whatever temperatures the two parts of water were. The mean temperature therefore always agreeing with the mean altitude, the expansion must be in proportion to the heat. The same is found to be true of mercury, and of spirits of wine. Mercury is best, as being less subject to freeze.

It is found by experiment, that fluids imbibe heat in proportion to their densities. Hence, if $H$, $h$, be the heats of two fluids, $Q$, $q$, their respective quantities of matter; when mixed together, the heat of the mixture $= \dfrac{HQ+hq}{Q+q}$.

## Prop. LXXX.

*To fill a Thermometer.*

(159.) The bore of the tube is so small that the fluid cannot be poured in; therefore to get in the fluid, heat the bulb, by blowing the flame of a lamp against it with a blow-pipe, and you will expel the air from within; then dip the open end of the tube into the fluid, and it will rise up into the tube and bulb, by the pressure of the air upon the surface of the fluid into which you dip it, there being a vacuum, or nearly so, within the tube and bulb. If it do not fill the first time, repeat the operation till it does; and if there be any air-bubbles, tie a string to the end of the tube, and whirl it about till the bubbles escape. Having thus filled the tube, hold it over the lamp till it boils, and in that state, let it be hermetically sealed, and upon the descent of the fluid, when it grows cold, the space above must be a vacuum.

Or the tube may be thus filled. Put the upper end of the tube into the stem of a funnel, and pour water into the funnel; then hold the bulb over the flame of lamp or candle, and the heat will expel the air from the tube. Then take the bulb from the flame, and the pressure of the air on the water will force it into the tube.

If the air be all expelled, and you turn the tube with the bulb upwards, the fluid will descend to the end of the tube. This is a test of the goodness of the exhaustion of the air. The ball should be large in proportion to the tube.

A thermometer tube ought to be a perfect cylinder. To find whether it be so, put in a small quantity of mercury, and let this run along the tube, and if the length of the mercury be the same in every part, the tube is a perfect cylinder.

To have the thermometer very sensible, the ball should be large in proportion to the tube; and also it should be made flat, that the heat may sooner diffuse itself through the mercury, as the effect will then sooner take place, from a variation of temperature. Thermometers are made to extend to different degrees of heat according to the uses for which they are intended.

## Prop. LXXXI.

*To graduate a Thermometer according to* Fahrenheit's *scale.*

(160.) Having filled the tube of the thermometer, and fixed it against a frame upon which the graduations are to be made, put it into water just freezing, and against the surface put 32; then put it into boiling water, and against the fluid put 212*.

* This is not true for all states of the air, as under different pressures of the air, water will boil at different degrees of heat.

Divide this interval into 180 equal parts, and also continue the same divisions down below 32 to the bulb. Then will 98 be blood heat, 76 summer heat, and 55 temperate. If the tube and scale be continued upwards to 600, it will give the heat of boiling mercury; and if it be continued downwards to 40 below 0, it will give the cold of freezing mercury. Or a thermometer may be graduated by comparing it with another, in this manner. Put them both into water, first of one temperature and then of another, and mark the ungraduated one in these two cases, according to the graduated one; then this interval may be subdivided, and the graduation continued both ways.

(161.) Hence, a thermometer may be graduated for any other scale. In Sir I. Newton's scale, freezing water is 0, and boiling water 34; and the other points may be found by proportion from the other scale. For instance, to find blood heat on this scale, we may observe, that in Fahrenheit's thermometer, from freezing to boiling water is 180, and to blood heat 66; and in this scale, from freezing to boiling water is 34; hence, 180 : 66 :: 34 : $12\frac{7}{15}$ the point of blood heat on this scale.

(162.) The pressure of the atmosphere against the outside of the bulb, not being counteracted by any air within, affects it's magnitude, diminishing it as the pressure is increased. The variation, however, which this causes on the scale is never above one-tenth of a degree. Thermometers are generally made with spirits of wine or mercury, because linseed oil is found to adhere to the sides of the tube, which prevents it from showing suddenly any change of temperature. It is better to make the bulb flat than globular, because all the fluid will then be sooner affected by a variation of temperature.

(163.) For the increase of 1 degree of heat, according to Fahrenheit's Thermometer,

Air expands about $\frac{1}{435}$ part of it's bulk.

Water expands about $\frac{1}{6666}$ part of it's bulk.

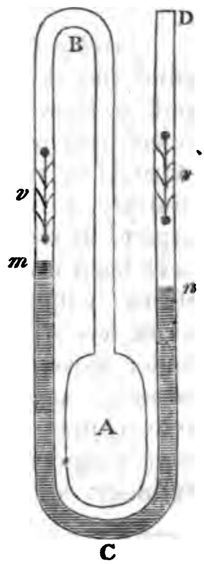
Mercury expands about $\frac{1}{9600}$ part of it's bulk.

Air rarefied by an heat that makes a retort red hot, increases in bulk 3 times; by the heat of boiling water, about $\frac{2}{3}$; by the heat of the human body, about $\frac{1}{4}$.

## Prop. LXXXII.

*To describe* Six's *Thermometer*.

*A* is a glass bulb, from which goes a tube *BCD*; the bulb and tube is filled with spirits of wine as far as *m*; from *m* to *n* the tube is filled with mercury; and the end *D* is either open to the air, or a small quantity is left in and *D* is closed up. Above the mercury on each side are two indexes, *v*, *v*, which are thus formed. A small cylindrical iron wire, with a small globule at each end, nearly the size of the tube, has fixed to it fine glass threads pressing against the sides of the tube, and being very elastic, they support the indexes; and by means of a magnet held against them on the outside, they will move with the magnet; thus the indexes may be both brought down with their lower ends just to touch the mercury; but this must be done very gently, that the globule may not be immersed in the mercury. The thermometer be-




ing thus constructed, it's use is to shew the greatest degree of heat or cold, during the absence of the observer. For this purpose, bring the two indexes down to the mercury; then if the heat of the air increase, the spirits of wine will expand, force down the mercury on the side *BC*, and of course elevate it on the

side *CD* carrying the index with it; but when the heat of the air begins to decrease, the spirits will contract, and the air in *Dn* will, by it's elastic force, push down the mercury on the side *DC*, thus making it rise on the other side and follow the spirits, and the index on the side *DC* will remain suspended and shew the greatest degree of heat. But on the increase of cold, the mercury on the side *BC* will rise, and continue so to do, as long as the cold increases; and upon the decrease of cold, the mercury on that side will begin to fall, and leave the index suspended, shewing the greatest degree of cold.

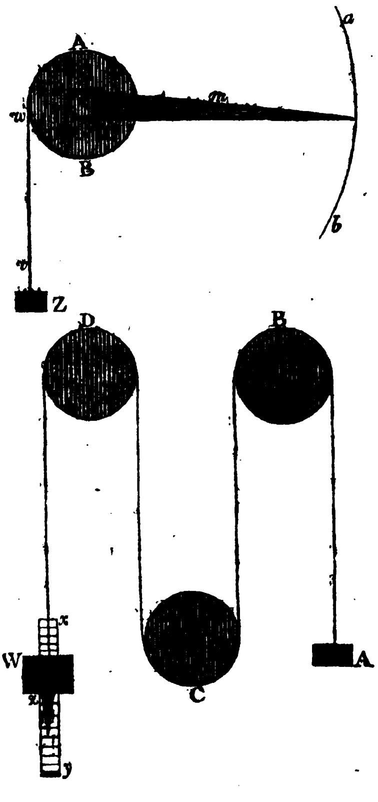
## Prop. LXXXIII.

### *To construct an Hygrometer.*

(164.) An Hygrometer is an instrument to determine the degrees of moisture and dryness of the air, and is formed by those substances which will expand or contract upon any alteration of the moisture. Wood expands by moisture, and contracts by dryness; on the contrary, cord, catgut, &c. contract by moisture*, and expand by dryness. Various mechanical contrivances have been invented to render sensible the smallest variations in the lengths of those substances. We will describe two of them. Let *AB* be the section of a cylinder moveable about it's axis, which is parallel to the horizon; at the end of which there is an index moveable against a graduated arc *ab*; about this cylinder some catgut is wound, one end of which is fixed to the cylinder, and the other end to something immoveable

* As an instance of this, the following story is told. An architect at *Rome* having fixed a very high pedestal, wanted to raise up it's obelisk to be set upon it; but when he had fixed his machines, hung on it's weight, and drawn it up as far as he could, his ropes stretched so much, that the obelisk could not be drawn up to it's necessary height; on which he was desired to wet the ropes; which being done the ropes contracted, and raised the obelisk to it's true place.

at *Z*. As the moisture of the air increases, the catgut contracts and turns the cylinder, and the motion of the index shows the increase of the moisture; and as the air decreases in moisture, the catgut will lengthen, and the weight of the index will carry the cylinder back, and the index will show the corresponding decrease of moisture. In the second figure, the catgut is




fixed at *A*, and goes over the pullies *B*, *C*, *D*, and at the other end a weight *W* is fixed, having an index *z* which moves against a graduated scale *xy*, that shows the increase and decrease of the length of the string, and consequently the state of the air in respect to it's moisture. Various other contrivances, upon the same principle,

have been invented, but it would be foreign to the plan of this Work to enter into a particular description of every instrument which has been constructed for this purpose.

(165.) Mr. De Luc has made a great many experiments, in order to find out such substances as expand most nearly in proportion to the quantity of moisture imbibed. The result was, that whalebone and box, cut across the fibres, increase very nearly in proportion to the quantity of moisture, and more nearly so than any other substances which he tried. This he found, by taking a quantity of shavings of each substance, and weighing them at the time when he measured the increase of the length of a slip of each, cut as above described, the increase of weight being always in proportion to the increase of length. In his construction of an Hygrometer, he preferred the whalebone, first, on account of it's steadiness, in always coming to the same point at extreme moisture; secondly, on account of it's greater expansion, it increasing in length above one-eighth of itself, from extreme dryness to extreme moisture; lastly, it is more easily made thin and narrow.

It is a little extraordinary, that when he took threads of some substances in the direction of the fibres, they first increased as the quantity of moisture increased, and afterwards upon a further increase of moisture, they decreased in length. See the *Phil. Trans.* for 1791.

It must be observed, that of whatever substance an hygrometer is made, in the course of time it will cease to act as it ought, and must be supplied with a fresh substance.

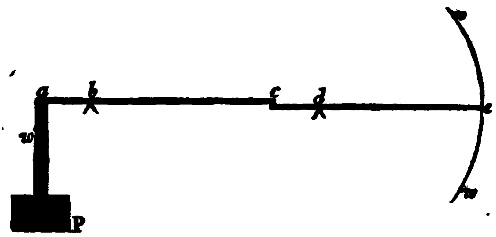
An accurate Hygrometer may be formed by *vitriolic acid*, in the following manner. In a pair of scales having a very sensible beam, put in one scale a plate with vitriolic acid in it, and balance it by weights in the other scale. Then as the acid imbibes the moisture from the air, that scale will preponderate, and

after a given time, find what weight will restore the equilibrium. Repeat this at different times, and the additional weights added will shew how much moisture has been imbibed, which must be in proportion to the moisture of the atmosphere.

## Prop. LXXXIV.

### *To construct a Pyrometer.*

(166.) A *Pyrometer* is an instrument invented to show the expansion and contraction of metals by heat and cold. Various machines have been constructed for this purpose; but as it would not be consistent with the plan of this Work, to enter into a particular description of each, we shall here only explain the general principle. Let *abc* be a lever, whose fulcrum is *b*, acting upon another lever *cde*, whose fulcrum is *d*; and




let *w* be a metallic rod, one end of which rests against an immoveable obstacle *P*, and the other end against the lever *abc* at *a*. If a lamp be put under this rod, the heat will increase it's length, and put the levers in motion; now

$$\begin{array}{l} \text{vel. of } a : \text{vel. of } c :: ab : bc \\ \text{vel. of } c : \text{vel. of } e :: cd : de \\ \hline \therefore \text{vel. of } a : \text{vel. of } e :: ab \times cd : bc \times de. \end{array}$$

Hence, if *bc* and *de* be very great in proportion to *ab* and *cd*, a small increase in the length of *w* will produce a considerable motion in the point *e*, which may be measured upon the graduated arc *vw*.

For example, if $ab : bc :: 1 : 25$, and $cd : de :: 1 : 40$, then $ab \times cd : bc \times de :: 1 \times 1 : 25 \times 40 :: 1 : 1000$; hence, whilst the rod increases the 1000th part of an inch, the end $e$ will describe 1 inch. On this principle the least increase of the length of the rod becomes visible. Instead of putting the lamp immediately under the rod $w$, this rod is laid upon another piece of metal, called the heater, the rod $w$ being laid upon it when the lamp has given the heater it's greatest degree of heat.

(167.) In this manner Mr. Muschenbroek made experiments to determine the proportion of the expansions of different metals, by applying a different number of lamps, and found the results as follows:

| Lamps. | Iron. | Steel. | Copper. | Brass. | Tin. | Lead. |
|---|---|---|---|---|---|---|
| 1 | 80 | 85 | 89 | 110 | 153 | 155 |
| 2 | 117 | 123 | 115 | 220 | * | 274 |
| 3 | 142 | 168 | 193 | 275 | * | * |
| 4 | 211 | 270 | 270 | 361 | * | * |
| 5 | 230 | 310 | 310 | 377 | * | * |

Tin melted with two lamps, and lead with three. With this kind of pyrometer, Mr. Ferguson found the expansion of metals to be in the following proportion; iron and steel 3, copper $4\frac{1}{2}$, brass 5, tin 6, lead 7. An iron rod 3 feet long is about one 70th of an inch longer in summer than in winter.

(168.) If a metal be put into water, and the water be heated, the metal expands as the heat of the water increases. By this method Mr. Smeaton determined the expansion of different metals; for by means of a mercurial thermometer immersed in the water, he could always ascertain the degree of heat. He found that in equal intervals of time, the expansions were in geometric progression. By this, he was enabled to get the measure of the bar before it was applied to the instrument. This will be best understood by explaining an experiment. The time elapsed between applying the bar to the instrument and taking the

first measure, was ½ a minute; therefore the intervals between taking the succeeding measures were ½ a minute also. The first measure was 208; the second 214,5; the third 216,5; the fourth 217,5. The differences of these are 6, 5; 2; 1. Now these three numbers are nearly equal to 6, 3; 2, 25; 0, 8, which form a geometrical progression whose common ratio is 2, 8*. As therefore we may suppose the expansion, from the instant the bar was applied to the time of taking the first measure, followed the same law, we can find the expansion in the first ½ minute (at the end of which the first measure was taken) by continuing back the progression, or multiplying 6, 3 by 2, 8, which gives 17,7 for the expansion the first ½ minute; hence, 208 − 17,7 = 190,3 for the measure before the bar was applied. The following expansions are selected from Mr. SMEATON's table, showing how much a foot, in length, of each increases in decimals of an inch, by an increase of heat corresponding to 180 degrees of FAHRENHEIT's thermometer from freezing to boiling water. See Mr. SMEATON's account in the *Phil. Trans.* 1754.

| | |
|---|---|
| White glass barometer tube - | ,01 |
| Hard Steel - - - - - | ,0147 |
| Iron - - - | ,0151 |
| Copper hammered - - | ,0204 |
| Cast brass - - - - | ,0225 |
| Grain tin - - - - | ,0298 |
| Lead - - - - | ,0344 |
| Zinc - - - - | ,0353 |

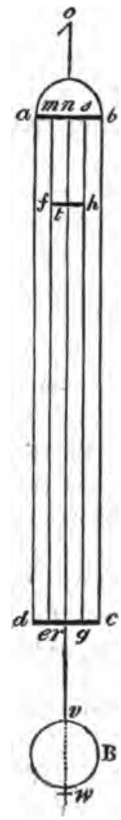
(169.) Metals being thus subject to expansion by heat, a pendulum made with a single rod of metal will continually be subject to a variation in it's length, from the variation of the temperature of the air. To correct

* The conclusions drawn from this supposition are manifestly inaccurate.

this, Mr. HARRISON invented a pendulum, called a gridiron pendulum, composed of rods of steel and rods of brass or zinc, so connected together, that the brass expands upwards when the steel expands downwards; and by a proper adjustment of these rods, the distance from the point of suspension to the centre of oscillation, may be rendered subject but to a very small variation. Mr. GRAHAM invented another method of preserving the length of the pendulum the same in different temperatures. He took a glass, or metallic tube, and put in some mercury; and the heat, which expands the glass or metal downwards, expands the mercury upwards; by the adjustment therefore of a proper quantity of mercury, he could make these effects in altering the length of the pendulum, nearly destroy each other. He found the errors of a clock of this sort to be only about $\frac{1}{8}$ of the errors of the best clock of the common sort.

Mr. HARRISON'S pendulum is thus constructed in the most simple manner, *ad*, *bc*, are two steel rods, fixed to two pieces *ab*, *cd*; and *me*, *sg*, are two zinc rods, fixed at *e*, *g* to the piece *dc*, but at liberty to expand upwards through holes made in the rod *ab*. A cross piece *fh* of steel is attached to the two zinc rods at *f*, *h*, and fixed to a steel rod *ntrv* at *t*, which rod passes freely through a hole in *dc* at *r*, and in *ab* at *n*; and *B* is the bob, which by a screw at *w* on the rod, can be elevated or depressed. Conceive now the temperature of the air to be increased; then the rods *ad*, *bc*, expanding, will let down the piece *dc*: that will let down the rods *em*, *gs*, which will sink the point *t*, and consequently throw the bob *B* further from the point of suspension *o*, or lengthen the pendulum. But as the zinc rods *em*, *gs*, expand upwards, they carry upwards the piece *fh*, which carries upwards the point *t*, and therefore brings the bob *B* nearer the point of suspension *o*, or shortens the pendulum. When therefore these two effects become equal, the pendulum will remain of the same length, notwith-

standing the change of temperature; and these may be made equal by changing the situation of *fh*, so that the parts *ef*, *gh*, of the zinc rods may expand upwards as much as the steel rods *ad*, *bc*, expand downwards.




For common clocks, a deal rod for a pendulum is better than an iron one; provided the deal be well dried, covered over with melted bees-wax, then painted and varnished or gilt, so effectually as to exclude the moisture of the air from getting at the deal. A deal rod alters it's length in the direction of it's fibres but little from a variation of moisture; excluding therefore the moisture, the variation of length will be very small, and much less than an iron rod would be from the variation of temperature of the air, the effect of that on the deal rod being comparatively very small. This kind of pendulum was first made by the Earl of ILAY.

# SECT. XI.

## ON WINDS, SOUND, VAPOURS, AND THE FORMATION OF SPRINGS.

### PROP. LXXXV.

*To explain the Causes of the various Winds.*

(170.) WIND is a current of air, and it's direction is denominated by that point of the compass *from* which it blows. The principal, if not the only cause of wind, is a partial rarefaction of the air by heat. When the air is heated, it becomes rarer, and therefore ascends; and the surrounding cold air rushing in to supply it's place, forms a current in some one direction. Winds may be divided into *constant*, or those which always blow in the same direction; *periodical*, or those which blow half a year in one direction, and half a year in the contrary direction; these are called *monsoons;* and *variable*, which are subject to no rules. The two former are also called *Trade* winds. We shall here give the principal phænomena of winds, from Dr. HALLEY's History thereof, in the *Philosophical Transactions*.

1st. In the *Atlantic* and *Pacific Ocean*, under the Equator there is a constant East wind.

2dly. To about 28° on each side of the Equator, the wind on the *north* side declines toward the north-east; and the more so, the further you recede from the Equator: and on the *south* side, it declines in like manner towards the south-east. The limits of

these winds are greater in the Atlantic Ocean, on the American, than on the African side, extending in the former case to about 32°, and in the latter, to about 28°. And this is true likewise to the southward of the Equinoctial, for near the *Cape of Good Hope*, the limits of the trade winds are 3° or 4° nearer the line, than on the coast of *Brasil*.

3dly. Towards the *Caribee Islands*, the aforesaid north-east wind becomes more and more easterly, so as sometimes to be east, and sometimes east by south; but mostly northwards of the east, a point or two.

4thly. On the coast of Africa, from the *Canaries* to about 10°. N. latitude, the wind sets in towards the north-west; then it becomes south-west, approaching more to the south as you approach the *Cape*. But away from the coasts, the winds are perpetually between the south and the east; on the African side they are more southerly; on the Brasilian, more easterly, so as to become almost due east. Upon the coast of Guinea, they are subject to frequent calms, and violent sudden gusts, called *Tornados*, from all points of the compass.

5thly. In the *Indian* Ocean, the winds are partly *constant*, and partly *periodical*. Between *Madagascar* and *New Holland*, from 10° to 30° latitude, the wind blows south-east by east. During the months of *May*, *June*, *July*, *August*, *September*, *October*, the aforesaid south-east winds extend to within 2° of the Equator; then for the other six months, the contrary winds set in, and blow from 3° to 10° S. latitude. From 3° S. latitude over the *Arabian* and *Indian* seas, and *Bay of Bengal*, from *Sumatra* to the coast of *Africa*, there is another monsoon, blowing from *October* to *April* on the north-east point, and in the other half year from the opposite direction. Between *Madagascar* and *Africa*, a south-south-west wind blows from *April* to *October*, which, as you go more northerly, becomes more westerly, till it falls in with the west-south-west winds; but the Dr. could not obtain a satisfactory account,

how the winds are in the other half year. To the eastward of *Sumatra* and *Malacca*, on the north side of the Equator along the coast of *Cambodia* and *China*, the monsoons blow and change at the same time as before-mentioned; but their directions are more northerly and southerly. These winds reach to the *Philippine Islands* and to *Japan*. Between the same meridians, on the south side of the Equator, from *Sumatra* to *New Guinea*, the same monsoons are observed. The shifting of these winds is attended with great hurricanes.

(171.) The east wind about the Equator is thus explained. The sun moving from east to west, the point of greatest rarefaction of the air, by the heat of the sun, must move in the same direction; and the point of greatest rarefaction following the sun, the air must continually rush in from the east and make a constant east wind.

(172.) The constant north-east wind on the north side of the Equator, and south-east wind on the south side, may be thus accounted for. The air towards the poles being denser than that at the Equator, will continually rush towards the Equator; but as the velocity of the different parts of the earth's surface, from it's rotation, increases as you approach the Equator, the air which is rushing from the north towards the Equator will not continue upon the same meridian, but it will be left behind; that is, in respect to the earth's surface, it will have a motion from the east, and these two motions combined, produce a north-east wind on the north side of the Equator. And in like manner, there must be a south-east wind on the south side. The air which is thus continually moving from the Poles to the Equator, being rarefied when it comes there, ascends to the top of the atmosphere, and then returns back to the Poles. This solution is given by Mr. Hadley in the *Phil. Trans.* vol. 39. and is capable of the following experimental proof. On the center of a circular board which is attached to the

whirling table, fix the center of an upright cylindrical vessel; draw a line from the center of the board to the circumference, and let this represent a meridian, the center of the board being the pole; over this line make a small hole in the vessel, and fill it with water; then the water will spout in the direction of the meridian, and fall upon it. But upon turning the board about its center, the fluid will deviate from the meridian, and fall behind it; thus the direction of the fluid will make an angle with the meridian. Considering then the board to represent the northern hemisphere of the Earth, and as turning from the west to the east, the fluid in respect to the meridian will move from a north-easterly direction; and this is exactly similar to the case of the air beginning its motion from the northern parts and moving towards the equator. The reason the water deviates from the meridian, is this, that the linear circular motion of the point from which the water flows is less than the linear circular motion of the meridian more remote from the center, and therefore the water in respect to the meridian must be left behind. For this very ingenious experiment, we are indebted to the Rev. Mr. ABBOT, formerly Fellow and Tutor of St. John's College, Cambridge.

(173.) The *periodical* winds, are supposed to be owing to the course of the sun northward and southward of the Equator. Dr. HALLEY explains them thus, "Seeing that so great continents do interpose and break the continuity of the ocean, regard must be had to the nature of the soil and the position of the high mountains, which I suppose the two principal causes of the several variations of the winds, from the former general rule: for if a country lying near the sun prove to be flat, sandy, low land, such as the *Deserts of Libya* are usually reported to be, the heat occasioned by the reflection of the sun's beams, and the retention thereof in the sand, is incredible to those that have not felt it; whereby the air being exceedingly rarefied, it is necessary that the cooler and more dense

air should run thitherward to restore the equilibrium. This I take to be the cause, why near the coast of *Guinea* the wind always sets in upon the land, blowing westerly instead of easterly, there being sufficient reason to believe, that the inland parts of *Africa* are prodigiously hot, since the northern borders thereof were so intemperate, as to give the ancients cause to conclude, that all beyond the *Tropic* was made uninhabitable by excess of heat. From the same cause it happens, that there are so constant calms in that part of the ocean, called the *Rains*. For this tract being placed in the middle, between the westerly winds blowing on the coast of *Guinea*, and the easterly trade winds blowing to the westwards thereof, the tendency of the air here is indifferent to either, and so stands in equilibrio between both; and the weight of the incumbent atmosphere being diminished by the continual contrary winds blowing from hence, is the reason that the air here holds not the copious vapour it receives, but lets it fall in so frequent rains.

"As the cool and dense air, by reason of it's greater gravity, presses upon the hot and rarefied, 'tis demonstrative that this latter must ascend in a continual stream as fast as it is rarefied, and that being ascended, it must disperse itself to preserve the equilibrium, that is, by a contrary current, the upper air must move from those parts where the greatest heat is: So by a kind of circulation, the N.E. trade wind below, will be attended with a S.W. above, and the S.E. with a N.W. wind above. And that this is more than a bare conjecture, the almost instantaneous change of the wind to the opposite point, which is frequently found in passing the limits of the trade winds, seems to assure us; but that which above all confirms this hypothesis, is, the phenomenon of the monsoons, by this means most easily solved, and without it hardly explicable. Supposing therefore such a circulation as above, 'tis to be considered, that to the northward of the *Indian Ocean* there is every where land within the usual limits

of the latitude of $30^{o}$, viz. *Arabia, Persia, India,* &c. which for the same reason as the Mediterranean parts of *Africa*, are subject to insufferable heats when the sun is to the north, passing nearly vertical, but yet are temperate enough when the sun is removed towards the other tropic; because of a ridge of mountains at some distance within the land, said to be frequently in winter covered with snow, over which the air, as it passes, must needs be much chilled. Hence it comes to pass, that the air coming, according to the general rule, out of the N. E. in the *Indian* seas, is sometimes hotter, sometimes colder than that which by this circulation is returned out of S.W. and by consequence, sometimes the under current or wind is from N.E, sometimes from the S.W. as is clear from the times wherein these winds set in, *viz.* in *April*, when the sun begins to warm those countries to the north, the S.W. monsoon begins, and blows during the heats till *October*, when the sun being retired, and all things growing cooler northward, and the heat increasing to the south, the N.E. winds enter and blow all the winter till *April* again.

"And it is undoubtedly from the same principle, that to the southward of the Equator, in part of the *Indian Ocean*, the N.W. wind succeeds the S.E. when the sun draws near the Tropic of *Capricorn*. But I must confess that in this latter occurs a difficulty not well to be accounted for, which is, why this change of the monsoons should be any more in this ocean, than in the same latitudes in the *Ethiopic*, where there is nothing more certain than a S. E. wind all the year.

" 'Tis likewise very hard to conceive, why the limits of the trade wind should be fixt about the $30^{th}$ deg. of latitude all round the globe; and that they should so seldom transgress or fall short of those bounds; as also that in the *Indian* sea, only the northern part should be subject to the changeable monsoons, and in the southern there be a constant S. E."

(174.) We may further add, that the causes men-

tioned in the last article, must here also operate. There may perhaps be some cases of these periodical winds, which we cannot see altogether a correct solution of; but if all the circumstances of situation, heat, cold, &c. were known, there is no reason to doubt but that they might be accounted for from the principles here delivered.

(175.) We may further observe in respect to the direction in which winds blow, that if a current set off in any one direction, north-east for instance, and move in a great circle, it will not continue to move on that point of the compass, because a great circle will not cut all the meridians at the same angle, the meridians not being parallel. This circumstance must therefore enter into our consideration in estimating the direction of the wind. High mountains are also observed to turn the winds into a particular course. On the lake of *Geneva*, there are only two winds, that is, either up or down the valley. And the like is known to happen at other such places.

(176.) The *constant* and *periodical* winds blow only at sea; at land, the wind is always *variable*.

(177.) Besides the winds already mentioned, there are others called *Land* and *Sea Breezes*. The air over the land being hotter during the day, than the air over the sea, a current of air will set in from the sea to the land by day; but the air over the land being colder than that over the sea at night, the current at that time will be from the land to the sea. This is very remarkable in islands situated between the tropics.

(178.) Mr. Clare exemplifies this by the following experiment. In the middle of a vessel of water, place a water-plate of warm water, the water in the vessel representing the ocean, and the plate, the island rarefying the air over it. Then hold a lighted candle over the cold water, and blow it out, and the smoke will move towards the plate. But if the plate be cold, and the ambient fluid be warm, the smoke will move in the contrary direction.

(179.) Dr. DERHAM, from repeated observations upon the motion of light downy feathers, found that the greatest velocity of wind was not above 60 miles in an hour. But Mr. BRICE justly observes, that such experiments must be subject to great inaccuracy, as the feathers cannot proceed in a straight line; he therefore estimates the velocity by means of the shadow of a cloud over the earth; by which he found, that in a great storm, the wind moves 63 miles in an hour; when it blows a fresh gale, at the rate of 21 miles an hour; and in a small breeze, at the rate of about 10 miles in an hour. But this method takes for granted, that the clouds move as fast as the wind. It is probable that the velocity is something more than what is here stated.

The following account of the velocity of Wind is given by Mr. SMEATON, in the *Phil. Trans.* 1760.

| Miles in an Hour. | Feet in 1″. | |
|---|---|---|
| 1 | 1,47 | |
| 2 | 2,93 | Light Airs. |
| 3 | 4,40 | |
| 4 | 5,87 | Breeze. |
| 5 | 7,33 | |
| 10 | 14,67 | Brisk Gale. |
| 15 | 22, | |
| 20 | 29,34 | Fresh Gale. |
| 25 | 36,67 | |
| 30 | 44,01 | Strong gale. |
| 35 | 51,34 | |
| 40 | 58,68 | Hard Gale. |
| 45 | 66,01 | |
| 50 | 73,35 | Storm. |
| 60 | 88,03 | |
| 80 | 117,36 | Hurricane, overturning trees, houses, &c. |
| 100 | 146,7 | |

## Prop. LXXXVI.

*To explain the Nature of Sound.*

(180.) *Sound* is a sensation excited by the vibrations of the air upon the tympanum or drum of the ear. That the air is the instrument by which sound is conveyed from the sonorous body, is manifest from hence, that no sound can be produced if the body be in a vacuum, or if there be a vacuum between the body and the ear.

(181.) By percussion, the parts of a sonorous body, as a bell, a musical string, &c. are put into a state of vibration, and as long as the vibrations are continued, corresponding vibrations are communicated to the air; and sound is heard, as long as the vibrations are strong enough to produce the sensation. All sonorous bodies are therefore elastic. The manner in which the vibrations are excited in the air is so clearly described by Mr. Cotes, that I cannot do better than give the account in his own words. "The parts of the sonorous body, being put into a tremulous and vibrating motion, are by turns moved forwards and backwards. Now as they go forwards, they must of necessity press upon the parts of the air to which they are contiguous, and force them also to move forwards in the same direction with themselves; and consequently those contiguous parts will at that time be condensed; then as the parts of the sonorous body return back again, the parts of the air which were just before condensed, will be permitted to return with them, and by returning they will again expand themselves. It is manifest therefore, that the contiguous parts of the air will go forwards and backwards by turns, and be subject to the like vibrating motion with the part of the sonorous body.

"And as the sonorous body produces a vibrating motion in the contiguous parts of the air, so will

these parts thus agitated, in like manner produce a vibrating motion in the next parts, and those in the next, and so on continually. And as the first parts were condensed in their progress, and relaxed in their regress, so will the other parts, as often as they go forwards, be condensed, and as often as they go backwards, be relaxed. And therefore they will not all go forwards together, and all go backwards together; for then their respective distances would always be the same, and consequently they could not be rarefied and condensed by turns; but meeting each other when they are condensed, and going from each other when they are rarefied, they must necessarily one part of them go forwards whilst the other goes backwards, by alternate changes from the first to the last.

"Now the parts which go forwards, and by going forwards are condensed, constitute those pulses which strike upon our organs of hearing, and other obstacles they meet with; and therefore a succession of pulses will be propagated from the sonorous body. And because the vibrations of the sonorous body follow each other at equal intervals of time, the pulses which are excited by those several vibrations, will also succeed each other at the same equal intervals."

(182.) As, when a fluid is put in motion, that motion is communicated in all directions. Sound must be propagated in all directions from a sonorous body as a centre, in concentric superficies, or shells of air, called *Aerial Pulses*, or *Waves of Air*, analogous, as supposed by some, to the circular waves produced on the surface of water when a stone is thrown in. If the sound be impeded by a body which has a hole, the waves pass through, and diverge from it as a new centre, and the sound is heard on all parts on the other side of the body.

(183.) The law by which the force of sound decreases as you recede from the sonorous body, is not easy to be determined by theory. It has been usually

estimated, by dividing the surrounding air into shells of an equal thickness, and supposing these shells to act upon each other, as so many elastic bodies would; but it is probable, that this is a supposition very far distant from the truth. The utmost distance at which a sound has been heard, is about 200 miles. This was observed in the war between England and Holland, in the year 1672. The unassisted human voice has been heard from Old to New Gibraltar, a distance of 10 or 12 miles; the watch-word, *All's well,* given at the latter, in a still night, having been heard at the former. In both these cases, the sound passed over the water; and it is found, that sound will always be conveyed much further along a smooth, than a rough surface.

(184.) The velocity of sound, produced by all bodies, is found by experiment to be 1142 feet in a second, subject to a small variation from the course of the wind. Dr. Derham determined this very accurately, by placing cannon at different distances, and firing them, and observing the interval between the flash and the report. And thus he also found that sound (or rather the pulses of air which excite it) moves uniformly; it being always found, that the interval was in proportion to the distance. Sir I. Newton determined the velocity of sound by theory; with which, if the Reader wish to be acquainted, he may consult the *Principia*, Lib. 2. Prop. 47.

(185.) Having had an opportunity, when resident at *Ramsgate,* to see the flash of the evening-gun at the *Downs,* and to hear the report, I made a great many observations on the interval between the flash and the report, in order to discover how much the velocity of sound varied in different directions of the wind. When there was no sensible wind, the interval was 32″ of time. When a very strong wind, amounting to a hurricane, blew directly from the Downs to Ramsgate, the interval was 30″; and when it blew in the opposite

direction, it was 34″. It appears, therefore, that a very strong wind will increase or diminish the velocity of sound by a $\frac{1}{16}$ part of it's mean velocity, according as the direction of the wind coincides with or opposes that of the sound. When the wind blew at right angles to the direction of the sound, however strong the wind might be, it did not appear to produce any sensible effect upon the velocity of sound.

(186.) Sound is conveyed to the greatest distance by a trumpet, called a *Speaking* or *Stentorophonic* trumpet, the form of which is like that figure which would be generated by any part of the logarithmic curve revolving about it's axis, the mouth being applied to the smaller end. The theory, by which this is attempted to be proved, is subject to the objection mentioned in Art. 182.

(187.) The *same* sound is always excited, when the air is put into the *same* state of vibration; that is, if a bell and musical string make the air vibrate the *same number* of times in a second, they excite the *same* tone. And as the same sonorous body performs all it's vibrations, whether greater or less, in the same time, the same body will always give the same tone, whether the percussive stroke be greater or less. The slower the vibration, the deeper or graver is the tone. But we mean not here to enter into the investigation of the times of vibration of musical strings; a subject of considerable difficulty, and therefore not proper for an elementary treatise. If the Reader wish for any information upon the subject, we refer him to Mr. Parkinson's *Hydrostatics*; or Dr. Smith's *Harmonics*.

Sounds of different tones move with the same velocity. For at all distances, a peal of bells are heard in the same order in which they are rung. Whereas, if the velocities of the different tones were different, there would be nothing but confusion.

(188.) The reflection of the vibrations of the air from any fixed object to the ear, will cause a sound

distinct from that which is caused by the vibrations coming directly to the ear; and this is called an *Echo.* If the distance of the object which returns the echo be great, it will return several syllables. A single syllable will not be clearly returned, unless the distance of the object be at least 120 feet; and so in proportion. Hence, an echo returning ten syllables must come from an object 1200 feet distant. More syllables however will be returned by night than by day, because the air being then colder, is denser, in which case, the return of the vibrations become slower, and consequently more syllables may be heard.

## Prop. LXXXVI.

*To explain the ascent of Vapours, and the origin of Springs.*

(189.) *Vapours* are raised from the surface of the water; the principal cause of which is, probably, the heat of the sun, the evaporation being always greatest when the heat is the greatest. The difficulty of solving the phænomenon arises from hence, that we find a heavier fluid (water) suspended in a lighter fluid (air), contrary to our foregoing principles.

(190.) Dr. Halley supposed, that by the action of the sun upon the surface of the water, the aqueous particles become formed into hollow bubbles filled with warm, and rarefied air, so as to make the whole bulk specifically lighter than the air, in which case the particles will (Art. 46.) ascend. But there is a great difficulty in conceiving how this can be effected. And if bubbles could be *at first* thus formed, when they ascend, the air within would very soon be reduced to the same temperature of the air without, and they would immediately descend upon that effect taking place. Another opinion is, that the particles of water are separated by a repulsive force, which is increased in proportion as the heat is increased, and thus they are dispersed through the air; but the same

argument may be used against this hypothesis, as against the last, that is, that this effect could not continue in the cold part of the atmosphere, where the clouds are suspended. The most probable supposition is, that evaporation is a chemical solution of water in air. We know that metals are dissolved in menstruums, and their particles diffused and suspended in the fluid, although their specific gravity be greater than that of the fluid. Heat promotes this solution; in the day-time therefore, the heat causes a more perfect solution than what can, *cæteris paribus*, take place in the night, when the air is colder, when the heat is frequently not sufficient to keep the water in a state of solution, and it falls in fogs and dews. The vapours, thus raised by heat, ascend into the cold regions of the atmosphere, and not being there kept in a state of solution, they appear in the form of clouds; and when driven together by the agitation of the air, the particles run together into drops, and fall down in rain. If they be frozen before they form themselves into drops, they descend in snow; but if the drops of rain themselves be frozen, they descend in hail. See HAMILTON on the *Ascent of Vapours*.

(191.) MARRIOTTE supposed *Springs* to be owing to rain-water and melted snow, which penetrating the surfaces of hills, and running by the side of clay or rocks which it cannot penetrate, at last comes to some place where it breaks out. This would account for the phænomenon, provided the supply from these causes was sufficient; but D. SIDELEAU, and others, making an estimate of the quantity of rain and snow which falls in the space of a year, to see whether it would afford a quantity of water, equal to that which is annually discharged into the sea by the rivers (which are supplied by springs) found that it would *not*. But Dr. HALLEY discovered the cause of a sufficient supply; for he has proved by experiment, that the vapours which are raised, afford a much greater supply

than is necessary. We will give the account in his own words.

(192.) "We took a pan of water (salted to the degree as is common sea-water, by the solution of about a fortieth part of salt) about four inches deep, and 7 inches $\frac{8}{10}$ diameter, in which we placed a thermometer, and by means of a pan of coals, we brought the water to the same degree of heat which is observed to be that of the air in our hottest summers; the thermometer nicely showing it. This done, we affixed the pan of water, with the thermometer in it, to one end of the beam of the scales, and exactly counterpoised it with weights in the other scale; and by the application or removal of the pan of coals, we found it very easy to maintain the water in the same degree of heat precisely. Doing thus, we found the weight of the water sensibly to decrease; and at the end of two hours we observed, that there wanted half an ounce troy, all but 7 grains, or 233 grains of water, which in that time had gone off in vapour; though one could hardly perceive it smoak, and the water was not sensibly warm. This quantity in so short a time seemed very considerable, being little less than 6 ounces in 24 hours, from so small a surface as a circle of 8 inches diameter. To reduce this experiment to an exact *calculus*, and determine the thickness of the skin of water that had so evaporated, I assume the experiment alledged by Dr. EDW. BERNARD to have been made in the *Oxford Society*, viz. that the cube foot *English* of water weighs exactly 76 pounds troy; this divided by 1728, the number of inches in a foot, will give 253$\frac{1}{3}$ grains, or half ounce 13$\frac{1}{3}$ grains for the weight of a cube inch of water; wherefore the weight of 233 grains is $\frac{233}{253}$, or 35 parts of 38 of a cube inch of water. Now the area of the circle, whose diameter is 7$\frac{8}{10}$ inches, is 49 square inches; by which dividing the quantity of water evaporated,

viz. $\frac{35}{38}$ of an inch, the quota $\frac{35}{1862}$ or $\frac{1}{53}$, shews that the thickness of the water evaporated was the 53[d] part of an inch: but we will suppose it only the 60[th] part, for the facility of calculation. If therefore water as warm as the air in summer, exhales the thickness of a 60[th] part of an inch in two hours from it's whole surface, in 12 hours it will exhale $\frac{1}{10}$ of an inch; which quantity will be found abundantly sufficient to serve for all the rains, springs, and dews, and account for the *Caspian Sea* being always at a stand, neither wasting nor overflowing; as likewise for the current said to set always in, at the *Straights* of *Gibraltar*, though those *Mediterranean Seas* receive so many, and so considerable rivers.

(193.) "To estimate the quantity of water arising in vapours out of the sea, I think I ought to consider it only for the time the sun is up, for that the dews return in the night as much if not more vapours than are then emitted; and in summer the days being longer than twelve hours, this excess is balanced by the weaker action of the sun, especially when rising before the water is warmed; so that if I allow $\frac{1}{10}$ of an inch of the surface of the sea to be raised *per diem* in vapours, it may not be an improbable conjecture.

"Upon this supposition, every 10 square inches of the surface of the water yields in vapour *per diem* a cube inch of water; and each square foot, half a wine pint; every space of 4 feet square, a gallon; a mile square, 6914 tons; a square degree, suppose of 69 *English* miles, will evaporate 33 millions of tons; and if the *Mediterranean* be estimated at 40 degrees long and 4 broad, allowances being made for the places where it is broader by those where it is narrower, (and I am sure I guess at the least) there will be 160 square degrees of sea; and consequently the whole *Mediter-*

*ranean* must lose in vapour, in a summer's day, at least 5280 millions of tons. And this quantity of vapour, though very great, is as little as can be concluded from the experiment produced: and yet there remains another cause, which cannot be reduced to the rule, I mean the winds, whereby the surface of the water is licked up, somewhat faster than it exhales by the heat of the sun, as it is well known to those that have considered those drying winds which blow sometimes.

"The *Mediterranean* receives these considerable rivers; the *Iberus*, the *Rhone*, the *Tiber*, the *Po*, the *Danube*, the *Niester*, the *Borysthenes*, the *Tanais*, and the *Nile*, all the rest being of no great note, and their quantity of water inconsiderable. We will suppose each of these nine rivers to bring down ten times as much water as the river *Thames*, not that any of them is so great in reality, but to comprehend with them all the small rivulets that fall into the sea, which otherwise I know not how to allow for.

"To calculate the water of the *Thames*, I assume that at *Kingston Bridge*, where the flood never reaches, and the water always runs down, the breadth of the channel is 100 yards, and it's depth 3, it being reduced to an equality, (in both which suppositions I am sure I take with the most.) Hence the profile of the water in this place is 300 square yards: this multiplied by 48 miles, (which I allow the water to run in 24 hours, at 2 miles in an hour) or 84480 yards, gives 25344000 cubic yards of water to be evacuated every day, that is 20300000 tons *per diem*; and I doubt not but in the excess of any measure of the channel of the river, I have made more than sufficient allowance for the waters of the *Brent*, the *Wandel*, the *Lea*, and *Darwent*, which are all worth notice, that fall into the *Thames* below *Kingston*.

"Now if each of the aforesaid nine rivers yield ten times as much water as the *Thames* doth, 'twill follow that each of them yields but 203 millions of tons *per*

*diem*, and the whole nine but 1827 millions of tons in a day; which is but little more than $\frac{1}{3}$ of what is proved to be raised in vapours out of the *Mediterranean* in twelve hours time."

(194.) Besides the *constant* Springs, there are others which *ebb* and *flow* alternately, which may thus be accounted for. The water, before it breaks out, may meet with a large cavity on the side of the hill, and the water, upon the overflowing of this reservoir, may find an aperture, and make it's escape; in case of dry weather, therefore, the supply of water may not be sufficient to keep it full, in which case, the spring will cease to flow, and continue dry, till a supply causes it to overflow, and produce again the spring.

For the other important articles in this branch of philosophy—The motion of bodies in resisting mediums; the resistances of all kinds of bodies moving in mediums; the investigation of the solid of least resistance; the general law of the variation of density of the atmosphere, upon any law of gravity; and the times in which vessels empty themselves;—the Reader is referred to the *Principles* of *Fluxions*.

THE END.

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# THE ELEMENTS

OF

# ASTRONOMY:

DESIGNED FOR THE USE OF STUDENTS IN THE UNIVERSITY.


BY THE

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FOURTH EDITION.


CAMBRIDGE:

*Printed by J. Smith, Printer to the University;*

AND SOLD BY DEIGHTON & SONS, NICHOLSON & SON, CAMBRIDGE; AND J. MAWMAN, 39, LUDGATE-STREET, LONDON.

1816.

# CONTENTS.

# A SYSTEM OF ASTRONOMY.

## CHAPTER I.

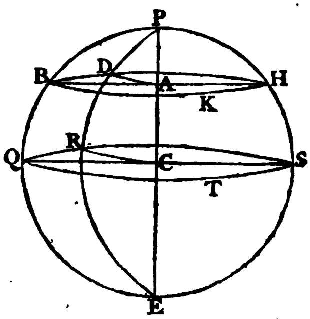
### DEFINITIONS.

Art. 1. Astronomy is that branch of Natural Philosophy which treats of the heavenly bodies. The determination of their magnitudes, distances, and the orbits which they describe, is called *plane* or *pure* Astronomy; and the investigation of the causes of their motions is called *physical* Astronomy. The *former* discoveries are made from observations on their apparent magnitudes and motions; and the *latter* from analogy, by applying those principles and laws of motion by which bodies on and near the earth are governed, to the other bodies in the system. The principles of plane Astronomy only are what we here propose to treat of, and we shall begin with the explanation of such terms as are the foundation of the science.

(2.) A *great* circle $QRST$ of a sphere is one whose plane passes through it's center $C$; and a *small* circle $BDHK$ is that whose plane does not pass through it's center.

(3.) A diameter $PCE$ of a sphere, perpendicular to any great circle $QRST$, is called the *axis* of that great circle; and the extremities $P$, $E$, of the axis, are called it's *Poles*.

(4.) Hence, the pole of a great circle is 90° from every point of it upon the sphere; because every angle $PCR$ being a right angle, the arc $PR$ is every where 90°. And as the axis $PE$ is perpendicular to the circle $QRST$ when it is perpendicular to any two radii $CQ$, $CR$, a point on the surface of the sphere 90° distant from two points of a great circle wherever taken, will be the pole.




(5.) All angular distances on the surface of a sphere, to an eye at the center, are measured by the arcs of *great* circles.

(6.) Hence, all the triangles formed on the surface of a sphere, for the solution of spherical problems, must be formed by the arcs of *great* circles.

(7.) Any two great circles bisect each other; for both passing through the center of the sphere, their common section must be a diameter of each, and every diameter bisects a circle.

(8.) *Secondaries* to a great circle, are great circles which pass through it's poles; thus, $PRE$ is a secondary to $QRST$.

(9.) Hence, secondaries must be perpendicular to their great circles; for if one line be perpendicular to a plane, any plane passing through that line will

also be perpendicular to it; therefore as the axis *PE* of the great circle *QRST* is perpendicular to it, and is the common diameter of all the secondaries, they must all be perpendicular to the great circle. Hence also, every secondary, bisecting it's great circle (*7), must bisect every small circle *BDHK* parallel to it; for the plane of the secondary passes not only through the center *C* of the great circle, but also the center *A* of the small circle parallel to it.

(10.) Hence, a great circle passing through the poles of two great circles, must be perpendicular to each; and vice versâ, a great circle perpendicular to two other great circles must pass through their poles.

(11.) If an eye be in the plane of a circle, that circle appears a straight line; hence, in the representation of the surface of a sphere upon a plane, those circles whose planes pass through the eye, are represented by straight lines.

(12.) The angle formed by the circumferences of two great circles on the surface of a sphere, is equal to the angle formed by the planes of those circles; and is measured by the arc of a great circle intercepted between them, and which is a secondary to each.

For let *C* be the center of the sphere, *PQE*, *PRE* two great circles; then as the circumferences of these circles at *P* are perpendicular to the common intersection *PCE*, the angle at *P* between them is equal to the angle between the planes, by Euc. B. XI. Def. 6. Now draw *CQ*, *CR* perpendicular to *PCE*, then as these lines are respectively parallel to the directions of the circumferences *PQ*, *PR*, at the point *P*, the angle *QCR* is equal to the angle at *P* formed by the two circles, Euc. B. XI. Prop. 10.; and the angle *QCR* is measured by the arc *QR* of a great circle whose pole is *P*, because *PQ*, *PR* are each 90°.

* The figures in the parentheses refer to the Articles.

(13.) If at the intersection $P$ of two great circles as a pole, a great circle $QRST$ be described, and also a small circle $BDHK$ parallel to it, the arcs $QR$, $BD$ of the great and small circles intercepted between the two great circles, contain the same number of degrees.

For $C$ and $A$ are the centers of the respective circles, and $QC$ is parallel to $BA$, and $RC$ is parallel to $DA$; therefore by Euc. B. XI. Prop. 10. the angle $BAD$ is equal to the angle $QCR$, consequently the arcs $BD$, $QR$ contain the same number of degrees.

Hence, the arc $BD$ of such a small circle measures the angle at the pole between the two great circles. Also, $QR : BD :: QC : BA ::$ radius : cos. $BQ$.

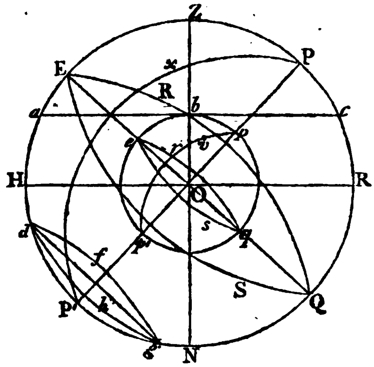
(14.) The *axis* of the earth $pep'q$ is that diameter $pOp'$ about which it performs it's diurnal rotation; and the extremities $p$, $p'$, of this diameter, are called it's *Poles*.

(15.) The *terrestrial Equator* is a great circle $erqs$ of the earth perpendicular to it's axis. Hence, the axis and poles of the earth are the axis and poles of it's equator. That half of the earth (suppose $epq$) which lies on the side of the equator which we inhabit, is called the *northern hemisphere*, and the other $ep'q$ the *southern*; and the poles are respectively called the *north* and *south* poles.

(16.) The *Latitude* of a place on the earth's surface, is it's angular distance from the equator, measured upon a secondary to it; thus, the arc $eb$ measures the latitude of $b$. The secondaries to the equator are called *Meridians*.

(17.) The *Longitude* of a place on the earth's surface, is an arc of the equator intercepted between the meridian passing through the place, and another, called the *first* meridian, passing through that place from which you begin to measure; thus, the longitude of the place $v$ on the meridian $prp'$ measured from the first meridian $pep'$, is $er$.

(18.) If the plane of the *terrestrial* equator *e r q s* be produced to the sphere of the fixed stars, it marks




out a circle *E R Q S* called the *celestial* equator; and if the axis of the earth *pOp′* be produced in like manner, the points *P*, *P′*, in the Heavens to which it is produced, are called *Poles*, being the poles of the celestial equator. The star nearest to each pole is called the *Pole star*.

(19.) Secondaries, as *PxP′*, to the celestial equator are called *Circles of Declination*; of these, 24 which divide the equator into equal parts, each containing 15°, are called *hour* circles.

(20.) Small circles, as *d f g h*, parallel to the celestial equator, are called *Parallels of Declination*.

(21.) The *sensible* horizon is that circle *a b c* in the heavens whose plane touches the earth at the spectator *b*. The *rational* horizon is a great circle *H O R* in the heavens, passing through the earth's center, parallel to the sensible horizon.

(22.) *Almacanter* is a small circle parallel to the horizon.

(23.) If the radius *O b* of the earth at the place *b* where the spectator stands, be produced both ways to the heavens, that point *Z* vertical to him is called the *Zenith*, and the opposite point *N* the *Nadir*. Hence, the *zenith* and *nadir* are the poles of the

*rational* horizon (3); for the radius produced being perpendicular to the sensible, must also be perpendicular to the rational horizon.

(24.) Secondaries to the horizon are called *vertical* circles; and being (9) perpendicular to the horizon, the altitude of an heavenly body is measured upon them.

(25.) A secondary $PEHP'$ common to the celestial equator and the horizon of any place $b$, and which therefore (10) passes through the poles $P$, $Z$, of each, is the *celestial Meridian* of that place. Hence, the plane of the celestial meridian of any place, conicides with the plane of the terrestrial meridian of the same place.

(26.) The meridian $ZPRH$ cuts the horizon in the point $R$, called the *north* point, and in the point $H$, called the *south* point; $P$ being the north pole.

(27.) The meridian of any place divides the heavens into two hemispheres lying to the east and west; that lying to the east is called the *eastern* hemisphere, and the other, lying to the west, is called the *western* hemisphere.

(28.) The vertical circle which cuts the meridian of any place at right angles, is called the *prime* vertical; and the points where it cuts the horizon are called the *east* and *west* points. Hence, the east and west points are 90° distant from the north and south points. These four are called the *cardinal* points.

(29.) If a body be referred to the horizon by a secondary to it, the distance of that point of the horizon from the north or south points, is called it's *Azimuth*. The *Amplitude* is the distance from the east or west point.

(30.) The *Ecliptic* is that great circle in the heavens which the sun appears to describe in the course of a year.

(31.) The ecliptic and equator being great circles must (7) bisect each other, and their inclination is

called the *obliquity of the eliptic*; also, the points where they intersect are called the *equinoctial* points. The times when the sun comes to these points are called the *Equinoxes*.

(32.) The ecliptic is divided into 12 equal parts, called *Signs:* Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricornus ♑, Aquarius ♒, Pisces ♓. The order of these is according to the motion of the sun. The first point of Aries coincides with one of the equinoctial points, and the first point of Libra with the other. The first six signs are called *northern*, lying on the *north* side of the equator; and the last six are called *southern*, lying on the *south* side. The signs ♑, ♒, ♓, ♈, ♉, ♊, are called *ascending*, the sun approaching our (or the north) pole whilst it passes through them; and ♋, ♌, ♍, ♎, ♏, ♐, are called *descending*, the sun receding from our pole as it moves through them.

(33.) When the motion of the heavenly bodies is according to the order of the signs, it is called *direct*, or *in consequentia*; and when the motion is in the contrary direction, it is called *retrograde*, or *in antecedentia*. The *real* motion of all the planets is according to the order of the signs, but their *apparent* motion is sometimes in an opposite direction.

(34.) The *Zodiac* is a space extending on each side of the ecliptic, within which the motions of all planets are performed.

(35.) The *right ascension* of a body is an arc of the equator intercepted between the first point of Aries and a declination circle passing through the body, measured according to the order of the signs.

(36.) The *oblique ascension* is an arc of the equator intercepted between the first point of Aries and that point of the equator which rises with any body, measured according to the order of the signs.

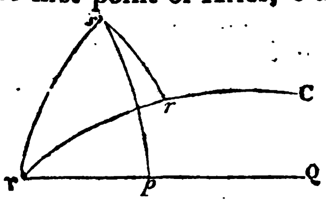
(37.) The *ascensional difference* is the difference between the right and oblique ascension.

(38.) The *Declination* of a body is it's angular distance from the equator, measured upon a secondary to the equator drawn through the body.

(39.) The *Longitude* of a star is an arc of the ecliptic intercepted between the first point of Aries and a secondary to the ecliptic passing through the star, measured according to the order of the signs. If the body be in our system, and seen from the *sun*, it is called the *heliocentric* longitude; but if seen from the *earth*, it is called the *geocentric* longitude; the body in each case being referred perpendicularly to the ecliptic in a plane passing through the eye.

(40.) The *Latitude* of a star is it's angular distance from the ecliptic, measured upon a secondary to the ecliptic drawn through the star. If the body be in our system, it's angular distance from the ecliptic seen from the *earth* is called the *geocentric* latitude; but if seen from the *sun*, it is called the *heliocentric* latitude.

(41.) Thus, if ♈ *Q* be the equator, ♈ *C* the ecliptic, ♈ the first point of Aries, *s* a star, and the




great circles *s r*, *sp* be drawn perpendicularly to ♈ *C* and ♈ *Q*; then ♈ *p* is it's right ascension, *sp* it's declination, *s r* it's latitude, and ♈ *r* its longitude. The circle *s r* is called a *Circle of Latitude*.

Hence, if we know the right ascension ♈ *p*, and declination *ps* of a body *s*, we know it's place; for, take ♈ *p* = to the given right ascension, draw the meridian *p s*, and take *ps* = to the given declination, and *s* is the place of the body. In like manner, if we know the longitude ♈ *r* of a body *s*, and latitude *r s*, we know the place of the body.

(42.) The *Tropics* are two parallels of declination

touching the ecliptic. One, touching it at the beginning of Cancer, is called the *Tropic of Cancer*; and the other, touching it at the beginning of Capricorn, is called the *Tropic of Capricorn*. The two points where the tropics touch the ecliptic, are called the *Solstitial* points.

(43.) The *Colures* are two secondaries to the celestial equator; one, passing through the equinoctial points, is called the *equinoctial* colure; and the other, passing through the solstitial points, is called the *solstitial* colure. The times when the sun comes to the solstitial points are called the *Solstices*.

(44.) The *Arctic* and *Antarctic* circles are two parallels of declination, the former about the north and the latter about the south pole, the distances of which from the two poles are equal to the distances of the tropics from the equator. These are also called *polar* circles.

(45.) The two tropics, and two polar circles, when referred to the earth, divide it into five parts, called *Zones*; the two parts within the polar circles are called the *frigid* zones; the two parts between the polar circles and the tropics are called the *temperate* zones; and the part between the tropics, is called the *torrid* zone. Small circles in the heavens are referred to the earth, and the contrary, by lines drawn to the earth's center.

(46.) A body is in *Conjunction* with the sun, when it has the same longitude; in *Opposition*, when the difference of their longitudes is 180°; and in *Quadrature*, when the difference of their longitudes is 90°. The conjunction is marked thus ☌, the opposition thus ☍, and quadrature thus □.

(47.) *Syzygy* is either conjunction or opposition.

(48.) The *Elongation* of a body from the sun, is it's angular distance from the sun when seen from the earth.

(49.) The *diurnal parallax* is the difference between the apparent places of a body in our system

when referred to the fixed stars, if seen from the center and surface of the earth. The *annual* parallax is the difference between the apparent places of a body in the heavens, when seen from the opposite points of the earth's orbit.

(50.) The *Argument* is a term used to denote any quantity by which another required quantity may be found. For example, the argument of a planet's latitude is it's distance from it's node, because upon that the latitude depends.

(51.) The *Nodes* are the points where the orbits of the primary planets cut the ecliptic, and where the orbits of the secondaries cut the orbits of their primaries. That node is called *ascending* where the planet passes from the south to the north side of the ecliptic; and the other is called the *descending* node. The ascending node is marked thus ☊, and the descending node thus ☋. The line which joins the nodes is called the *line of the nodes*.

(52.) If a perpendicular be drawn from a planet to the ecliptic, the angle at the sun between two lines, one drawn from it to that point where the perpendicular falls, and the other to the earth, is called the angle of *Commutation*.

(53.) The angle of *Position* is the angle at an heavenly body formed by two great circles, one passing through the pole of the equator, and the other through the pole of the ecliptic.

(54.) *Apparent* noon is the time when the sun comes to the meridian.

(55.) *True* or *mean* noon is 12 o'clock, by a clock adjusted to go 24 hours in a mean solar day.

(56.) The *Equation of time* at noon, is the interval between *true* and *apparent* noon.

(57.) A star is said to rise or set *Cosmically*, when it rises or sets at sun-rising; and when it rises or sets at sun-setting, it is said to rise or set *Achronically*.

(58.) A star rises *Heliacally*, when, after having been so near to the sun as not to be visible, it emerges

out of the sun's rays, and just appears in the morning; and it sets *Heliacally*, when the sun approaches so near to it, that it is about to immerge into the sun's rays and to become invisible in the evening.

(59.) The *curtate* distance of a planet from the sun or earth, is the distance of the sun or earth from that point of the ecliptic where a perpendicular to it passes through the planet.

(60.) *Aphelion* is that point in the orbit of a planet which is furthest from the sun.

(61.) *Perihelion* is that point in the orbit of a planet which is nearest the sun.

(62.) *Apogee* is that point of the earth's orbit which is furthest from the sun, or that point of the moon's orbit which is furthest from the earth.

(63.) *Perigee* is that point of the earth's orbit which is nearest the sun, or that point of the moon's orbit which is nearest the earth.

The terms aphelion and perihelion are also applied to the earth's orbit.

(64.) *Apsis* of an orbit, is either it's aphelion or perihelion, apogee or perigee; and the line which joins the apsides is called *the line of the apsides.*

(65.) *Anomaly* (*true*) of a planet, is it's angular distance at any time from it's aphelion, or apogee—(*mean*) is the angular distance from the same point at the same time, if it had moved uniformly with it's mean angular velocity.

(66.) *Equation of the center* is the difference between the *true* and *mean* anomaly; this is sometimes called the *prosthapheresis.*

(67.) *Nonagesimal degree* of the ecliptic, is that point which is highest above the horizon.

(68.) The *mean* place of a body, is the place where a body (not moving with an uniformly angular velocity about the central body) *would* have been, if it had moved with it's mean angular velocity. The *true* place of a body, is the place where the body actually is at any time.

(69.) *Equations* are corrections which are applied to the *mean* place of a body, in order to get it's *true* place.

(70.) A *Digit* is a twelfth part of the diameter of the sun or moon.

(71.) Those bodies which revolve about the sun in orbits nearly circular, are called *Planets*, or *primary* planets for the sake of distinction; and those bodies which revolve about the *primary* planets are called *secondary* planets, or *Satellites*.

(72.) Those bodies which revolve about the sun in very elliptic orbits are called *Comets*. The sun, planets, and comets, comprehend all the bodies in what is called the *Solar System*.

(73.) All the other heavenly bodies are called *Fixed Stars*, or simply *Stars*.

(74.) *Constellation* is a collection of stars contained within some assumed figure, as a ram, a dragon, an hercules, *&c.* the whole heaven is thus divided into constellations. A division of this kind is necessary, in order to direct a person to any part of the heavens which we want to point out.

*Characters used for the Sun, Moon and Planets.*

| | | | |
|---|---|---|---|
| ☉ | The Sun. | ⚴ | Pallas. |
| ☽ | The Moon. | ⚵ | Juno. |
| ☿ | Mercury. | | Vesta. |
| ♀ | Venus. | ♃ | Jupiter. |
| ⊖ | The Earth. | ♄ | Saturn. |
| ♂ | Mars. | ♅ | Georgian. |
| ⚳ | Ceres. | | |

*Characters used for the Days of the Week.*

| | | | |
|---|---|---|---|
| ☉ | Sunday. | ♃ | Thursday. |
| ☽ | Monday. | ♀ | Friday. |
| ♂ | Tuesday. | ♄ | Saturday. |
| ☿ | Wednesday. | | |

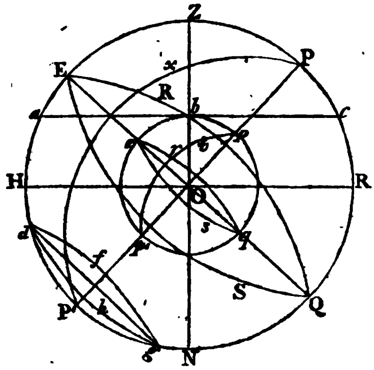
# CHAP. II.

## ON THE DOCTRINE OF THE SPHERE.

(75.) A SPECTATOR upon the earth's surface conceives himself to be placed in the center of a concave sphere in which all the heavenly bodies are situated; and by constantly observing them, he perceives that by far the greater number never change their relative situations, each rising and setting at the same interval of time, and at the same points of the horizon, and are therefore called *fixed* stars; but that a few others, called *planets*, together with the *sun* and *moon*, are constantly changing their situations, each continually rising and setting at different points of the horizon, and at different intervals of time. Now the determination of the times of the rising and setting of all the heavenly bodies; the finding of their position at any given time in respect to the horizon or meridian, or the time from their position; the causes of the different lengths of days and nights, and the changes of seasons; the principles of dialling, and the like, constitute the *doctrine of the sphere.* And as the apparent diurnal motion of all the bodies has no reference to any particular system, or disposition of the planets, but may be solved, either by supposing them actually to perform those motions every day, or by supposing the earth to revolve about an axis, we will suppose this latter to be the case, the truth of which will afterwards appear.

(76.) Let $pep'q$ represent the earth, $O$ it's center, $b$ the place of a spectator, $HZRN$ the sphere of the fixed stars; and although the fixed stars do not lie in the concave surface of a sphere of which the center

of the earth is the center, yet, on account of the immense distance even of the nearest of them, their



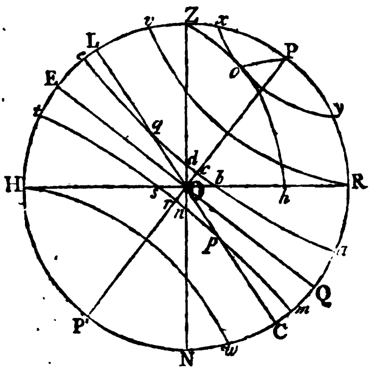

relative situations from the motion of the earth, and consequently the place of a body in our system referred to them, will not be affected by this supposition. The plane *a b c* touching the earth in the place of the spectator, is called (21) the *sensible* horizon, as it divides the visible from the invisible part of the heavens; and a plane *H O R* parallel to *a b c*, passing through the centre of the earth, is called the *rational* horizon; but in respect to the sphere of the fixed stars, these may be considered as coinciding, the angle which the arc *Ha* subtends at the earth becoming then insensible, from the immense distance of the fixed stars. Now if we suppose the earth to revolve daily about an axis, all the heavenly bodies must successively rise and set in that time, and appear to describe circles whose planes are perpendicular to the earth's axis, and therefore parallel to each other, because each body continues at the same distance from the equator, during the revolution of the earth about it's axis. Thus, all the stars will appear to revolve daily about the earth's axis, as if they were placed in the concave surface of a sphere having the earth in the center. Let therefore *pp'* be that diameter of the earth about which it must

revolve in order to give the apparent diurnal motion to the heavenly bodies, then $p$, $p'$, are called it's poles; and if $pp'$ be produced both ways to $P$, $P'$, in the heavens, these points are called (18) the poles of the heavens, and the star nearest to each of these is called the pole star. Now, although the earth, from it's motion in it's orbit, continually changes it's place, yet as the axis always continues parallel to itself, the points $P$, $P'$, will not, from the immense distance of the fixed stars, be sensibly altered; we may therefore suppose these to be fixed points*. Produce $Ob$ both ways to $Z$ and $N$, and $Z$ is the zenith, and $N$ the nadir (23). Draw the great circle $PZHNR$, and it will be the celestial meridian (25), the plane of which coincides with the terrestrial meridian $pbp'$ passing through the place $b$ of the spectator. Let $erqs$ represent a great circle of the earth perpendicular to it's axis $pp'$, and it will be the equator (15); and if the plane of this circle be extended to the heavens, it marks out a great circle $ERQS$ called the celestial equator (18). Hence, for the same reason that we may consider the points $P$, $P'$, as fixed, we may consider the circle $ERQS$ as fixed. Now as the latitude of any place $b$ on the earth's surface is measured by the degrees of the arc $be$ (16), it may be measured by the degrees of the arc $ZE$; hence, as the equator, zenith, and poles in the heaven, correspond to the equator, place of the spectator, and poles of the earth, we may leave out the consideration of the earth in our further enquiries upon this subject, and only consider the equator, zenith, and poles in the heavens, and $HR$ the rational horizon to the spectator.

(77.) Let the annexed figure represent the position

* This is not accurately true, the earth's axis varying a little from its parallelism from the action of the moon. This is called the *Nutation* of the earth's axis, and was discovered by Dr. Bradley.

of the heavens to $Z$ the zenith of a spectator in *north* latitude, $EQ$ the equator, $P$, $P'$, it's poles, $HOR$

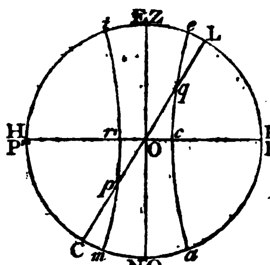



the rational horizon, $PZHP'R$ the meridian of the spectator, and draw the great circle $ZON$ perpendicular to $ZPRH$, and it is the prime vertical (28); $R$ will be the north point of the horizon, and $H$ the south (26), and $O$ will be the east or west point, (28) according as this figure represents the eastern or western hemisphere. Draw also a great circle $POP'$ perpendicular to the meridian. We must therefore conceive this figure to represent half a globe, and all the lines upon it to represent circles; and if we conceive the eye to be vertical to the middle point $O$ of the figure, all the circles which pass through that point will appear right lines; therefore the right lines $ZON$, $POP'$, $EOQ$, $HOR$, must be considered as semicircles. Now as each circle $HR$, $EQ$, $ZN$, $PP'$ is perpendicular to the meridian, it's pole must be in each (8, 9), therefore their common intersection $O$ is the pole of the meridian. Draw also the small circles $wH$, $mt$, $ae$, $Rv$, $yx$, parallel to the equator; and as the great circle $POP'$ bisects $EQ$ in $O$, it must also bisect the small circles $mt$, $ae$, in $r$ and $c$; for as $EO=90°$, $tr$ and $ec$ are each $=90°$ (13); and as $QO=90°$, $mr$ and $ac$ are each $=90°$; hence, $ac=ce$, and $mr=rt$.

(78.) As all the heavenly bodies, in their diurnal motion, describe either the equator, or small circles parallel to the equator, according as the body is in or out of the equator; if we conceive this figure to represent the eastern hemisphere, *QE*, *ae*, *mt*, may represent their apparent paths from the meridian under the horizon to the meridian above, and the points *b*, *O*, *s*, are the points of the horizon where they rise. And as *ae*, *QE*, *mt*, are bisected in *c*, *O*, *r*, *eb* must be greater than *ba*, *QO* equal to *OE*, and *ts* less than *sm*. Hence, a body on the *same* side of the equator with the spectator, will be longer above the horizon than below, because *eb* is greater than *ba*; a body *in* the equator will be as long above as below, because $QO = OE$; and a body on the *contrary* side will be longer below than above, because *ms* is greater than *st*. And the further *ae*, or *mt*, are from the equator, the greater will be the difference of *ab*, *be*, and *ms*, *st*, or of the times of continuing above and below the horizon; and the further they will rise from *O*. The bodies describing *ae*, *mt*, rise at *b* and *s*; and as *O* is the east point of the horizon, and *R* and *H* are the north and south points, a body, on the *same* side of the equator with the spectator, rises between the east and the north, and a body on the *contrary* side rises between the east and the south, the spectator being supposed to be in the *north* latitude; and a body *in* the equator rises in the east at *O*. When the bodies come to *d* or *n*, they are in the prime vertical, or in the east; hence, a body on the *same* side of the equator with the spectator comes to the east *after* it is risen, and a body on the *contrary* side, *before* it rises. The body which describes the circle *Rv*, or any circle nearer to *P*, never sets; and such circles are called circles of *perpetual apparition*; and the stars which describe them are called *circumpolar* stars. The body which describes the circle *wH*, just becomes visible at *H*, and then it instantly descends below the horizon; but the bodies which describe the circles nearer to *P′* are never

visible. Such is the apparent diurnal motion of the heavenly bodies, when the spectator is situated any where between the equator and poles; and this is called an *oblique* sphere, because all the bodies rise and set obliquely to the horizon. As this figure may also represent the western hemisphere, the same circles *ea*, *tm* will represent the motions of the heavenly bodies as they descend from the meridian above the horizon to the meridian under. Hence, a body is at the greatest altitude above the horizon, when on the meridian, and at equal altitudes when equidistant on each side, from it, if the body have not changed it's declination.

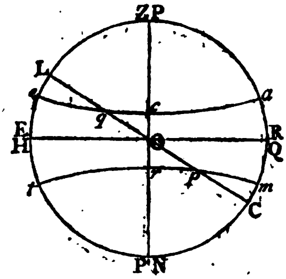
(79.) If the spectator be at the *equator*, then *E* coincides with *Z*, and consequently *EQ* with *ZN*, and




therefore *PP'* with *HR*. Hence, as the equator *EQ* is perpendicular to the horizon, the circles *ace*, *mrt*, parallel to *EQ* must also be perpendicular to it; and as these circles are always bisected by *PP'*, they must now be bisected by *HR*. Hence, all the heavenly bodies are as long above the horizon as below, and rise and set at right angles to it, on which account this is called a *right* sphere.

(80.) If the spectator be at the *pole*, then *P* coincides with *Z*, and consequently *PP'* with *ZN*, and therefore *EQ* with *HR*. Hence, the circles *mt*, *ae*, parallel to the equator, are also parallel to the horizon; therefore as a body in it's diurnal motion describes a circle parallel to the horizon, those fixed bodies in the heavens, which are above the horizon, must always

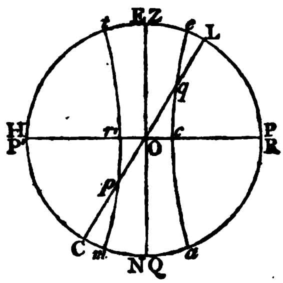
continue above, and those which are below must always continue below. Hence, none of the bodies,




by their *diurnal* motion, can either rise or set. This is called a *parallel* sphere, because the diurnal motion of all the bodies is parallel to the horizon. These apparent diurnal motions of the fixed stars remain constant, that is, each always describes the same parallel of declination.

(81.) The *ecliptic*, or that great circle in the heavens which the sun appears to describe in the course of a year, does not coincide with the equator, for during that time it is found to be only twice in the equator; let therefore *COL* represent half the ecliptic, or half the sun's apparent annual motion; *C* the first point of Capricorn, and *L* the first point of Cancer; and this being a great circle, must cut the equator into two equal parts (7). Hence, as the apparent motion of the sun is nearly uniform, the sun is nearly as long on one side of the equator as on the other. (See Fig. in p. 16.) When therefore the sun is at *q*, on the *same* side of the equator with the spectator, describing the parallel of declination *ae* by it's apparent diurnal motion, the days are longer than the nights, and it rises at *b* to the north of the east point; but when it is on the *contrary* side, at *p*, describing *mt*, the days are shorter than the nights, and it rises at *s* to the south of the east point, the spectator being on the *north* side of the equator; but when the sun is *in* the equator, at *O*, describing *QE*, the days

and nights are equal, and it rises in the east, at $O$ *. If $ae$, $mt$ be equidistant from $EQ$, then will $be=ms$, and $ab=st$; hence, when the sun is in these opposite parallels, the length of the day in one is equal to the length of the night in the other; therefore the *mean* length of a day at every place is 12 hours. Hence, at every place, the sun, in the course of a year, is half a year above, and half a year below the horizon †. It is manifest, also, that the days increase from the time the sun leaves $C$ the beginning of Capricorn, till he comes to $L$ the beginning of Cancer; and that they decrease from the time the sun leaves the beginning of Cancer till he comes to the beginning of Capricorn. When the spectator is at the *Equator*, the sun at $p$ or $q$ describing the circles $mt$, $ae$, by it's apparent *diurnal*

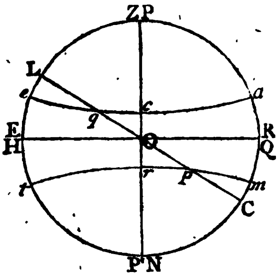



motion, and these being bisected by the horizon, the sun will be always as long above as below the horizon,

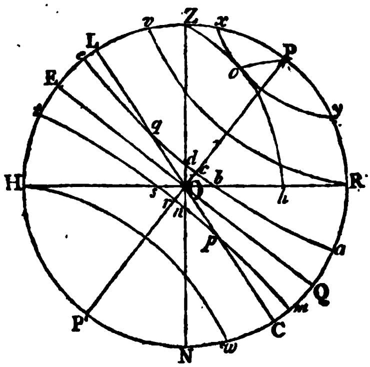
* The different degrees of heat in summer and winter, do not altogether arise from the different times which the sun is above the horizon, but partly from the different altitudes of the sun above the horizon; the higher the sun is above the horizon, the greater is the number of rays which fall on any given space, and the greater also is the force of the rays. From all these circumstances arise the different degrees of heat in summer and winter. The increase of heat also as you approach the equator, arises from the two latter circumstances.

† This is not accurately true, because the sun's motion in the ecliptic is not quite uniform, on which account it is not exactly as long on one side of the equator as on the other. If the major axis of the earth's orbit coincided with the line joining the equinoctial points, the times would be equal. This happened at the Creation.

and consequently the days and nights will be always 12 hours long. There will however be some variety of seasons, as the sun will recede 23°. 28′ on each side from the spectator. In this situation of the spectator, the sun will be vertical to him at noon when it is in the equator. And when the spectator is any where between the tropics, the sun will be vertical to him at noon, when it's declination is equal to the latitude of the place, and of the same kind, that is, when they are both north, or both south. When the spectator is at the *Pole*, the sun at *p* or *q* is carried, by it's apparent




*diurnal* motion, in the circles *mpt, aqe,* parallel to the horizon; hence, it never sets when it is in that part *OL* of the ecliptic which is above the horizon, nor rises when in that part *OC* which is below; consequently there is half a year day, and half a year




night. As the sun illuminates one half of the earth, or 90° all round about that place to which he is ver-

tical *; when he is *in* the equator, he will illuminate as far as each pole; when he is on the north side of the equator, the north pole will be within the illuminated part, and the south pole will be in the dark part; and when the sun is on the south side of the equator, the south pole will be within the illuminated part, and the north pole in the dark part. And when the sun is at the tropic, he illuminates 23° . 28′ beyond one pole, and the other pole is 23° . 28′ within the dark part. Hence, the variety of seasons arises from the axis of the earth, which coincides with *PP′*, not being perpendicular to the plane of the ecliptic *LOC*, for if it were, the ecliptic and equator would coincide, and the sun would then be always in the equator, and consequently it would never change it's position in respect to the surface of the earth. If $QR = EH =$ 23° . 28′, the sun's greatest declination, then on the longest day the sun describes the parallel *Rv*, which just touching the horizon at *R*, shows that the sun does not descend on that day below the horizon, and therefore that day is 24 hours long. But when the sun comes to its greatest declination on the other side of *EQ*, it describes *wH*, and consequently does not ascend above the horizon for 24 hours, and therefore that night is 24 hours long. This therefore happens when *EH*, the complement of *EZ* the latitude (16), is 23° . 28′, or in latitude 66° . 32′. If *EH*, the complement of latitude, be less than 23° . 28′, the sun will be above the horizon in summer, and below in winter, for more than 24 hours, and the longer above or below, as you approach the pole, where, as was before observed, it will be six months above, and as long below the horizon. The orbits of all the planets, and of the moon, are also inclined to the equator, as appears by tracing their motions amongst the fixed

* This is not accurately true, because, as the sun is greater than the earth, he will illuminate beyond 90°, by a quantity which is nearly equal, in minutes of a degree, to his apparent semidiameter.

stars; therefore, in the time in which each makes one revolution in its orbit, the same appearances will take place, as in the sun. All these different appearances in the motion of the moon, must therefore happen in every month. It is also evident, that these variations of rising and setting must be greater or less, as the orbits are more or less inclined to the equator, as appears by Art. 78. Hence, they must be greater in the moon than in the sun*. The apparent annual motion of the sun, and the real motion of the moon and planets, is from west to east, and therefore contrary to their apparent diurnal motion.

(82.) Hitherto we have considered the motion of the heavenly bodies in the eastern hemisphere; but if the figure represent the western hemisphere, all the reasoning will equally apply. The bodies will be just as long in descending from the meridian to the horizon, as in ascending from the horizon to the meridian; the paths described will be similar; and they will set in the same situation in respect to the west point of the horizon, as they rise in respect to the east; that is, if a body rise to the north or south of the east, it will set at the same distance from the west towards the north or south.

(83.) Having thus explained all the apparent diurnal motions of the heavenly bodies, with the cause of the variety of seasons, we shall proceed in the next place to show the method of determining the positions of the different circles, and the situation of the bodies in respect to the horizon, meridian, or any other circles, at any given time; and having given their situation, to find the time; for the understanding of

* On account of the continual change of declination of the sun, moon, and planets, their apparent diurnal motions will not be accurately parallel to the equator; in those cases therefore, where the declination alters sensibly in the course of a day, and where great accuracy is required, we must in our computations, take into consideration, the change of declination.

which, a knowledge of plane and spherical trigonometry is all that is requisite.

(84.) *The altitude* PR *of the pole above the horizon, is equal to the latitude of the place.*

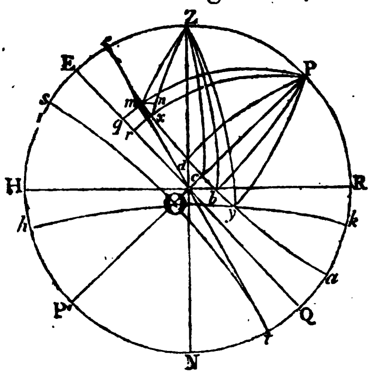
For the arc $ZE$ is (16) the measure of the latitude; but $PE = ZR$, each being $= 90^0$; take away $ZP$ which is common to both, and $EZ = PR$*.

(85.) *To find the latitude of a place, observe the greatest and least altitude of a circumpolar star, and apply the correction for refraction, in order to get the true altitudes, and half the sum will be the altitude of the pole.*

For if $yx$ be the true circle described by the star, then, as $Px = Py$, $PR = \frac{1}{2} \times \overline{Ry + Rx}$. See the last figure.

*The latitude may also be thus found.*

Let $eOt$ be the ecliptic; then when the sun comes to $e$, it's declination is the greatest, and $eH$ is the




greatest meridian altitude; when the sun comes to the ecliptic at $t$, let $ts$ be the parallel described on that

* From hence arises the method of measuring the circumference of the earth; for if a man travel upon a meridian till the height of the pole has altered one degree, he must then have travelled one degree; hence, by measuring that distance and multiplying it by 360, we get the circumference of the earth. This was undertaken by our countryman Mr. *Norwood*, who measured the distance between London and York, and observed the different altitudes of the pole at those places. Afterwards, the French mathematicians measured

day, and then $sH$ is the least meridian altitude; and as $Ee = Es$, $\frac{1}{2} \times \overline{He + Hs} = HE$ the complement of the latitude.

(86.) *Half the difference of the sun's greatest and least meridian altitudes, is equal to the inclination of the ecliptic to the equator.*

For half $\overline{He - Hs}$, or half $se$, is equal to $Ee$ which (12) measures the angle $EOe$, the inclination of the ecliptic to the equator.

(87.) *The angle which the equator makes with the horizon, or the altitude of that point of the equator which is on the meridian, is equal to the complement of the latitude.*

For $ZH$ is $90^0$, and therefore $EH$ is the complement of $EZ$; and as $OE = OH = 90^0$, $EH$ measures (12) the angle $EOH$ *.

(88.) Let $abcdxe$ be a parallel of declination described by an heavenly body in the eastern hemisphere, and draw the circles of declination $Pb$, $Pc$, $Pd$, $Px$, and the circles of altitude $Zb$, $Zc$, $Zd$, $Zx$. Now, as has been already explained, when the body comes to $b$, it rises; at $c$ it is at the middle point between $a$ and $e$; and at $d$ it is due east; and let $x$ be it's place at any other time. Let us suppose this body to be the sun, and not to change it's declination in it's passage from $a$ to $e$, and let us suppose a clock to be adjusted to go 24 hours in one apparent diurnal revolution of the sun, or from the time it leaves any meridian till it returns to it again, then the sun will always approach the meridian, or any other circle of declination, at the

measured a degree. *Cassini* measured one in France. After that, *Clairaut*, *Maupertuis*, and several other mathematicians went to Lapland, and measured a degree, the length of which appears to be 69,2 English miles in the latitude of 45°; for the earth being a spheroid, the degrees in different latitudes are different.

* See my *Treatise on Plane and Spherical Trigonometry*, Art. 173. This is the Trigonometry referred to in the future part of this Work.

rate of $15^{\circ}$ in an hour; also, the angle which the sun describes about the pole will vary at the same rate, because (13) an arc $xe$, which the sun at $x$ has to describe before it comes to the meridian, measures the angle $xPe$, called the *hour* angle. If therefore we suppose the clock to show 12 when the sun is on the meridian at $a$ or $e$, it will be 6 o'clock when he is at $c$. And as the sun describes angles about the pole $P$ at the rate of $15^{\circ}$ in an hour, the angle between any circle, $Px$, of declination passing through the sun at $x$, and the meridian $PE$, converted into time at the rate of $15^{\circ}$ for an hour, will give the time from *apparent* noon, or when the sun comes to the meridian.

(89.) *Given the sun's declination, and latitude of the place, to find the time of his rising, and azimuth at that time.*

The sun rises at $b$; and in the triangle $bZp$, $bZ = 90^{\circ}$, $bP$ = co-dec. $PZ$ = co-lat. Now when one side of a triangle $= 90^{\circ}$, it may be solved by the circular parts, taking the angles adjacent to the side $= 90^{\circ}$, and the complements of the other three parts, for the circular parts. Hence, (Trig. Art. 215.) rad. $\times$ cos. $ZPb$ = cot. $bP \times$ cot. $ZP$, or, rad. $\times$ cos. hour angle = tan. dec. $\times$ tan. lat. therefore (Trig. Art. 213).

*Log. tan. dec. + log. tan. lat. − 10, = log. cos. hour ang. from app. noon;*

which converted into time, at the rate of $15^{\circ}$ for an hour, (see Table I. at the end), and subtracted from 12 o'clock, gives the apparent time of rising. Also, (Trig. Art. 215.) rad. $\times$ cos. $bP$ = sin. $ZP \times$ cos. $PZb$, or, rad. $\times$ sin. dec. = cos. lat. $\times$ cos. azi. therefore

10, + *log. sin. dec. − log. cos. lat. = log. cos. azi. from North.*

Ex. Given the latitude of Cambridge $52^{\circ}.12'.35''$, to find the time of the sun's rising on the longest day, and azimuth at that time, assuming the greatest declination of the sun $23^{\circ}.28'$.

| | | | |
|---|---|---|---|
| Dec. 23° : 28′. 0″. | - | tan. | 9,6376106 |
| Lat. 52° : 12′. 35. | - | tan. | 10,1104699 |
| Hour ∠ 124°. 2′. 47″ * | - | cos. | 9,7480805 |

Convert this into time (Tab. I.) and it gives 8*h*. 19′.6″, which subtracted from 12, gives ~~3*h*. 40′. 54″~~, 3h 43m 49s the time when the sun's center is upon the rational horizon on the longest day; Also,

| | | | |
|---|---|---|---|
| Dec. 23°. 28′. 0″. | - | 10, + sin. | 19,6001181 |
| Lat. 52. 12. 35. | - - | cos. | 9,7872996 |
| Azi. 49. 28. 9. | - - | cos. | 9,8128185 |

Hence, on the longest day, the sun rises ~~40°. 31′. 51″~~ from the east towards the north. 49°28′ 9″

(90.) *To find the sun's altitude at six o'clock.*

The sun is at $c$ at 6 o'clock, and the angle $ZPc$ is a right one; hence, (Trig. Art. 212.) rad. × cos. $Zc$ = cos. $ZP$ × cos. $Pc$, or rad. × sin. alt. = sin. lat. × sin. dec. therefore

*Log. sin. lat. + log. sin. dec. − 10, = log. sin. alt.*

Ex. Taking the data of the last example, we have,

| | | | |
|---|---|---|---|
| Lat. 52°. 12′. 35″ | - - | sin. | 9,8977695 |
| Dec. 23. 28. 0 | - - | sin. | 9,6001181 |
| Alt. 18. 20. 32. | - - | sin. | 9,4978876 |

(91.) *To find the time when the sun comes to* d *the prime vertical, and it's altitude at that time.*

In this case, the angle $dZP = 90°$; hence, (Trig. Art. 112.) rad. × cos. $dP$ = cos. $ZP$ × cos. $Zd$, or, rad. × sin. dec. = sin. lat. × sin. alt. therefore

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* This log. 9,7480805 is found in the tables to be the log. cosine of 55°. 57′. 13″, but as the angle is manifestly greater than 90°, we must take its supplement. In the solution of spherical triangles, ambiguous cases will frequently arise; for the determination of which, where the case is not evident, the reader is referred to my *Treatise on Trigonometry.*

10, + *log. sin. dec.* − *log. sin. lat.* = *log. sin. alt.*

Also, (Trig. Art. 212) rad. × cos. $ZPd$ = cot. $Pd$ × tan. $PZ$, or, rad. × cos. hour angle = tan. dec. × cot. lat. therefore

*log. tan. dec.* + *log. cot. lat.* − 10, = *log. cos. hour angle*;

which, converted into time (Tab. I.), gives the time from *apparent* noon.

Ex. Taking the data of the last example, we have,

| | | |
|---|---|---|
| Dec. 23°. 28′. 0″ | - | 10 +, sin. 19,6001181 |
| Lat. 52 . 12 . 35 | - - | sin. 9,8977695 |
| Alt. 30 . 15 . 31 | - - | sin. 9,7023486 |
| Dec. 23 . 28 . 0 | - - | tan. 9,6376106 |
| Lat. 52 . 12 . 35 | - - | cot. 9,8895301 |
| Hour ∠ 70. 19. 44. | - - | cos. 9,5271407 |

This angle 70°. 19′. 44″, converted into time, gives 4*h*. 41′. 19″ the time from *apparent* noon.

(22.) *Given the latitude of the place, the sun's declination, and altitude, to find the hour, and his azimuth.*

Let $x$ be the sun's place; then, (Trig. Art. 239) sin. $Px$ × sin. $PZ$ : rad.² :: sin. $\frac{1}{2} \times \overline{Px + PZ + Zx}$ × sin. $\frac{1}{2} \times \overline{Px + PZ - Zx}$ : cos. $\frac{1}{2}\, ZPx^2$; hence, $ZPx$ is known, which converted into time (Tab. I.) gives the time from *apparent* noon. Also, (Trig. Art. 239) sin. $Zx$ × sin. $ZP$ : rad.² :: sin. $\frac{1}{2} \times \overline{Zx + ZP + Px}$ × sin. $\frac{1}{2} \times \overline{Zx + ZP - Px}$ : cos. $\frac{1}{2}\, xZP^2$; hence, the azimuth $xZP$ from the north is known.

Ex. Given the lat. 34°. 55′ N, sun's declination 23°. 22′. 57″ N, and true altitude 36°. 59′. 39″, to find the apparent time.

Here, $ZP$ = 55°. 5′, $Zx$ = 53°. 0′. 21″, $Px$ = 67°. 37′. 3″: hence (Trig. Art. 239)

| | | |
|---|---|---|
| $Px = 67^\circ.37'.\ 3''$ | - ar. co. sin. | 0,034019 |
| $ZP = 55.\ 5.\ 0$ | - ar. co. sin. | 0,086193 |
| $Zx = 53.\ 0.\ 21$ | | |
| Sum 175. 42. 24 | | |
| ½ Sum 87. 51. 12 | - - - sin. | 9,999694 |
| $Zx = 53.\ 0.\ 21$ | | |
| Diff. 34. 50. 51 | - - - sin. | 9,756932 |
| | | 2)19,876838 |
| | | 9,938419 |

the cosine of $29^\circ.\ 47'.\ 44''$, half the angle $ZPx$, $\therefore ZPx = 59^\circ.35'.\ 28''$, which reduced into time gives 3 *h.* $58'.\ 22''$, the time from *apparent* noon. By the very same process, the angle $xZP$ is found.

(93.) *Given the error in altitude, to find the error in time.*

Let $mn$ be parallel to the horizon, and $nx$ represent the error in altitude; then, as the calculation of the time is made upon supposition that there is no error in the declination, we must suppose the body to be at $m$ instead of $x$, and consequently the angle $mPx$, or the arc $qr$, measures the error in time.

Now $nx : xm :: sin.\ nmx : rad.$ (Trig. Art. 125)

$\underline{xm : qr :: cos.\ rx : rad.}$ (Art. 13).

hence, $nx : qr ::$ sin. $nmx \times$ cos. $rx$ : rad.$^2$ $\therefore qr = nx \times \frac{\text{rad.}^2}{\text{sin. } nmx \times \text{cos. } rx}$; but $ZxP = nmx$, $nxm$ being the complement of both; also, (Trig. Art. 221.) sin. $ZxP$, or $nmx$, : sin. $ZP$ :: sin. $xZP$ : sin. $xP$, or cos. $rx$, $\therefore$ sin. $nmx \times$ cos. $rx =$ sin. $ZP \times$ sin. $xZP$; hence, $qr =$

$$nx \times \frac{\text{rad.}^2}{\text{sin.} ZP \times \text{sin.} xZP} = nx \times \frac{\text{rad.}^2}{\text{cos.lat.} \times \text{sin. azim.}}.$$

Hence, the error is least on the prime vertical. All altitudes therefore, for the purpose of deducing the

time, ought to be taken on, or as near to, the prime vertical as possible.

Ex. In lat. 50°. 12′, if the error in alt. at an azim. 44°. 22′ be 1′, then $qr = 1' \times \frac{1^2}{,612 \times ,690} = 2',334$ of a degree $= 9'',336$ in time.

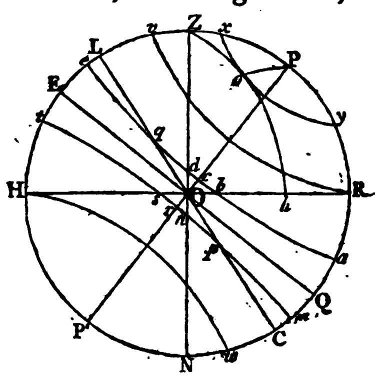
Hence, the perpendicular ascent of a body is quickest when it is on the prime vertical; for $nx$ varies as sin. azim. when $qr$ and the lat. are given.

(94.) *Given the lat. of the place, and the sun's declination, to find the time when twilight begins.*

Twilight is here supposed to begin when the sun is 18° below the horizon; draw therefore the circle $hyk$ parallel to the horizon, and 18° below it, and twilight will begin when the sun comes to $y$, and $Zy = 108°$; hence, (Trig. Art. 239) sin. $Py$ × sin. $PZ$ : rad.$^2$ :: sin. $\frac{1}{2} \times \overline{PZ + Py + 108°} \times$ sin. $\frac{1}{2} \times \overline{PZ + Py - 108°}$ : $\overline{\text{cos. } \frac{1}{2}\, yPZ}^2$; therefore $yPZ$ is known, which converted into time (Tab. I.), gives the time from *apparent* noon. The operation is the same as that in Art. 92.

(95.) *To find the time when the apparent diurnal motion of a fixed star, is perpendicular to the horizon.*

Let $yx$ be the parallel described by the star; draw the vertical circle $Zh$, touching it at $o$, and when the

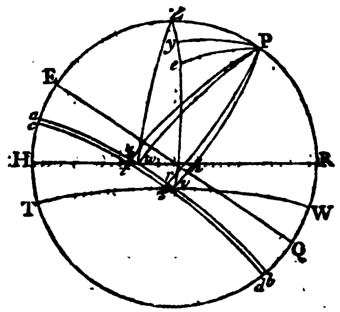



star comes to $o$, it's motion is perpendicular to the horizon; and as $ZoP$ is a right angle, we have (Trig.

Art. 212) rad. × cos. $ZPo$ = tan. $Po$ × cot. $PZ$, or rad. × cos. hour angle = cot. dec. × tan. lat. therefore, *log. cot. dec.* + *log. tan. lat.* − 10, = *log. cos. hour angle*; which converted into time (Tab. I.), gives the time from the star's being on the meridian. Hence, the time of the star's coming to the meridian being known, the time required will be known.

(96.) *To find the time of the shortest twilight.*

Let $ab$ be the parallel of the sun's declination at the time required, draw $cd$ indefinitely near and parallel to it, and $TW$ a parallel to the horizon 18° below it; then $vPw$, $sPt$ measure the duration of twilight on each parallel of declination, and when the twilight is shortest, the increment of the duration is = 0, and these must be equal; hence, $vPr = wPz$, therefore $vr = wz$; and as $rs = tz$, and $r$ and $z$ are right angles, $rvs = zwt$; but $Pvr = 90° = Zvs$, take $Zvr$ from both, and $PvZ = rvs$; for the same reason $PwZ = zwt$; hence, $PvZ = PwZ$. Take $ve = wZ = 90°$, and join $Pe$; and as $Pv = Pw$, $ve = wZ$, and $Pve = PwZ$, we




have $Pe = PZ$; and if $Py$ be perpendicular to $eZ$, then will $Zy = ye$. Now, (Trig. Art. 224) cos. $Pv$ : cos. $Pe$, or $PZ$, :: cos. $vy$ : cos. $ey$, that is, sin. dec. : sin. lat. :: sin. $ey$ : cos. $ey$ :: tan. $ey$ = 9° : rad. or, *rad.* : *sin. lat.* :: *tan.* 9° : *sin. of the sun's declination* at the time of the shortest twilight; and the logarithmic operation is,

*log. sin. lat.* + *log. tan.* 9° − 10, = *log. sin. dec.*

Because $PZ$ is never greater than 90°, and $Zy$ = 9°,

therefore $Py$ is never greater than $90^0$, and it's cosine is positive; also, $vy$ is always greater than $90^0$, therefore it's cosine is negative; hence, (Trig. Art. 212. rad. being unity) cos. $Pv$ (= cos. $Py \times$ cos. $vy$) is negative; consequently $Pv$ is greater than $90^0$; therefore the sun's declination is *south*.

If, instead of taking $RW = 18^0$, we take it = the sun's diameter ($2s$), we shall get the time of the year when the body of the sun is the least time in ascending above the horizon; hence,

$$log.\ sin.\ lat. + log.\ tan.\ s - 10, = log.\ sin.\ dec.$$

Thus we get the declination when the sun is the least time in rising; and as the declination must be always very small, this event must happen when the sun is very near the equinox.

(97.) *To find the duration of the shortest twilight.*

As $wPz = vPe$, therefore $ZPe = vPw$, which measures the shortest time. Now (Trig. Art. 212) rad. $\times$ sin. $Zy$ = sin. $PZ \times$ sin. $ZPy$, or, rad. $\times$ sin. $9^0$ = cos. lat. $\times$ sin. $ZPy$, therefore,

$$10, + log.\ sin.\ 9^\circ - log.\ cos.\ lat. = log.\ sin.\ ZPy,$$

which doubled gives $ZPe$, or $vPw$, which, converted into time (Tab. I.), gives the duration of the shortest twilight.

Ex. To find the time of the year at *Cambridge*, when the twilight is shortest; and the length of that twilight.

| | | | |
|---|---|---|---|
| Lat. $52^0.\ 12'.\ 35''$ | - - - - - | sin. | 9,8977695 |
| $9^0$ | - - - - - - | tan. | 9,1997125 |
| Dec. $7^0.\ 11'.\ 25''$ | - - - - - | sin. | 9,0974820 |

This declination of the sun gives the time about March 2, and October 11.

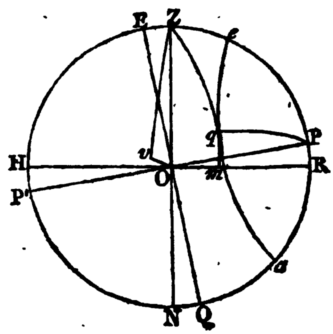
| | | | |
|---|---|---|---|
| $9^0.\ 0'.\ 0''$ | - - | 10, + sin. | 19,1943324 |
| Lat. 52. 12. 35 | - - - | cos. | 9,7872996 |
| $ZPy$ 14. 47. 27 | - - - | sin. | 9,4070328 |

The double of this gives 29°. 34′. 54″, which, converted into time, gives 1*h*. 58′. 20″ for the duration of the shortest twilight, it being supposed to end when the sun is 18° below the horizon.

(98.) *To find the sun's declination, when it is just twilight all night.*

Here the sun at *a* (Fig. p. 25.) must be 18° below the horizon; therefore 18° + dec. $Qa = RQ = EH =$ comp. of lat. of place; hence, the sun's dec. = comp. lat. − 18°; look therefore into the *Nautical Almanack*, and see on what days the sun has this declination, and you have the time required. The sun's greatest declination being 23°. 28′ it follows, that if the complement of latitude be greater than 41°. 28′, or if the latitude be less than 48°. 32′, there can never be twilight all night. If the sun be on the other side of the equator, then it's dec. = 18°. − comp. lat.

(99.) If the sun's declination *Ee* be greater than *EZ*, then the sun comes to the meridian at *e* to the *north* of the zenith *Z* of the spectator; and if we draw the secondary *Zqm* touching the parallel *ae* of declination described by the sun, then *Rm* is the greatest azimuth from the north which the sun has that day, the




azimuth increasing till the sun comes to *q*, and then decreasing; for a circle from *Z* to any other point of *ea* will cut *RO* nearer to *R*, and it will also cut *ea* in two points which have the same azimuth, they being in the same vertical circle; in this case, therefore, the

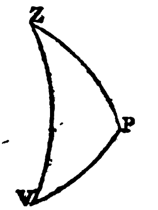
sun has the same azimuth twice in the morning. If, therefore, we draw the straight line $Zv$ perpendicular to the horizon, the shadow of this line, being always opposite to the sun, will, in the morning, first recede from the south point $H$, and then approach it, and therefore will go backwards upon the horizon. But if we consider $PP'$ as a straight line, or the earth's axis produced, the shadow of that line will not go backwards upon the horizon, because the sun always revolves about that line, whereas it does not revolve about the perpendicular $Zv$, it never getting to the south of it. Hence it appears, that the shadow of the sun upon a dial can never go backwards, because the gnomon of a dial is parallel to $PP'$, and therefore the sun must always revolve about the gnomon.

The time when the azimuth is greatest is found from the right angled triangle $PqZ$; for (Trig. Art. 212) rad. $\times$ cos. $ZPq$ = tan. $qP$ $\times$ cot. $PZ$, or, rad. $\times$ cos. hour angle = cot. dec. $\times$ tan. lat.; therefore,
*log. cot. dec.* + *log. tan. lat.* – 10, = *log. cos. hour angle* from *apparent* noon.

(100.) It has hitherto been supposed, that it is 12 o'clock when the sun comes to the meridian $ZHN$ (Fig. p. 20) and that the clock goes just 24 hours in the interval of the sun's passage from any meridian till it returns to it again. But if a clock be thus adjusted for one day, it will not continue to show 12 o'clock every day when the sun comes to the meridian, because it is found by observation, that the intervals of time from the sun's leaving any meridian till it returns to it again, are not always equal; this difference between the sun and the clock is called the *Equation of Time*, as will be explained in Chap. IV. Hence, when the clock does not agree with the sun, and the sun is at $x$, any arc $xe$ is not the measure of the time from 12 o'clock, but from the time when the sun comes to the meridian, or from *apparent* noon, as it is called.

(101.) The method of finding the hour angle for the time at which a body rises, has been upon the sup-

position that the body is upon the rational horizon at the instant it appears, or $90°$ from the zenith; but all bodies in the horizon are elevated by refraction $33'$ above their true places; this therefore would make them appear when they are $33'$ below the rational horizon, or $90° + 33'$ from the zenith; also, all the bodies in our system are depressed below their true places by parallax, as will be afterwards explained; therefore from this cause they would not appear till they were elevated above the rational horizon by a quantity equal to their horizontal parallax, or when distant from the zenith $90°$ – hor. par. Hence, from

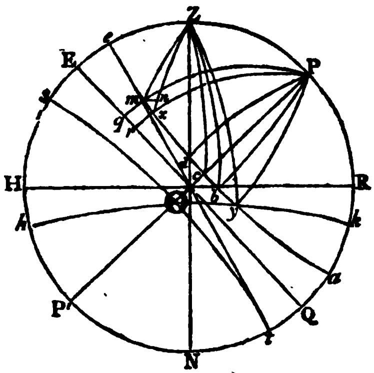



both causes together, a body becomes visible when it's distance $ZV$ from the zenith $= 90° + 33' -$ hor. parallax; $V$ being the place of the body when it becomes visible, $Z$ the zenith, and $P$ the pole; hence, knowing $ZV$, also $ZP$ the complement of latitude, and $PV$ the complement of declination, we can find the hour angle $ZPV$. A fixed star has no parallax, therefore in this case $ZV = 90°. 33'$.

### *To find the Time in which the Sun passes the Meridian, or the horizontal Wire of a Telescope.*

(102.) Let $mx$ be the diameter of $d''$ of the sun, estimated in seconds of a great circle; then, as the seconds in $mx$, considered as a small circle, must be increased in proportion as the radius is diminished, because (Trig. Art. 75) when the arc is given, the angle is inversely as the radius, we have, sin. $Px$, or

cos. dec. $rx$, : rad. :: seconds $d''$ in $mx$ of a great circle : the seconds in $mx$ of the small circle $ea$, which (13) is



equal to the seconds in $qr$, or, in the angle $rPq$, and therefore the angle $rPq = d''$ divided by cos. dec. (rad. being unity) $= d'' \times$ sec. dec., which measures the time in which the sun passes over a space equal to it's diameter, and consequently the time the diameter will be in passing over the meridian; hence, 15″ in space (corresponding to 1″ in time) : $d'' \times$ sec. dec. in space :: 1″ in time : the time in seconds of passing the meridian $= \frac{d'' \times \text{sec. dec.}}{15''}$.

(103.) Hence, $qr$, the sun's diameter in right ascension, is equal to $d'' \times$ sec. dec. If, therefore, the sun's diameter $= 32' = 1920''$, and it's dec. $= 20°$, it's diameter in right ascension $= 1920'' \times 1{,}064 = 34'.2,''88$. The same is true for the moon, if $d''$ = it's diameter.

(104.) By Art. 93. $qr = nx \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}$

$= (\text{if } nx = d'', \text{the sun's diameter}) d'' \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}$

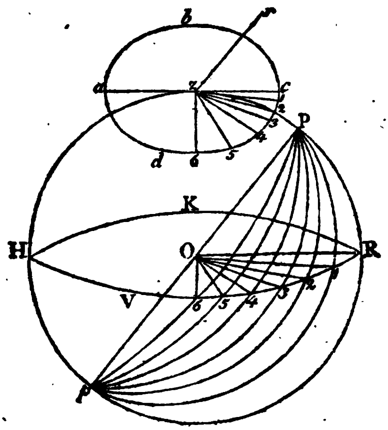
hence, as before, the time of describing $qr$, or the time in which the sun ascends perpendicularly through a space equal to it's diameter, or the time of passing an horizontal wire, is equal to $\frac{d''}{15''} \times \frac{\text{rad.}^2}{\text{cos. lat.} \times \text{sin. azim.}}$

The same expression must also give the time which the body of the sun is in ascending above the horizon.

If $d''=1980''$ the horizontal refraction, then $d''$ divided by $15''=132''$; hence, refraction accelerates the rising of the sun by $132''\times\frac{\text{rad.}^2}{\text{cos. lat.}\times\text{sin. azim.}}$.

*On the Principles of Dialling.*

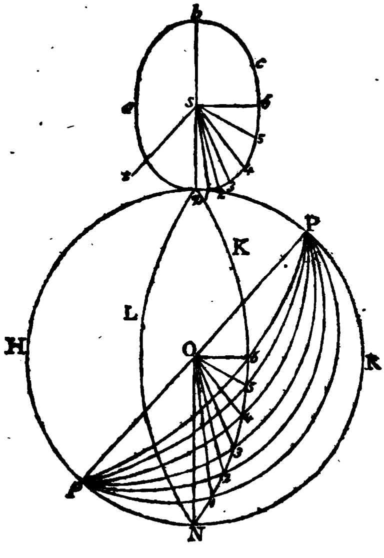
(105.) As the apparent motion of the sun about the axis of the earth, is at the rate of $15°$ in an hour, very nearly, let us suppose the axis of the earth to project it's shadow into the meridian opposite to that in which the sun is, and then this meridian will move at the rate of $15°$ in an hour. Hence, let $zPRpH$ represent a meridian




on the earth's surface, $POp$ the earth's axis, $z$ the place of the spectator, $HKRV$ a great circle, of which $z$ is the pole; draw the meridians $P1p$, $P2p$, &c. making angles with $PRp$ of $15°$, $30°$, &c. respectively, then, supposing $PR$ to be the meridian into which the shadow of $OP$ is projected at 12 o'clock, $P1$, $P2$, &c. are the meridians into which it is projected at 1, 2, &c. o'clock, and the shadow will be projected on the plane $HKRV$ into the lines $OR$, $O1$, $O2$, &c. and the angles $RO1$, $RO2$, &c. will be the angles between the 12 o'clock line and the 1, 2, &c. o'clock lines. Now in the right

angled triangle $PR1$, we have (84) $PR$ the latitude of the place, and the angle $RP1 = 15°$; hence, (Trig. Art. 210) *rad.* : *tan.* 15° :: *sin.* PR : *tan.* R1; in the same manner we may calculate the arcs $R2$, $R3$, &c. In this case, we make the earth's axis the gnomon, and the shadow is projected upon the plane $HKRV$. Take a plane $abcd$ at $z$, parallel to $HKRV$, and it is the sensible horizon (21), and draw $zr$ parallel to $POp$; then, on account of the great distance of the sun, we may conceive it to revolve about $zr$ in the same manner as about $PO$, and consequently the shadow will be projected upon the plane $abcd$, in the same manner as the shadow of $PO$ is projected upon the plane $HKRV$, and therefore the hour angles are calculated by the same proportion. This is an *horizontal* dial.

(106.) Now let $NLzK$ be a great circle perpendicular to $PRpH$, and consequently perpendicular to the




horizon at $z$, and the side next to $H$ is full south. Then, for the same reason as before, if the angles $Np1$, $Np2$, &c. be 15°, 30°, &c. the shadow of $pO$ will be

projected into the lines $O1$, $O2$, &c. at 1, 2, &c. o'clock, and the angles $NO1$, $NO2$, &c. will be measured by the arcs $N1$, $N2$, &c. Hence, in the right angled triangle $pN1$, $pN$ = the complement of the latitude, and the angle $Np1 = 15°$; therefore (Trig. Art. 210) *rad.* : *tan.* 15° :: *sin.* pN : *tan.* N1; in the same manner we find $N2$, $N3$, &c. Hence, for the same reason as for the horizontal dial, if *zabc* be a plane coinciding with *NLzK*, and *st* be parallel to *Op*, *st* will project it's shadow in the same manner on the plane *zabc*, as *Op* does on the plane *NLzK*, and therefore the hour angles from the 12 o'clock line are computed by the same proportion. This is a *vertical south* dial. In the same manner the shadow may be projected upon a plane in any position, and the hour angles calculated.

(107.) In order to fix an horizontal dial, we must be able to tell the exact time of the sun's coming to the meridian; for which purpose, find the time (92) by the sun's altitude when it is at the solstices, that being the best time of the year for the purpose, because then the declination does not vary, and set a well-regulated watch to that time; then, when the watch shows 12 o'clock, the sun is on the meridian; at that instant, therefore, set the dial, so that the shadow of the gnomon may coincide with the 12 o'clock line, and it stands right.

(108.) Hence, we may easily draw a meridian line upon an horizontal plane. Suspend a plumb line so that the shadow of it may fall upon the plane, and when the watch shows 12, the shadow of the plumb line is the true meridian. The common way is to describe several concentric circles upon an horizontal plane, and in the center to erect a gnomon perpendicularly to it, with a small round well defined head, like the head of a pin; make a point upon any one of the circles where the shadow of the head falls upon

it in the morning, and again where it falls upon the same circle in the afternoon; draw two radii from these two points, and bisect the angle between them, and the bisecting line will be a meridian line. This should be done when the sun is at the tropic, when it does not sensibly change it's declination in the interval of the observations; for if it do, the sun will not be equidistant from the meridian at equal altitudes. But this method is not capable of very great accuracy; for the shadow not being very accurately defined, it is not easy to say at what instant of time the shadow of the head of the gnomon is bisected by the circle. If, however, several circles be made use of, and the mean of the whole number of meridians so taken, be drawn, the meridian may be found with sufficient accuracy for all common purposes.

(109.) To find whether a wall be full south for a vertical south dial, erect a gnomon perpendicularly to it, and hang a plumb line from it; then when the watch, as above adjusted, shows 12, if the shadow of the gnomon coincide with the plumb line, the wall is full south.

# CHAP. III.

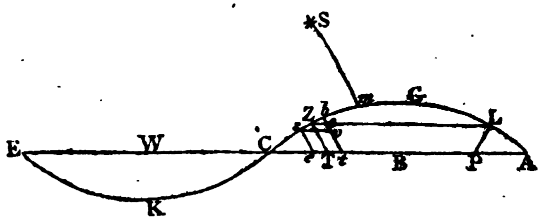
## TO DETERMINE THE RIGHT ASCENSION, DECLINATION, LATITUDE AND LONGITUDE OF THE HEAVENLY BODIES.

(110.) THE foundation of all Astronomy is to determine the situation of the fixed stars, in order to find, by a reference to such fixed objects, the places of the other bodies at any given time, and thence to deduce their proper motions. The positions of the fixed stars are found from observation, by knowing their right ascensions and declinations (41); and these are found by means of the transit telescope and astronomical quadrant, as explained in my *Treatise on Practical Astronomy*; and then, by computation, their latitudes and longitudes may be found.

(111.) As the earth revolves uniformly about it's axis, the apparent motion of all the heavenly bodies, arising from this motion of the earth, must be uniform; and as this motion is parallel to the equator (76), the intervals of the times, in which any two stars pass over the meridian, must be in proportion to the arc of the equator intercepted between the two secondaries passing through them, because (13) this arc of the equator contains the same number of degrees as the arc of any small circle parallel to it, and comprehended between the same secondaries; and therefore, if one increase uniformly, the other must. Hence, the right ascension of stars passing the meridian at different times, will differ in proportion to the difference of the times of their passing, that is, if one star pass the meridian 1 hour before another, the difference of their right ascensions is 15°. Hence, if the clock be supposed to go uniformly, we have the

following rule: *As the interval of the times of the succeeding passages of any one fixed star over the meridian : the interval of the passages of any two stars :: 360° : their difference of right ascensions**. By the same method we may find the difference of right ascensions of the sun or moon, when they pass the meridian, and a star, and therefore if that of the star be known, that of the sun or moon will; which conclusion will be more exact, if we compare them with several stars, and take the mean.

(112.) Now to determine the right ascension of a fixed star, Mr. *Flamstead* proposed a method, by comparing the right ascension of the star with that of the sun when near the equinoxes, the sun having the same declination each time; and as this method has not been noticed by any writers, we shall give an explanation. Let *AGCKE* be the equator, *ABCWE* the ecliptic, *S* the place of the star, *Sm* a secondary to the equator, and let the sun be at *P*, near to *A*, when it is on the meridian, and take *CT*=*PA*, and draw *PL*, *TZ*, perpendicular to *AGC*, and *ZL* is parallel to *AC*, and the sun's declination is the same at *T* as at *P*. Observe the meridian altitude of the sun when at *P*, and also the time of the passage of it's




center over the meridian; observe also at what time the star passes over the meridian, and then (111) find

* A small correction must here be applied for the aberration of the star, in order to get the true difference of right ascensions, as will be explained; because there is a small difference between the true and apparent places.

the apparent difference $Lm$ of their right ascensions. When the sun approaches near to $T$, observe it's meridian altitude for several days, so that on one of them, at $t$, it may be greater, and on the next day, at $e$, it may be less than the meridian altitude at $P$, so that in the intermediate time it must have passed through $T$; and drawing $tb$, $es$, perpendicular to $AGCE$, observe, on these two days, the difference $bm$, $sm$ of the sun's right ascension and that of the star; draw also $sv$ parallel to $Zo$. Then, to find $Zb$, we may consider the variation both of the right ascension and declination, to be uniform for a small time, and consequently to be proportional to each other; hence, $vb$ (the change of meridian altitudes in one day) : $ob$ (the difference of the meridian altitudes at $t$ and $T$, or the difference of declination) :: $sb$ (the difference of $sm$, $bm$ found by observation) : $Zb$, which added to $bm$, or subtracted from it, according to the situation of $m$, gives $Zm$, to which add $Lm$, or take their difference, according to circumstances, and we get $ZL$, which subtracted from $AGC$, or $180^0$, half the remainder will be $AL$, the sun's right ascension at the first observation, to which add $Lm$, and we get the star's right ascension at the same time. Instead of finding $bZ$, we might have found $sZ$, by taking $TZ - es$ for the second term, and thence we should have got $Zm$. Thus we should get the right ascension of a star, upon supposition that the position of the equator had remained the same, and the apparent place of the star had not varied in the interval of the observations. But the intersection of the equator with the ecliptic has a retrograde motion, called the Precession of the Equinoxes; also, the inclination of the equator to the ecliptic is subject to a variation, called the Nutation; and from the aberration of the star, it's apparent place is continually changing; these must therefore be allowed for, by considering how much they have varied in the interval of the observations; but these are not subjects to be treated of in an elementary treatise.

Having thus determined the right ascension of one star, that of the rest may be found from it (111).

(113.) The practical method of finding the right ascension of a body from that of a fixed star, by a clock adjusted to *sidereal* time *, is this: Let the clock begin it's motion from $0h\ 0'.\ 0''$ at the instant the first point of Aries is on the meridian; then, when any star comes to the meridian, the clock will show the apparent right ascension of the star, the right ascension being estimated in the time, at the rate of 15° for an hour, provided the clock is subject to no error, because it will then show, at any time, how far the first point of Aries is from the meridian. But as the clock is necessarily liable to err, we must be able, at any time, to ascertain what it's error is, that is, what is the difference between the right ascension shown by the clock, and the right ascension of that point of the equator which is at that time on the meridian. To do this, we must, when a star, whose apparent right ascension is known, passes the meridian, compare it's apparent right ascension with the right ascension shown by the clock, and the difference will show the error of the clock. For instance, let the apparent right ascension of *Aldebaran* be $4h.\ 23'.\ 50''$ at the time when it's transit over the meridian is observed by the clock, and suppose the time shown by the clock to be $4h.\ 23'.\ 52''$, then there is an error of $2''$ in the clock, it giving the right ascension of the star $2''$ more than it ought. If the clock be compared with several stars, and the mean error taken, we shall have, more accurately, the error at the mean time of all the observations. These observations being repeated every day, we shall get the rate of the clock's going, that is, how fast it gains or loses. The error of the clock, and the rate of it's

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* A clock is said to be adjusted to *sidereal* time, when it is adjusted to go 24 hours from the time a fixed star leaves the meridian till it returns to it, or it is the time of a revolution of the earth about it's axis.

going, being thus ascertained, if the time of the true transit of any body be observed, and the error of the clock at the time be applied, we shall have the right ascension of the body. This is the method by which the right ascension of the sun, moon, and planets are regularly found in observatories.

(114.) The right ascension of the heavenly bodies being thus ascertained, the next thing to be explained is, the method of finding their declinations. Take the apparent altitude of the body, when it passes the meridian, by an astronomical quadrant, as explained in my *Treatise on Practical Astronomy*; correct it for parallax and refraction (Chap. VI. and VII.) and you get the true meridian altitude, *Ht*, or *He*, (Fig. page 25), the difference between which and the altitude *HE* of the equator (which, by Art. 87, is equal to the complement of the latitude previously determined) is the declination *Et*, or *Ee*, required.

Ex. On April 27, 1774, the zenith distance of the moon's lower limb, when it passed the meridian at Greenwich, was 68°. 19′. 37″,3; it's parallax in altitude was 56′. 19″,2, allowing for the spheroidical figure of the earth; the barometer stood at 29, 58, and the thermometer at 49; to find the declination.

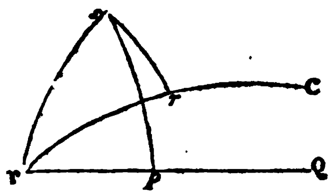
| | |
|---|---|
| Observed zenith distance of L.L. | 68°. 19′. 37″3 |
| Refr. cor. for bar. and ther. - | + 2. 23 |
| | 68. 22. 00,3 |
| Parallax - - - - - - - - - | − 56. 19,2 |
| True zenith distance of L.L. - | 67. 25. 41,1 |
| Semidiameter - - - - - - - - | − 16. 35 |
| True zenith distance of the center | 67. 9. 6,1 |
| Latitude - - - - - - - - - - | 51. 28. 40 |
| Declination *south* - - - - - - | 15. 40. 26,1 |

The *horizontal* parallax and semidiameter may be taken from the Nautical Almanack; and the parallax

in altitude may be found, as will be explained when we come to treat of the parallax.

*Given the Right Ascension and Declination of an Heavenly Body, and the Obliquity of the Ecliptic, to find the Latitude and Longitude.*

(115.) Let $s$ be the body, $\Upsilon C$ the ecliptic, $\Upsilon Q$ the equator, $sr$, $sp$ perpendicular to $\Upsilon C$, $\Upsilon Q$. Then (Trig. Art. 212) tan. $sp$ : rad. :: sin $\Upsilon p$ : cot. $s\Upsilon p$. Hence, $s\Upsilon p + Q\Upsilon C = s\Upsilon r$. Also,




cos. $s\Upsilon p$ : rad. :: tan. $p\Upsilon$ : tan. $s\Upsilon$ (Trig. Art. 219)
rad. : cos. $s\Upsilon r$ :: tan. $s\Upsilon$ : tan. $r\Upsilon$ (Trig. Art. 219)

∴ cos. $s\Upsilon p$ : cos. $s\Upsilon r$ :: tan. $p\Upsilon$ : tan. $r\Upsilon$ = $\dfrac{\text{cos. } s\Upsilon r \times \text{tan. } p\Upsilon}{\text{cos. } s\Upsilon p}$ the tangent of the *longitude*; and the logarithmic operation is,

*ar.co. log.cos.* $s\Upsilon p$ + *log.cos.* $s\Upsilon r$ + *log.tan.* $p\Upsilon$ − 10, = *log.tan.* $r\Upsilon$.

Also, (Trig Art. 210) rad. : sin. $r\Upsilon$ :: tan. $r\Upsilon s$ : tan. $sr$ the tangent of *latitude*; and the logarithmic operation is,

*log. sin.* $r\Upsilon$ + *log. tan.* $r\Upsilon s$ − 10, = *log. tan.* $sr$.

In this manner, the right ascensions and declinations of the fixed stars being found from observation, their latitudes and longitudes may be computed, and their places become determined (41); hence, a catalogue of the fixed stars may be made for any time.

If the latitude and longitude be given, the right ascension and declination may be found in the same manner; considering $\Upsilon C$ the equator, and $\Upsilon Q$ the ecliptic.

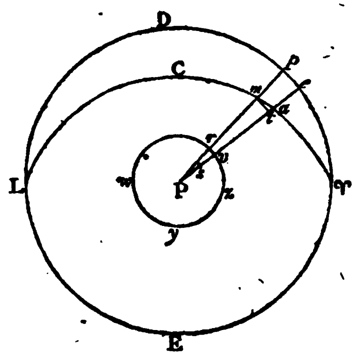
## CHAP. IV.

### ON THE EQUATION OF TIME.

(116.) HAVING explained, in the last Chapter, the practical methods of determining the place of any body in the heavens, we come next to the consideration of another circumstance not less important, that is, the irregularity of time as measured by the sun. The best measure of time which we have, is a clock regulated by the vibration of a pendulum. But with whatever accuracy a clock may be made, it must be subject to go irregularly, partly from the imperfection of the workmanship, and partly from the expansion and contraction of the materials by heat and cold, by which the length of the pendulum, and consequently the time of vibration, will vary. As no clock, therefore, can be depended upon for keeping time accurately, it is necessary that we should be able to ascertain, at any time, how much it is too fast or too slow, and at what rate it gains or loses. For this purpose, it must be compared with some motion which is uniform, or of which, if it be not uniform, you can ascertain the variation. The motions of the heavenly bodies have therefore been considered as most proper for this purpose. Now the earth revolving uniformly about it's axis, the apparent diurnal motion of the fixed stars about the axis must be uniform. If a clock, therefore, be adjusted to go 24 hours from the passage of any fixed star over the meridian till it returns to it again, it's rate of going may be at any time determined by comparing it with the transit of any fixed star, and observing whether the interval continues to be 24 hours; if not, the difference shows

how much it gains or loses in that time. A clock adjusted to go 24 hours in this interval, is said to be adjusted to *sidereal* time. But if we compare a clock with the sun, and adjust it to go 24 hours from the time the sun leaves the meridian on any day, till he returns to it the next day, which is a *true* solar day, the clock will not, even if it go uniformly, continue to agree with the sun, that is, it will not show 12 when the sun comes to the meridian.

(117.) For let $P$ be the pole of the earth, $vwyz$ it's equator, and suppose the earth to revolve about it's axis in the order of the letters $v\,w\,y\,z$; let ♈ $DLE$ be the celestial equator, and ♈ $CL$ the ecliptic, in which




the sun moves according to that direction. Let the sun be at $a$ when it is upon the meridian of any place on any one day, and $m$ the place when it is on the meridian the next day, and draw $Pvae$, $Prmp$, secondaries to the equator, and let the spectator be at $s$ on the meridian $Pv$, with the sun at $a$ on his meridian. Then when the earth has made one revolution about it's axis, $Psv$ is come again into the same position; but the sun having moved forward, the earth must continue to revolve, in order to bring the meridian $Psv$ into the position $Prm$, so that the sun at $m$ may be again in the spectator's meridian. Now the angle $vPr$ is measured by the arc $ep$, which is the increase of the sun's right ascension in a *true* solar day, the right ascension being measured upon the equator

$\Upsilon DLE$ (41); hence, the length of a *true* solar *day, is equal to the time of the earth's rotation about it's axis + the time of it's describing an angle equal to the increase of the sun's right ascension on a true solar day.* Now if the sun moved *uniformly*, and *in the equator* $\Upsilon DLE$, this increase, *ep*, would be always the same in the same time, and therefore the solar days would be always equal; but the sun moves in the ecliptic $\Upsilon CL$, and therefore, *if* it's motion were *uniform*, equal arcs (*am*) upon the ecliptic would not give equal arcs (*ep*) upon the equator *. But the motion of the sun is *not* uniform, and therefore *am*, described in any given time, is subject to a variation, which must, on this account also, necessarily make *ep* variable. Hence, the increase, *ep*, of the sun's right ascension in a day, varies from two causes, that is, from the ecliptic not coinciding with the equator, and from the unequal motion of the sun in the ecliptic; therefore the length of a true solar day is subject to a continual variation; consequently a clock, adjusted to go 24 hours for any one true solar day, will not continue to shew 12 when the suncmes to the meridian, because the intervals bythe clock will continue equal (the clock being supposed neither to gain nor lose), but the interval of the sun's passage over the meridian will vary.

(118.) As the sun moves through 360° of right ascension in $365\frac{1}{4}$ days very nearly, therefore $365\frac{1}{4}$ days : 1 day :: 360°. : 59′. 8″,2 the increase of right

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* For draw *mt* parallel to *ep*, and suppose *ma* to be indefinitely small; then by plain trigonometry,

*ma* : *mt* :: rad. : sin. *mat*, or $\Upsilon$ *ae* (Trig. Art. 125.)

*mt* : *ep* :: cos. *ae* : rad. (Art. 13)

$\therefore$ *ma* : *ep* :: cos. *ae* : sin. $\Upsilon$ *ae* :: (because sin. $\Upsilon ae = \frac{\text{cos. } a\Upsilon e \times \text{rad.}}{\text{cos. } ae}$ Trig. Art. 212) $\overline{\text{cos. } ae}^2$ : cos. $a\Upsilon e \times$ radius : hence, the ratio of *ma* to *ep* is variable; if therefore the sun's motion *ma* were uniform the corresponding increase *ep* of right ascension would not be uniform.

ascension in one day, *if* the increase were uniform, in which case the solar days would be equal, and these days are called *mean* solar days. If therefore a clock be adjusted to go 24 hours in a *mean* solar day, it cannot continue to coincide with the sun, that is, to show 12 when the sun is on the meridian; but the sun will pass the meridian, sometimes *before* 12 and sometimes *after*. This difference is called the *Equation of Time*. A clock thus adjusted, is said to be adjusted to *mean solar time**. The time shown by the clock is called *true* or *mean* time, and that shown by the sun is called *apparent* time.

(119.) A clock adjusted to go 24 hours in a *mean* solar day, would coincide with an imaginary star moving uniformly in the equator with the sun's mean motion 59′. 8″,2 in right ascension, if the star were to set off from any given meridian when the clock shows 12; that is, the clock would always show 12 when the star came to the meridian, because the interval of the passages of this star over the meridian would be a *mean* solar day. This star, therefore, if we reckon it's motion from the meridian, in time, at the rate of 1 hour for 15°, would always coincide with the clock; that is, when the clock shows 1 hour, the star's motion would be 1 hour in right ascension, reckoned in time at the rate of 15° for an hour; when the clock shows 2 hours, the star's motion would be 2 hours; and so on. Hence, this star may be substituted instead of the clock; therefore, when the sun passes the given meridian, the difference between it's right ascension and that of the star, converted into time, is the difference between the time when the sun is on the me-

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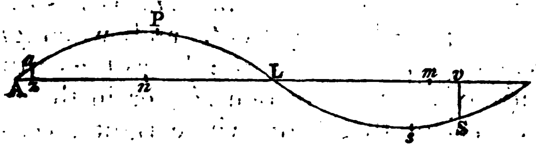
* As the earth describes an angle of 360°. 59′. 8″,2 about it's axis in a *mean* solar day of 24 hours, and an angle of 360° in a *sidereal* day, therefore 360°. 59′. 8″,2 : 360°. :: 24*h*. : 23*h*. 56′. 4,″098 the length of a sidereal day in mean solar time, or the time from the passage of a fixed star over the meridian, till it returns to it again.

ridian and 12 o'clock, or the equation of time; because the given meridian passes through the star at 12 o'clock, and it's motion, in respect to the star, is at the rate of 15° in an hour (121).

(120.) Now, to compute the equation of time, let *APLS* be the ecliptic, *ALv* the equator, *A* the first




point of aries, *P* the sun's apogee, *S* any place of the sun; draw *Sv* perpendicular to the equator, and take $An=AP$. When the sun departs from *P*, let the imaginary star set out from *n* with the sun's mean motion in right ascension, or in longitude, or at the rate of 59′. 8″,2 in a day, and when *n* passes the meridian, let the clock be adjusted to 12, as described in the last article; these are the corresponding positions of the clock and sun, as assumed by astronomers. Take $nm=Ps$, and when the star comes to *m*, the place of the sun, if it moved uniformly with it's mean motion, would be at *s*, but at that time let *S* be the place of the sun, and let the sun at *S*, and consequently *v*, be on the meridian; then as *m* is the place of the imaginary star at that instant, *mv* is the equation of time. Let *a* be the *mean* equinox*, and draw *az* perpendicular to *AL*; then *z* on the equator would have coincided with *a*, if the equinox had moved uniformly; therefore we must reckon the *mean* right ascension from *z*. Now $mv=Av-Am$; but $Am=Az+zm$ $=Aa\times\cos.\ aAz+zm=$ (because cos. *aAz* (23°. 28′) $=\frac{11}{12}$ very nearly) $\frac{11}{12}Aa+zm$; hence, $mv=Av-zm$

* The equinox has a retrograde motion, and that motion is not uniform; we here therefore suppose *a* to be the point where the equinox would have been, if it moved uniformly with it's *mean* velocity.

$-\frac{1}{4}\frac{1}{4}Aa$; but $Av$ is the sun's true right ascension, $xm$ is the mean right ascension, or mean longitude, and $\frac{1}{4}\frac{1}{4}Aa$ ($Ax$) is the equation of the equinoxes in right ascension; hence, the equation of time is equal to *the difference of the sun's true right ascension, and it's mean longitude corrected by the equation of the equinoxes in right ascension.* When $Am$ is less than $Av$, *mean* time precedes *apparent*, and when *greater*, *apparent* time precedes *mean*; for as the earth turns about it's axis in the direction $Av$, or in the order of right ascension, that body whose right ascension is least must come upon the meridian first; that is, when the sun's true right ascension is *greater* than it's mean longitude corrected as above, we must *add* the equation of time to the apparent, to get the mean time; and when it is *less*, we must *subtract*. To convert mean time into apparent, we must *subtract* in the *former* case, and *add* in the *latter*. This rule for computing the equation of time, was first given by Dr. *Maskelyne* in the *Phil. Trans.* 1764.

(121.) As a meridian of the earth, when it leaves $m$, returns to it again in 24 hours, it may be considered, when it leaves that point, as approaching a point at that time 360° from it, and at which time it arrives in 24 hours. Hence, the relative velocity with which a meridian accedes to or recedes from $m$, is at the rate of 15° in an hour. Therefore, when the meridian passes through $v$, the arc $vm$, reduced into time at the rate of 15° in an hour, gives the equation of time at that instant. Hence, the equation of time is computed for the instant of *apparent* noon, or when the sun is on the meridian. Now the time of apparent noon in mean solar time, for which we compute, can only be known by knowing the equation of time. To compute, therefore, the equation on any day, you must assume the equation the same as on that day four years before, from which it will differ but very little, and it will give the time of apparent noon, sufficiently accurate for the purpose of computing the

equation. If you do not know the equation four years before, compute the equation for noon mean time, and that will give apparent noon accurately enough.

Ex. To find the equation of time on July 1, 1792, for the meridian of Greenwich, by *Mayer's* Tables.

The equation on July 1, 1788, was, by the *Nautical Almanac*, 3′. 28″, to be added to apparent noon, to give the corresponding mean time; hence, for July 1, 1792, at 0*h*. 3′. 2″, compute the true longitude *.

* The reason of this operation will appear to those who understand the method of computing the place of the sun from the solar tables. The explanation of such matters comes not within the plan of this work. See my *Complete System of Astronomy*.

| | Mean Long. ☉ | Long. ☉'s Apog. | N°. 1. | N°. 2. | N°. 3. | N°. 4. |
|---|---|---|---|---|---|---|
| Epoch for 1792. | 9ˢ. 10°. 50′. 0″,7 | 3ˢ. 9°. 23′. 46″ | 241 | 227 | 123 | 478 |
| Mean Mot. July 1, | 5. 29. 23. 16,2 | 33 | 163 | 456 | 312 | 27 |
| 3′ | 7,4 | | | | | |
| 28″ | 1,1 | | | | | |
| Mean Longitude | 3. 10. 13. 25,4 | 3. 9. 24. 19 | 404 | 683 | 435 | 505 |
| Equat. of Center | − 1. 37,1 | 3. 10. 13. 25,4 | | | | |
| Equat. ☽ I. | + 4,5 | | | | | |
| ♃ II. | − 4,7 | 49. 6,4 | Mean Anomaly. | | | |
| ♀ III. | + 3,65 | | | | | |
| ☊ IV. | − 0,6 | | | | | |
| True Longitude | 3. 10. 11. 51,15 | | | | | |

With this true longitude, and obliquity 23°. 27′. 48″,4 of the ecliptic, the true right ascension of the

sun is found to be $3^s.\ 11^\circ.\ 5'.\ 41'',25$; also, the equation of the equinoxes in longitude $= -0'',6$; hence,

| | |
|---|---|
| The mean longitude - - | $3^s.\ 10^\circ.\ 13'.\ 25'',4$ |
| $+\frac{11}{12}$ of $-0'',6$ - - - - - - - | $-\ 0,55$ |
| Mean longitude corrected | 3. 10. 13. 24,85 |
| True right ascension - - | 3. 11. 5. 41,25 |
| Equation - - - - - - | 52. 16,4 |

Which converted into time, gives $3'.\ 29'',1$ for the true equation of time; which must be added to apparent, to give the true time, because the true right ascension is greater than the mean longitude.

(122.) The sun's apogee, $P$, has a progressive motion, and the equinoctial points, $A$, $L$, have a regressive motion; the inclination also of the equator to the ecliptic is subject to a constant variation. Hence, the same Table of the equation of time cannot continue to serve for the same degree of the sun's longitude. Also, the sun's longitude at noon at the same place is different for the same days on different years, and it is for apparent noon that the equation is computed. For these reasons, the equation of time must be computed a-new for every year.

(123.) The two inequalities are sometimes separately considered, thus: *First*, that arising from the obliquity of the ecliptic. Let the sun and the imaginary star set off together from $L$, and let us now assume $LS = Lm$; and let each move uniformly with the mean velocity, and then they will come to $S$ and $m$ together. Now when $LS$ is greater than $90^\circ$, the hypothenuse, $LS$, is less than the base, $Lv$ (Trig. Art. 197); therefore $Lm$ is less than $Lv$ (the case represented by the Figure); the star therefore, being left behind, comes upon the meridian first, and consequently true time precedes apparent. But when $LS$ is less than $90^\circ$, the hypothenuse, $LS$, is greater than the base, $Lv$; therefore $Lm$ is greater than $Lv$, and $m$ lies on the other side of $v$; therefore the sun comes upon the meridian

first; consequently apparent time precedes true. Hence, from equinox to tropic, apparent time precedes true; and from tropic to equinox, true time precedes apparent. *Secondly*, that arising from the unequal motion of the earth in it's orbit. Let us suppose the sun to move about the earth, instead of the earth about the sun, the effect here being just the same, and this supposition will render the explanation easier. Let the sun depart from the apogee, and let the imaginary star set off from thence at the same time, with the *mean* angular velocity of the sun. Now when the sun is at it's greatest distance, it's angular velocity it less than it's *mean* angular velocity. (Note to Art. 165), and consequently less than the velocity of the star; the star therefore getting forwarder than the sun, the sun comes upon the meridian first, as shown in Art. 120, and therefore apparent time precedes true; and this will continue till the sun comes to it's least distance, where, having performed half it's revolution, and the star also having performed half it's revolution, the sun and star will coincide, (see Art. 168). Hence, from apogee to perigee, apparent time precedes true. Now the sun and star departing together from perigee, the sun's velocity is greater than that of the star; the star therefore being left behind, comes upon the meridian first, and true time precedes apparent; and this will continue till the sun comes to the apogee, where they again coincide. Hence, from perigee to apogee, true time precedes apparent.

(124.) Whenever the time is computed from the sun's altitude, that time must be *apparent* time, because we compute it from the time when the sun comes to the meridian, which is noon, or 12 o'clock apparent time, and will differ from the time shown by a well-regulated watch or clock, by the equation of time. A clock or watch may therefore be regulated by a good dial; for if you apply the equation, as before directed, to the apparent time shown by the dial,

it will give the *mean* time, or that which the clock or watch ought to show.

(125.) The equation of time was known to, and made use of by, *Ptolemy*. *Tycho* employed only one part, that which arises from the unequal motion of the sun in the ecliptic; but *Kepler* made use of both parts. He further suspected, that there was a third cause of the inequality of solar days, arising from the unequal motion of the earth about it's axis. But the equation of time, as now computed, was not generally adopted till 1672, when *Flamstead* published a dissertation upon it, at the end of the works of *Horrox*.

# CHAP. V.

## ON THE LENGTH OF THE YEAR, THE PRECESSION OF THE EQUINOXES FROM OBSERVATION, AND OBLIQUITY OF THE ECLIPTIC.

(126.) FROM comparing the sun's right ascension every day with that of the fixed stars lying to the east and west, the sun is found constantly to recede from those on the west, and approach to those on the east; hence, it's apparent annual motion is found to be from west to east; and the interval of time from it's leaving any fixed star till it returns to it again, is called a *sidereal* year, being the time in which the sun completes it's revolution amongst the fixed stars, or in the ecliptic. But the sun, after it leaves either of the equinoctial points, returns to it again in a less time than it returns to the same fixed star, and this interval is called a *solar* or *tropical* year, because the time from it's leaving one equinox till it returns to it, is the same as from one tropic till it comes to the same again. This is the year on which the return of the season depends.

### *On the Sidereal Year.*

(127.) To find the length of a *sidereal* year, On any day when the sun is at $Z$ on the meridian, (Fig. page 42), take the difference, $Zm$, between the sun's right ascension when it passes the meridian, and that of a fixed star, $S$; and when the sun returns to the same part of the heavens the next year, compare it's right ascension with that of the same star for two days, one when their difference, $bm$, of right ascensions is

less, and the other when the difference, *sm*, is greater than the difference, *Zm*, before observed; then *bs* is the increase of the sun's right ascension in the time, *t*; and as the increase of right ascension may be considered as uniform for a small time, we have *bs* : *bZ* :: *t* : the time, *T*, in which the right ascension is increased from *b* to *Z*; this time, *T*, therefore, added to the time of the observed right ascension at *b*, gives the time when the sun is at the same distance, *Zm*, in right ascension from the star, which it was when observed at *Z* the year before; the interval of these times is therefore the length of a sidereal year. The best time for these observations is about March 25, June 20, September 17, December 20, the sun's motion in right ascension being then uniform. Instead of observing the difference of the right ascensions, you may observe that of their longitudes.

If, instead of repeating the second observations the year after, there be an interval of several years, and you divide the observed interval of time when the difference of their right ascensions was found to be equal, by the number of years, you will have the length of a sidereal year more exactly.

Ex. On April 1, 1669, at 0*h*. 3′. 47″, mean solar time, M. *Picard* observed the difference between the sun's longitude and that of *Procyon* to be 3ˢ. 8°. 59′. 36″, which is the most ancient observation of this kind, the accuracy of which can be depended upon; see *Hist. Celeste, par M. le Monnier*, page 37. And on April 2, 1745, M. *de la Caille* found, by taking their difference of longitudes on the 2d and 3d, that at 11*h*. 10′. 45″, mean solar time, the difference of their longitudes was the same as at the first observation. Now as the sun's revolution was known to be nearly 365 days, it is manifest that it had made seventy-six complete revolutions, in respect to the same fixed star, in the space of 76 years 1*d*. 11*h*. 6′. 58″. Now in these 76 years, there were 58 of 365 days and 18 bissextiles of 366 days; that interval therefore contains

27759*d.* 11*h.* 6′. 58″; which being divided by 76, the quotient is 365*d.* 6*h.* 8′. 47″. the length of a sidereal year. From the most accurate observations, the length of a sidereal year is found to be 365*d.* 6*h.* 6′. 11″,5.

## *On the Tropical Year.*

(128.) Observe the meridian altitude, $a$, of the sun on the day nearest to the equinox; then the next year take it's meridian altitude on two following days, one when it's altitude, $m$, is less than $a$, and the next when it's altitude, $n$, is greater than $a$, then $n-m$ is the increase of the sun's declination in 24 hours; also, when the declination has increased by the quantity $a-m$, from the time when the meridian altitude $m$, was observed, the declination will then become $a$; and as we may consider the increase of declination to be uniform for a day, we have $n-m : a-m ::$ 24 hours : the interval from the time when the sun was on the meridian on the first of the two days, till the sun has the same declination $a$, as at the observation the year before; and this time, added to the time when the sun's altitude $m$ was observed, gives the time when the sun's place in the ecliptic had the same situation in respect to the equinoctial points, which it had at the time of the observation the preceding year; and the interval of these times is the length of a tropical year.

If instead of repeating the second observation the next year, there be an interval of several years, and you divide the interval between the times when the declination was found to be the same, by the number of years, you will get the tropical year more exactly.

Ex. M. *Cassini* informs us, that on March 20, 1672, his father observed the meridian altitude of the sun's upper limb at the Royal Obervatory at Paris, to be 41°. 43′; and on March 20, 1716, he himself observed the meridian altitude of the upper limb to be 41°. 27′. 10″; and on the 21st to be 41°. 51′: therefore the difference of the two latter altitudes was 23′. 50″, and

of the two former 15′, 50″; hence, 23′. 50′.: 15′. 50″ :: 24 hours: 15*h*. 56′. 39″; therefore, on March 20, 1716, at 15*h*. 56′. 39″, the sun's declination was the same as on March 20, 1672. Now the interval between these two observations was 44 years, of which 34 consisted of 365 days each, and 10 of 366; therefore the interval in days was 16070; hence, the whole interval between the equal declinations was 16070 days 15*h*. 56′. 39″, which divided by 44, gives 365*d*. 5*h*. 49′. 0″. 53‴, the length of a tropical year from these observations. From the best observations, the length of a tropical year is found to be 365*d*. 5*h*. 48′. 48″.

### *To find the Precession of the Equinoxes from Observation.*

(129.) The sun returning to the equinox every year before it returns to the same point in the heavens, shows that the equinoctial points have a retrograde motion, and this arises from the motion of the equator, which is caused by the attraction of the sun and moon upon the earth, in consequence of it's spheroidical figure. The effect of this is, that the longitude of the stars must constantly increase; and by comparing the longitude of the same stars at different times, the motion of the equinoctial points, or the precession of the equinoxes, may be found.

(130.) *Hipparchus* was the first person who observed this motion, by comparing his own observations with those which *Timocharis* made 155 years before. From this he judged the motion to be one degree in about 100 years; but he doubted whether the observations of *Timocharis* were accurate enough to deduce any conclusion to be depended upon. In the year 128 before J. C. he found the longitude of *Virgin's Spike* to be $5^s. 24°$; and in the year 1750 its longitude was found to be $6^s. 20°. 1′$, the difference of

which is 26°. 21′. In the same year he found the longitude of the *Lyon's Heart* to be 3ˢ. 29°. 50′; and in 1750 it was 4ˢ. 26°. 21′, the difference of which is 26°. 31′. The mean of these two gives 26°. 26′ for the increase of longitude in 1878 years, or 50″. 40‴. in a year for the precession. By comparing the observations of *Albategnius*, in the year 878, with those made in 1738, the precession appears to be 51″. 9‴. From a comparison of 15 observations of *Tycho*, with as many made by M. *de la Caille*, the precession is found to be 50″. 20‴. But M. *de la Lande*, from the observations of M. *de la Caille*, compared with those in *Flamstead's* Catalogue, determines the secular precession to be 1°. 23′. 45″, or 50″, 25 in a year.

(131.) The precession being given, and also the length of a tropical year, the length of a sidereal year may be found by this proportion; 360° − 50″,25 : 360°. :: 365*d.* 5*h.* 48′. 48″ : 365*d.* 6*h.* 9′. 11″½ the length of the *sidereal* year.

## *On the Anomalistic Year.*

(132.) The year, called the *anomalistic* year, is sometimes used by astronomers, and is the time from the sun's leaving its apogee, till it returns to it. Now the progressive motion of the apogee in a year is 11″,75, and hence, the anomalistic must be longer than the sidereal year, by the time the sun takes in moving over 11″,75 of longitude at it's apogee; but when the sun is in it's apogee, it's motion in longitude is 58′. 13″ in 24 hours; hence, 58′. 13″ : 11″,75 :: 24 hours : 4′. 50″¾, which added to 365*d.* 6*h.* 9′. 11½″, gives 365*d.* 6*d.* 14′. 2″¼ the length of the anomalistic year. M. *de la Lande* determined this motion of the apogee, from the observations of M. *de la Hire* and those of Dr. *Maskelyne*. *Cassini* made it the same.

## *On the Obliquity of the Ecliptic.*

(133.) The method used by astronomers to determine the obliquity of the ecliptic, is that explained in Art. 86, by taking half the difference of the greatest and least meridian altitudes of the sun. The following is the obliquity, as determined by the different astronomers.

| | | | |
|---|---|---|---|
| *Eratosthenes* 230 years before *J. C.* | 23°. | 51′. | 20″ |
| *Hipparchus* 140 years before *J. C.* | 23. | 51. | 20 |
| *Ptolemy* 140 years after *J. C.* - | 23. | 51. | 10 |
| *Pappus* in the year 390 - - - | 23. | 30. | 0 |
| *Albategnius* in 880 - - - - | 23. | 35. | 40 |
| *Arzachel* in 1070 - - - - - - | 23. | 34. | 0 |
| *Prophatius* in 1300 - - - - - | 23. | 32. | 0 |
| *Regiomontanus* in 1460 - - - | 23. | 30. | 0 |
| *Waltherus* in 1490 - - - - | 23. | 29. | 47 |
| *Copernicus* in 1500 - - - - - | 23. | 28. | 24 |
| *Tycho* in 1587 - - - - - - | 23. | 29. | 30 |
| *Cassini* (the Father) in 1656 - | 23. | 29. | 2 |
| *Cassini* (the Son) in 1672 - - | 23. | 28. | 54 |
| *Flamstead* in 1690 - - - - - | 23. | 28. | 48 |
| *De la Caille* in 1750 - - - - | 23. | 28. | 19 |
| Dr. *Bradley* in 1750 - - - | 23. | 28. | 18 |
| *Mayer* in 1750 - - - - - - | 23. | 28. | 18 |
| Dr. *Maskelyne* in 1769 - - - | 23. | 28. | 8,5 |
| M. *de la Lande* in 1768 - - - | 23. | 28. | 0 |

The observations of *Albategnius*, an Arabian, are here corrected for refraction. Those of *Waltherus*, M. *de Caille* computed. The obliquity by *Tycho* is here put down as correctly computed from his observations. Also, the obliquity, as determined by *Flamstead*, is corrected for the nutation of the earth's axis. These corrections M. *de la Lande* applied.

(134.) It is manifest, from the above observations, that the obliquity of the ecliptic continually decreases;

and the irregularity which here appears in the diminution, we may ascribe to the inaccuracy of the observations, as we know that they are subject to greater errors than the irregularity of this variation. If we compare the first and last observations, they give a diminution of 70″ in 100 years. If we compare the last with that of *Tycho*, it gives 45″. The last, compared with that of *Flamstead*, gives 50″. If we compare that of Dr. *Maskelyne* with Dr. *Bradley's* and *Mayer's*, it gives 50″. The comparison of Dr. *Maskelyne's* determination, with that of M. *de la Lande*, which he took as the mean of several results, gives 50″. We may therefore state the secular diminution of the obliquity of the ecliptic, at this time, to be 50″, as determined from the most accurate observations. This result agrees very well with that deduced from theory.

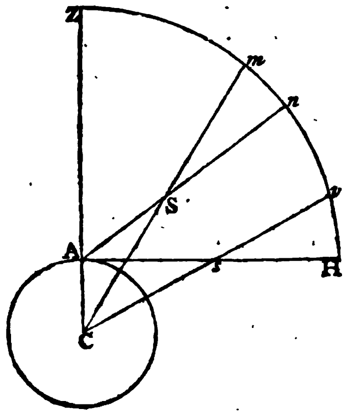
# Chap. VI.

## ON PARALLAX.

(135.) The center of the earth describes that circle in the heavens which is called the ecliptic; but as the same object would appear in different positions in respect to this circle, when seen from the center and surface, astronomers always reduce their observations to what they would have been, if they had been made at the center of the earth, in consequence of which, the places of the heavenly bodies are computed as seen from the ecliptic, and it becomes a fixed plane for that purpose, on whatever part of the earth's surface the observations are made.

(136.) Let $C$ be the center of the earth, $A$ the place of the spectator on it's surface, $S$ any object, $ZH$ the sphere of the fixed stars, to which the places of all the bodies in our system are referred; $Z$ the zenith, $H$ the horizon; draw $CSm$, $ASn$, and $m$ is the place of $S$ seen from the center, and $n$ from the surface. Now the plane $SAC$ passing through the center of the earth, must be perpendicular to it's surface, and consequently it will pass through the zenith $Z$; and the points, $m$, $n$, lying in the same plane, the arc of parallax, $mn$, must lie in a circle perpendicular to the horizon; and hence, the azimuth is not affected, if the earth be a sphere. Now the parallax, $mn$, is measured by the angle $mSn$, or $ASC$; and (Trig. Art. 128) $CS : CA :: \sin. SAC$, or $\sin. SAZ, : \sin. ASC$ the parallax $= \frac{CA \times \sin. SAZ}{CS}$. As $CA$ is constant, supposing the earth to be a sphere, the sine of the parallax varies as the sine of the apparent zenith distance directly, and

the distance of the body from the center of the earth inversely. Hence, a body in the zenith has no parallax, and at *s* in the horizon it is the greatest. And

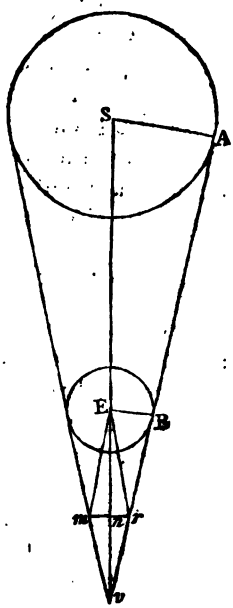



if the object be at an indefinitely great distance, it has no parallax; hence, the apparent places of the fixed stars are not altered by it. As *n* is the apparent place, and *m* is called the true place, the parallax depresses an object in a vertical circle. For different altitudes of the same body, the parallax varies as the sine, *s*, of the apparent zenith distance; therefore, if $p=$ the horizontal parallax, and radius be unity, we have $1 : s :: p : ps$, the sine of the parallax. To ascertain, therefore, the parallax at all altitudes, we must first find it at some given altitude.

(137.) *First* method, for the *sun. Aristarchus* proposed to find the sun's parallax, by observing it's elongation from the moon at the instant it is dichotomized, at which time the angle at the moon is a right angle; therefore we may find the angle which the distance of the moon subtends at the sun; which, diminished in the ratio of the moon's distance from the earth's center to the radius of the earth, would give the sun's horizontal parallax. But a very small error in the time when the moon is dichotomized (and it is impossible to be very accurate in this), will make so very great an error in the sun's parallax, that nothing

can be deduced from it to be depended upon. *Vendelinus* determined the angle of elongation when the moon was dichotomized, to be 89°. 45′, from which the sun's parallax was found to be 15″. But *P. Riccioli* found it to be 28″ or 30″ from like observations.

(153.) *Second* method. *Hipparchus* proposed to find the sun's parallax from a lunar eclipse, by the following method. Let $S$ be the sun, $E$ the earth, $Ev$ the length of it's shadow, $mr$ the path of the moon in a central eclipse. Observe the length of this eclipse, and then, from knowing the periodic time of the moon, the angle $mEr$, and consequently the half of that angle, or $nEr$, will be known. Now the horizontal parallax, $ErB$, of the moon being known, we have the angle $Evr = ErB - nEr$; hence, we know $EAB$*




$= AES - Evr = AES - ErB + nEr$; that is, the sun's horizontal parallax = the apparent semidiameter of the

* Supply the line $AE$.

sun − the horizontal parallax of the moon + the semi-diameter of the earth's shadow where the moon passes through it. The objection to this method, is, the great difficulty of determining the angle $nEr$, with sufficient accuracy; for any error in that angle will make the same error in the sun's parallax, the other quantities remaining the same. By this method, *Ptolemy* made the sun's horizontal parallax 2′. 50″. *Tycho* made it 3′.

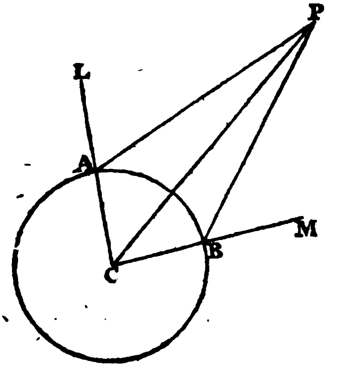
(139.) *Third* method, for the *moon*. Take the meridian altitudes of the moon, when it is at it's greatest north and south latitudes, and correct them for refraction; then the difference of the altitudes, thus corrected, would be equal to the sum of the two latitudes of the moon, if there were no parallax; consequently the difference between the sum of the two latitudes and the difference of the altitudes, will be the difference between the parallaxes at the two altitudes. Now, to find from thence the parallax itself, let $S$, $s$, be the sines of the greatest and least apparent zenith distances, $P$, $p$, the sines of the corresponding parallaxes; then as, when the distance is given, the parallax varies (136) as the sine of the zenith distance,

$$S : s :: P : p; \text{ hence, } S-s : s :: P-p : p = \frac{s \times \overline{P-p}}{S-s}$$

the parallax at the greatest altitude. This supposes that the moon is at the same distance in both cases; but as this will not necessarily happen, we must correct one of the observations, in order to reduce it to what it would have been, had the distance been the same, the parallax at the same altitude being inversely as the distance (136). If the observations be made in those places where the moon passes through the zenith of one of the observers, the difference between the sum of the two latitudes and the zenith distance at the other observation, will be the parallax at that altitude.

(140.) *Fourth* method. Let a body, $P$, be observed from two places, $A$, $B$, in the same meridian, then

the whole angle $APB$, is the sum of the two parallaxes of the two places. The parallax (136) $APC = hor.$ $par. \times$ sin. $PAL$, taking $APC$ for sin. $APC$,

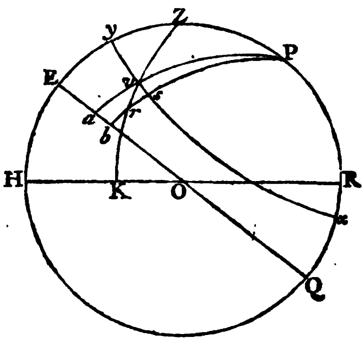



and the parallax $BPC = hor. par. \times \sin PBM$; hence, $hor. par. \times \overline{\sin PAL + \sin. PBM} = APB$, $\therefore$ $hor. par.$ $= APB$ divided by the sum of these two sines. If the two places be not in the same meridian, it does not signify, provided we know how much the altitude varies from the change of declination of the body in the interval of the passages over the meridians of the two observers.

Ex. On Oct. 5, 1751, M. *de la Caille*, at the Cape of Good Hope, observed *Mars* to be 1′. 25″,8 below the parallel of λ in Aquarius, and at 25° distance from the zenith. On the same day, at Stockholm, Mars was observed to be 1′. 57″,7 below the parallel of λ, and at 68°. 14′ zenith distance. Hence, the angle $APB$ is 31″,9, and the sines of the zenith distances being 0,4226 and 0,9287, the horizontal parallax was 23″,6. Hence, if the ratio of the distance of the earth from Mars to it's distance from the sun be found, we shall have the sun's horizontal parallax, the horizontal parallaxes being inversely as their distances from the earth (136).

(141.) *Fifth* method. Let $EQ$ be the equator, $P$ it's pole, $Z$ the zenith, $v$ the true place of the

body, and $r$ the apparent place, as depressed by parallax in the vertical circle $ZK$, and draw the secondaries,




$Pva$, $Prb$; then $ab$ is the parallax in right ascension, and $r\,s$ in declination. Now,

$vr : vs :: 1$ (rad,) : sin. $vrs$, or $ZvP$ (Trig. Art. 125)
$vs : ab ::$ cos. $va : 1$ (rad.) Art. 13.

$\therefore vr : ab ::$ cos. $va$ : sin. $ZvP$;

therefore $ab = \dfrac{vr \times \text{sin. } ZvP}{\text{cos. } va}$; but $vr = hor.\ par. \times$ sin. $vZ$ (136), and (Trig. Art. 221) sin. $vZ$ : sin. $ZP$ :: sin. $ZPv$ : sin. $ZvP = \dfrac{\text{sin.} ZP \times \text{sin.} ZPv}{\text{sin. } vZ}$; therefore, by substitution, $ab = \dfrac{hor.par. \times \text{sin.} ZP \times \text{sin.} ZPv}{\text{cos. } va}$.

Hence, for the same star, where the *hor. par.* is given, the parallax in right ascension varies as the sine of the hour angle. Also, the $hor.\ par. = \dfrac{ab \times \text{cos.} va}{\text{sin. } ZP \times \text{sin. } ZPv}$.

For the eastern hemisphere, the apparent place $b$, lies on the equator to the east of $a$, the true place, and therefore the right ascension is increased by parallax; but in the western hemisphere, $b$ lies to the west of $a$, and therefore the right ascension is diminished. Hence, if the right ascension be taken before and after the meridian, the whole change of parallax in right ascension between the two observations, is the sum ($s$) of the two parts before and after the meridian, and the

*hor. par.* $=\frac{s \times \cos. va}{\sin. ZP \times S}$, where $S$ = sum of the sines of the two hour angles. On the meridian there is no parallax in right ascension, for $ab=\frac{vr \times \sin. ZvP}{\cos. va}$, where the angle $ZvP$, and consequently it's sine, vanishes.

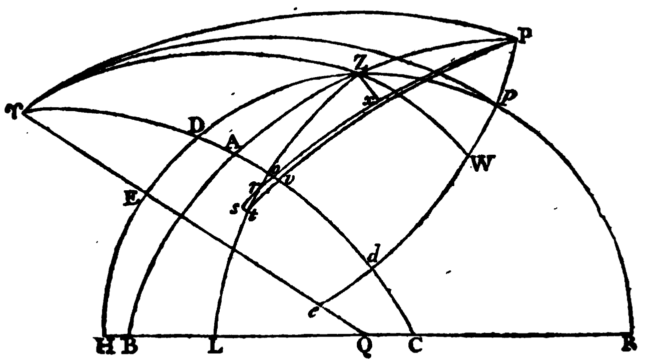
(142.) To apply this rule, observe the right ascension of the planet when it passes the meridian, compared with that of a fixed star, at which time there is no parallax in right ascension; about 6 hours after, take the difference of their right ascensions again, and observe how much the difference, $d$, between the apparent right ascensions of the planet and fixed star has changed in that time. Next, observe the right ascension of the planet for 3 or 4 days when it passes the meridian, in order to get it's true motion in right ascension; then, if it's motion in right ascension in the above interval of time, between the taking of the right ascensions of the fixed star and planet on and off the meridian, be equal to $d$, the planet has no parallax in right ascension; but if it be not equal to $d$, the difference is the parallax in right ascension; and hence, by the last article the horizontal parallax will be known. Or one observation may be made before the planet comes to the meridian, and one after, by which a greater difference will be obtained.

Ex. On Aug. 15, 1719, *Mars* was very near a star of the 5th magnitude in the eastern shoulder of Aquarius, and at 9*h*. 18′ in the evening, Mars followed the star in 10′. 17″, and on the 16th, at 4*h*. 21′ in the morning, it followed it in 10′. 1″, therefore, in that interval, the apparent right ascension of Mars had increased 16″ in time. But, according to observations made in the meridian for several days after, it appeared that Mars approached the star only 14″ in that time, from it's motion; therefore 2″ in time, or 30″ in motion, is the effect of parallax in the interval of the observations. Now the declination of Mars was 15°, the co-latitude 41°. 10′, and the two hour angles

$49°. 15'$, and $56°. 39'$; therefore the *hor. par.* $= \dfrac{30'' \times \cos. 15°}{\sin. 41°. 10' \times \sin. 49°. 15' + \sin. 56°. 39'} = 27\frac{1}{2}''$. But at that time, the distance of the earth from Mars was to it's distance from the sun, as 37 to 100, and therefore the sun's horizontal parallax comes out $10'',17$.

(143.) But, besides the effect of parallax in right ascension and declination, it is manifest that the latitude and longitude of the moon and planets must also be affected by it; and as the determination of this, in respect to the moon, is in many cases, particularly in solar eclipses, of great importance, we shall proceed to show how to compute it, supposing that we have given the latitude of the place, the time, and consequently the sun's right ascension, the moon's true latitude and longitude, with her horizontal parallax.

(144.) Let *HZR* be the meridian, ♈ *EQ* the equator, *p* it's pole; ♈ *C* the ecliptic, *P* it's pole; ♈




the first point of Aries, *HQR* the horizon, *Z* the zenith, *ZL* a secondary to the horizon passing through the true place *r*, and aparent place *t*, of the moon; draw *Pt*, *Pr*, which produce to *s*, drawing the small circle *ts*, parallel to *ov*; then *rs* is the parallax in *latitude*, and *ov* the parallax in *longitude*. Draw the great circles, ♈ *P*, *PZAB*, *Ppde*, and *ZW*, perpendicular to *pe*; then, as ♈ $P = 90°$, and ♈ $p = 90°$, ♈ must (4) be the pole of *Ppde*, and therefore $d$♈ $= 90°$; consequently *d* is one of the solstitial points, ♋ or ♑; also, draw *Zx* per-

pendicular to $Pr$, and join $Z♈$, $p♈$. Now $♈E$, or the angle $♈ pE$, or $Zp♈$, is the right ascension of the mid-heaven, which is known; $PZ=AB$ (because $AZ$ is the complement of both) the altitude of the highest point, $A$, of the ecliptic above the horizon, called the nonagesimal degree, and $♈A$, or the angle $♈PA$, is it's longitude; also, $Zp$ is the co-latitude of the place, and the angle $ZpW$ is the difference between the right ascension of the mid-heaven $♈pE$, and $♈e$. Now, in the right-angled triangle $ZpW$, (Trig. Art. 212) rad. $\times$ cos. $p=$tan. $pW \times$ cot. $pZ$; therefore,

$$log.\ tan.\ pW = 10, + log.\ cos.\ p - log.\ cot.\ pZ, \text{ (Trig. Art. 213)};$$

hence, $PW=pW \pm pP$, where the *upper* sign takes place when the right ascension of the mid-heaven is *less* than 180°, and the *lower* sign, when *greater*. Also, in the triangles $WZp$, $WZP$, we have (Trig. Art. 231) sin. $Wp$ : sin. $WP$ :: tan. $WPZ$ : tan. $Wpz$ :: (Trig. Art. 82) cot. $WpZ$ : cot. $WPZ$, or, tan. $AP♈$; therefore (Trig. Art. 49),

$$Log.tan.\ AP♈ = ar.co.\ lo.\ sin.\ Wp + log.\ sin.\ WP + lo.cot.\ WpZ - 10;$$

and as we know $♈o$, or $♈Po$, the true longitude of the moon, we know $APo$, or $ZPx$. Also, in the triangle $WPZ$, we have (Trig. Art. 219) cos. $WPZ$, or sin. $AP♈$, : rad. :: tan. $WP$ : tan. $ZP$; therefore,

$$Log.\ tan.\ ZP = 10, + log.\ tan.\ WP - log.\ sin.\ AP♈.$$

Again, in the triangle $ZPr$, we know $ZP$, $Pr$, and the angle, $P$, from which the angle $ZrP$, or $trs$, may be thus found. In the right-angled triangle $ZPx$, we know $ZP$, and the angle $P$; hence, (Trig. Art. 212) rad. $\times$ cos. $ZPx=$cot. $PZ \times$ tan. $Px$; therefore,

$$Log.\ tan.\ Px = 10, + cos.\ ZPx - log.\ cot.\ PZ;$$

therefore we know $rx$; hence, (Trig. Art. 231) sin. $rx$ : sin. $Px$ :: tan. $ZPx$ : tan. $Zrx$, or $trs$; therefore,

$$Log.\ tan.\ Zrx = ar.\ co.\ log.\ sin.\ rx + log.\ sin.\ Px + log.\ tan. ZPx - 10;$$

also, in the right-angled triangle $Zrx$, we have (Trig.

Art. 212) rad. × cos. $Zrx$ = cot. $Zr$ × tan. $rx$; therefore,

$$Log. cot. Zr = 10, + log. cos. Zrx - log. tan. rx.$$

With this *true* zenith distance $Zr$, find (136) the parallax, as if it were the *apparent* zenith distance, and it will give you the true parallax *nearly*; add this therefore to the *true* zenith distance, and you will get *nearly* the *apparent* zenith distance, to which compute again (136) the parallax, and thus you will get the true parallax, $rt$, extremely nearly; then, in the right-angled triangle $rst$, which may be considered as plane, we have (Trig. Art. 125) rad. : cos. $r$ :: $rt$ : $rs$, the parallax in *latitude*; therefore,

$$log. rs = log. rt + log. cos. r - 10, = log. par. latitude.$$

Also, rad. : sin. $r$ :: $rt$ : $ts$; therefore,

$$log. ts. = log. rt + log. sin. r - 10;$$

hence, (13) cos. $tv$ : rad. :: $ts$ : $ov$, the parallax in *longitude*; therefore,

$$log. ov = 10, + log. ts - log. cos. tv = log. par. longitude.$$

Ex. On January 1, 1771, at 9*h*. apparent time, in lat. 53° N. the moon's true longitude was 3ˢ. 18°. 27′. 35″, and latitude 4°. 5′. 30″ S. and it's horizontal parallax 61′. 9″; to find its parallax in latitude and longitude.

The sun's right ascension was 282°. 22′. 2″ by the Tables, and its distance from the meridian 135°; also, the right ascension ♈ $E$, of the mid-heaven, was 57°. 22′. 2″; hence, the whole operation for the solution of the triangles will stand thus:

| | | | | |
|---|---|---|---|---|
| Tri. $ZpW$ | $ZpW$ | = 32°. 37′. 58″ | 10, + cos. | 19.9253864 |
| | $Zp$ | = 37. 0. 0. | cot. | 10.1228856 |
| | $pW$ | = 32. 23. 57 | tan. | 9.8025008 |
| | $Pp$ | = 23. 28. 0 | | |
| | $PW$ | = 55. 51. 67 | | |

| | | | | |
|---|---|---|---|---|
| Tri. $WpZ, WPZ$ | $pW$ = 32°. 23'. 57" | - | ar. co. sin. | 0.2709855 |
| | $PW$ = 55. 51. 57 | - - - | sin. | 9.9178865 |
| | $ZpW$ = 32. 37. 58 | - - - | cot. | 10.1935941 |
| | $AP\gamma$ = 67. 29. 8 | - - - | tan. | 10.3824661 |
| | $oP\gamma$ = 108. 27. 35 | | | |
| | $oPA$ = 40. 58. 27 | | | |
| Tri. $WPZ$ | $WP$ = 55. 51. 57 | - | 10, + tan. | 20.1688210 |
| | $AP\gamma$ = 67. 29. 8 | - - | sin. | 9.9655700 |
| | $ZP$ = 57. 56. 36 | - - - | tan. | 10.2032510 |
| Tri. $ZPx$ | $ZPx$ = 40. 58. 27 | - | 10, + cos. | 19.8779500 |
| | $ZP$ = 57. 56. 36 | - - - | cot. | 9.7967445 |
| | $Px$ = 50. 19. 33 | - - - | tan. | 10.0812055 |
| | $Pr$ = 94. 5. 30 | | | |
| Tri. $Zpx, Zrx$ | $rx$ = 43. 45. 47 | - | ar. co. sin. | 0.1600743 |
| | $Px$ = 50. 19. 33 | - - - | sin. | 9.8863144 |
| | $ZPx$ = 40. 58. 27 | - - - | tan. | 9.9387676 |
| | $Zrx$ = 44. 1. 16 | - - - | tan. | 9.9851563 |
| Tri. $Zrx$ | $Zrx$ = 44. 1. 16 | - | 10, + cos. | 19.8567795 |
| | $rx$ = 43. 45. 57 | - - - | tan. | 9.9812846 |
| | $Zr$ = 53. 6. 10 | - - - | cot. | 9.8754949 |
| | $Zr$ = 53. 6. 10 | - - - | sin | 9.9029362 |
| | Hor. par. = 61'. 9" = 3669" | - | log. | 3.5645477 |
| | $rt$ uncorrected = 2934" = 48'.54" | | log. | 3.4674839 |
| | App. zen. dist. $Zt$ = 53°.55'.4" nearly, | | sin. | 9.9075042 |
| | Hor. par. = 61'. 9" = 3669" | - | log. | 3.5645477 |

Tri. *trs*
Par. *rt* cor. = 2965″ = 49′. 25″ — log. 3.4720519
*trs* = 44°. 1′. 16″ - - - - - cos. 9.8567795

*rs* par. in *lat.* = 2132″ = 35′. 32″ log. 3.3288314

Tri. *trs*
*rt* cor. = 2965″ - - - - - - log. 3.4720519
*trs* = 44°. 1′. 16″ - - - - - sin. 9.8419369

*ts* = 2061″ = 34′. 21″ - - - - log. 3.3139888
True lat. *ro* = 4°. 5′. 30″
App. lat. *tv* = *ro* + *rs* = 4°. 41′. 2″ cos. 9.9985472
*ts* = 2061″ - - - - - - 10, + log. 13.3139888

*ov* par. in *long.* = 2067″ = 34′. 27″ log. 3.3154416

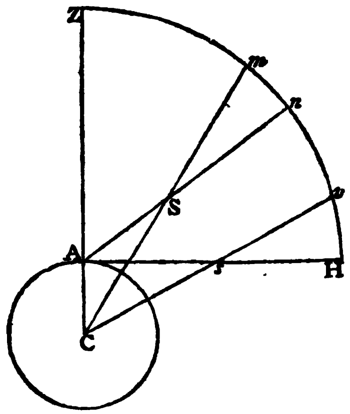
The value of *tv* is *ro* − or + *rs*, according as the moon has N. or S. latitude.

The order of the signs being from West to East, from *A* towards *C* is eastward, and from *A* towards ♈ is westward; now the parallax depressing the body from *r* to *t*, increases the longitude from *o* to *v*; but if the point *o* had been on the other side of *A*, *ov* would have lain the contrary way; hence, when the body is to the *East* of the nonagesimal degree, the parallax *increases* the longitude; and when it is to the *West*, it *diminishes* the longitude.

(145.) According to the Tables of *Mayer*, the greatest parallax of the moon (or when she is in her perigee and in opposition) is 61′. 32″; the least parallax (or when in her apogee and conjunction) is 53′. 52″, in the latitude of Paris; the arithmetical mean of these is 57′. 42″; but this is not the parallax at the mean distance, because the parallax varies inversely as the distance, and therefore the parallax at the mean distance is 57′. 24″, an harmonic mean between the two. M. *de Lambre* re-calculated the parallax from the same observations from which *Mayer* calculated it, and found it did not exactly agree with *Mayer's*. He made the equatorial parallax 57′. 11″,4. M. *de la Lande* makes it 57′. 5″ at the equator, 56′. 53″,2 at the

pole, and 57′. 1″ for the mean radius of the earth, supposing the difference of the equatorial and polar diameters to be $\frac{1}{300}$ of the whole. From the formula of *Mayer*, the equatorial parallax is 57′. 11″,4.

(146.) To find the mean distance, *Cs*, of the moon, we have *AC*, the mean radius ($r$) of the earth, : *Cs*,




the mean distance ($D$) of the moon from the earth, :: sin. 57′. 1″=*AsC* (145) : radius :: 1 : 60,3; consequently $D=60{,}3r$; but $r=3964$ miles; hence, $D=$ 239029 miles.

(147.) According to M. *de la Lande*, the horizontal semidiameter of the moon : it's horizontal parallax for the mean radius ($r$) of the earth :: 15′ : 54′. 57″,4, or very nearly as 3 : 11; hence, the semidiameter of the moon is $\frac{3}{11}r = \frac{3}{11} \times 3964 = 1081$ miles; and as the magnitude of spherical bodies are as the cubes of their radii, we have the magnitudes of the moon and earth as $3^3 : 11^3$, or as 1 : 49 nearly.

# Chap. VI.

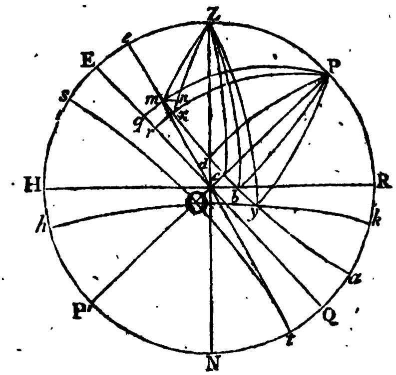
## ON REFRACTION.

(148.) When a ray of light passes out of a vacuum into any medium, or out of any medium into one of greater density, it is found to deviate from it's rectilinear course towards a perpendicular to the surface of the medium into which it enters. Hence, light passing out of a vacuum into the atmosphere will, where it enters, be bent towards a radius drawn to the earth's center, the top of the atmosphere being supposed to be spherical and concentric with the center of the earth; and as, in approaching the earth's surface, the density of the atmosphere continually increases, the rays of light, as they descend, are constantly entering into a denser medium, and therefore the course of the rays will continually deviate from a right line, and describe a curve; hence, at the surface of the earth, the rays of light enter the eye of the spectator in a different direction from that in which they would have entered, if there had been no atmosphere; consequently the apparent place of the body from which the light comes, must be different from the true place. Also, the refracted ray must move in a plane perpendicular to the surface of the earth; for, conceiving a ray to come in that plane before it is refracted, then the refraction being always in that plane, the ray must continue to move in that plane. Hence, the refraction is always in a vertical circle. The ancients were not unacquainted with this effect. *Ptolemy* mentions a difference in the rising and setting of the stars in different states of the atmosphere; he makes, however, no allowance for it in his computations from his observa-

tions; this correction, therefore, must be applied, where great accuracy is required. *Archimedes* observed the same in water, and thought the quantity of refraction was in proportion to the angle of incidence. *Alhazen*, an Arabian optician, in the eleventh century, by observing the distance of a circumpolar star from the pole, both above and below, found them to be different, and such as ought to arise from refraction. *Snellius*, who first observed the relation between the angles of incidence and refraction, says, that *Waltherus*, in his computation, allowed for refraction; but *Tycho* was the first person who constructed a table for that purpose, which, however, was very incorrect, as he supposed the refraction at 45° to be nothing. About the year 1660, *Cassini* published a new table of refractions, much more correct than that of *Tycho*; and, since his time, astronomers have employed much attention in constructing more correct tables, the niceties of modern astronomy requiring their utmost accuracy.

To find the quantity of refraction.

(149.) *First method.* Take the altitude of the sun, or a star whose right ascension and declination are known, and note the time by the clock; observe also the times of their transits over the meridian, and that interval gives the hour angle. Now, in the triangle




$PZx$, we know $PZ$ and $Px$, the complements of latitude and declination, and the angle $xPZ$, to find the

side $Zx$ (92), the complement of which is the *true* altitude, the difference between which and the *observed* altitude, is the refraction at that altitude.

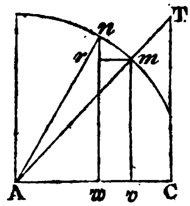
Ex. On May 1, 1738, at 5$h$. 20′ in the morning, *Cassini* observed the altitude of the sun's center at Paris to be 5°. 0′. 14″, and the sun passed the meridian at 12$h$. 0′. 0″, to find the refraction, the latitude being 48°. 50′. 10″, and the declination was 15°. 0′. 25″. The sun's distance from the meridian was 6$h$. 40′, which gives 100° for the hour angle $xPZ$; also, $PZ$ $=41°. 9'. 50''$, and $Px=74°. 59'. 35''$; hence, (Trig. Art. 233) $Zx=85°. 10'. 8''$, consequently the true altitude was 4°. 49′. 52″. Now to 5°. 0′. 14″, the apparent altitude, add 9″ for the parallax, and we have 5°. 0′. 23″ the apparent altitude corrected for parallax; hence, $5°. 0'. 23'' - 4°. 49'. 52'' = 10'. 31''$, the refraction at the apparent altitude 5°. 0′. 14″.

(150.) *Second method.* Take the greatest and least altitudes of a circumpolar star which passes through, or very near, the zenith, when it passes the meridian above the pole; then the refraction being nothing in the zenith, we shall have the true distance of the star from the pole at that observation, the altitude of the pole above the horizon, being previously determined; but when the star passes the meridian under the pole, we shall have it's distance affected by refraction, and the difference of the two observed distances, above and below the pole, gives the refraction at the apparent altitude below the pole.

Ex. M. *de la Caille,* at Paris, observed a star to pass the meridian within 6′ of the zenith, and consequently, at the distance of 41°. 4′ from the pole; hence, it must pass the meridian under the pole at the same distance, or at the altitude 7°. 46′; but the observed altitude at that time was 7°. 52′. 25″; hence, the refraction was 6′. 25″ at that apparent altitude.

(151.) Let $CAn$ be the angle of incidence, $CAm$ the angle of refraction, and consequently $mAn$ the quantity of refraction; let $TC$ be the tangent of $Cm$, $mv$

its sine, $nw$ the sine of $Cn$, and draw $rm$ parallel to $vw$; then, as the refraction of air is very small, we



may consider $mrn$ as a rectilinear triangle; and hence, by similar triangles, $Av : Am :: rn : mn = \frac{Am \times rn}{Av}$; but $Am$ is constant, and as the ratio of $mv$ to $nw$ is constant by the laws of refraction, their difference, $rn$, must vary as $mv$; hence, $mn$ varies as $\frac{mv}{Av}$; but $CT = \frac{Am \times mv}{Av}$, which varies as $\frac{mv}{A}$, because $Am$ is constant; hence, the refraction, $mn$, varies as $CT$, the tangent of the apparent zenith distance of the star, because the angle of refraction, $CAm$, is the angle between the refracted ray and the perpendicular to the surface of the medium, which perpendicular is directed to the zenith. Whilst, therefore, the refraction is very small, so that $rmn$ may be considered as a rectilinear triangle, this rule will be sufficiently accurate *.

(152.) The twilight in the morning and evening, arises both from the refraction and reflection of the sun's rays by the atmosphere.

It is probable that the reflection arises principally from the exhalations of various kinds with which the lower parts of the atmosphere are charged; for the twilight lasts till the sun is further below the horizon in the evening, than it is in the morning when it

* For further information on this subject, see my *Complete System of Astronomy*.

begins; and it is longer in summer than in winter. Now, in the *former* case, the heat of the day has raised the vapours and exhalations; and in the *latter*, they will be more elevated from the heat of the season; therefore, upon supposition that the reflection is made by them, the twilight ought to be longer in the evening than in the morning, and longer in summer than in winter.

(153.) Another effect of refraction is, that of giving the sun and moon an oval appearance, by the refraction of the lower limb being greater than that of the upper, whereby the vertical diameter is diminished. For suppose the diameter of the sun to be 32′, and the lower limb to touch the horizon, then the mean refraction at that limb is 33″, but the altitude of the upper limb being then 32′, it's refraction is only 28′. 6″, the difference of which is 4′, 54″, the quantity by which the vertical diameter appears shorter than that parallel to the horizon. When the body is not very near the horizon, the refraction diminishing nearly uniformly, the figure of the body is very nearly that of an ellipse.

# Chap. VIII.

## ON THE SYSTEM OF THE WORLD.

(154.) When any effect or phænomenon is discovered by experiment or observation, it is the business of Philosophy to investigate it's cause. But there are very few, if any, enquiries of this kind, where we can be led from the effect to the cause by a train of mathematical reasoning, so as to pronounce with certainty upon the cause. Sir *I. Newton*, therefore, in his *Principia*, before he treats on the System of the World, has laid down the following Rules to direct us in our researches into the constitution of the universe.

Rule I. No more causes are to be admitted, than what are sufficient to explain the phænomenon.

Rule II. Of effects of the same kind, the same causes are to be assigned, as far as it can be done.

Rule III. Those qualities which are found in all bodies upon which experiments can be made, and which can neither be increased nor diminished, may be looked upon as belonging to all bodies.

Rule IV. In Experimental Philosophy, propositions collected from phænomena by induction, are to be admitted as accurately, or nearly true, until some reason appears to the contrary.

The principles, upon which the application of these Rules is admitted, are, the supposition that the operations of nature are performed in the most simple manner, and regulated by general laws. And although their application may, in many cases, be very unsatisfactory, yet, in the instances to which we shall here want to apply them, their force is little inferior to

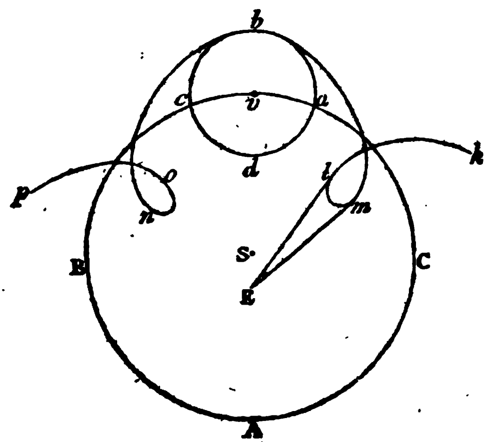
that of direct demonstration, and the mind rests equally satisfied as if the matter were strictly proved.

(155.) The diurnal motion of the heavenly bodies may be accounted for, either by supposing the earth to be at rest, and all the bodies daily to perform their revolutions in circles parallel to each other; or by supposing the earth to revolve about one of it's diameters as an axis, and the bodies themselves to be fixed, in which case their apparent diurnal motions would be the same. If we suppose the earth to be at rest, all the fixed stars must make a complete revolution, in parallel circles, every day. Now the nearest of the fixed stars cannot be less than 400000 times further from us than the sun is, and the sun's distance from the earth is not less than 93 millions of miles. Also, from the discoveries which are every day making by the improvement of telescopes, it appears that the heavens are filled with almost an infinity of stars, to which the number visible to the naked eye bears no proportion; and whose distances are, probably, incomparably greater than what we have stated above. But that an almost infinite number of bodies, most of them invisible, except by the best telescopes, at almost infinite distances from us and from each other, should have their motions so exactly adjusted, as to revolve in the same time, and in parallel circles, and all this without their having any central body, which is a physical impossibility, is an hypothesis, which, by the Rules we have here laid down, is not to be admitted, when we consider, that all the phænomena may be solved, simply by the rotation of the earth about one of it's diameters. If therefore we had no other reason, we might rest satisfied that the apparent diurnal motions of the heavenly bodies are produced by the earth's rotation. But we have other reasons for this supposition. Experiments prove that all the parts of the earth have a gravitation towards each other. Such a body, therefore, the greatest part of whose surface is a fluid, if it remain at rest, must, from the equal gra-

vitation of it's parts, form itself into a perfect sphere. But if we suppose the earth to revolve, the parts most distant from the axis must, from their greater velocity, have a greater tendency to fly off, and therefore that diameter which is perpendicular to the axis must be increased. That this must be the consequence appears from taking an iron hoop, and making it revolve swiftly about one of it's diameters, and that diameter will be diminished, and the diameter perpendicular to it will be increased. Now it appears from mensuration, that the earth is not a perfect sphere, but a spheroid, having the equatorial longer than it's polar diameter. That diameter therefore, about which the earth must revolve, in order to solve all the phænomena of the apparent revolution of the heavenly bodies, is the shortest; and as it necessarily must be the shortest, if the earth be supposed to revolve, this agreement affords a very satisfactory proof of the earth's rotation. Another reason for the earth's rotation is from analogy. The planets are opaque and spherical bodies, like to our earth; now all the planets, on which sufficient observations have been made to determine the matter, are found to revolve about an axis, and the equatorial diameters of some of them are visibly greater than their polar. When these reasons, all upon different principles, are considered, they amount to a proof of the earth's rotation about it's axis, which is as satisfactory to the mind, as the most direct demonstration could be. These, however, are not all the arguments which might be offered; the situations and motions of the bodies in our system necessarily require this motion of the earth.

(156.) Besides this apparent diurnal motion, the sun, moon, and planets, have another motion; for they are observed to make a complete revolution amongst the fixed stars, in different periods; and whilst they are performing these motions in respect to the fixed stars, they do not always appear to move in the same direction, or in that direction in which their

complete revolutions are made, but sometimes appear stationary, and sometimes to move in a contrary direction. We will here briefly describe and consider the different systems which have been invented, in order to solve these appearances. *Ptolemy* supposed the earth to be perfectly at rest, and all the other bodies, that is, the sun, moon, planets, comets and fixed stars, to revolve about it every day; but that, besides this diurnal motion, the sun, moon, planets and comets had a motion in respect to the fixed stars, and were situated, in respect to the earth, in the following order; the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn. These revolutions he first supposed to be made in circles about the earth, placed a little out of the center, in order to account for some irregularities of their motions; but as their retrograde motions and stationary appearances could not thus be solved, he supposed them to revolve in epicycloids, in the following manner. Let *ABC* be a circle, *S* the center, *E* the earth, *a b c d* another circle, whose center *v* is in the circumference of the circle *ABC*. Conceive the circumference of the circle *ABC* to be carried round the earth every twenty-four hours, according to the order of the letters,




and at the same time let the center *v* of the circle *abcd* have a slow motion in the opposite direction, and let

a body revolve in this circle in the direction *a b c d*; then it is manifest, that by the motion of the body in this circle, and the motion of the circle itself, the body will describe such a curve as is represented by *klmbnop*; and if we draw the tangents *El*, *Em*, the body would appear stationary at the points *l* and *m*, and it's motion would be *retrograde* through *lm*, and then *direct* again. Now to make Venus and Mercury always accompany the sun, the center *v* of the circle *abcd* was supposed to be always very nearly in a right line between the earth and the sun, but more nearly so for Venus than for Mercury, in order to give each it's proper elongation. This system, although it will account for all the motions of the bodies, yet it will not solve the phases of Venus and Mercury; for in this case in both conjunctions with the sun, they ought to appear dark bodies, and to lose their lights both ways from their greatest elongations; whereas it appears from observation, that in one of their conjunctions they shine with full faces. This system therefore cannot be true.

(157.) The system received by the Egyptians was this: That the earth is immoveable in the center, about which revolve in order, the Moon, Sun, Mars, Jupiter, and Saturn; and about the Sun revolve Mercury and Venus. This disposition will account for the phases of Mercury and Venus, but not for the apparent motions of Mars, Jupiter, and Saturn.

(158.) The next system which we shall mention, though posterior in time to the true, or *Copernican System*, as it is usually called, is that of *Tycho Brahe*, a Polish nobleman. He was pleased with the Copernican System, as solving all the appearances in the most simple manner; but conceiving, from taking the literal meaning of some passages in scripture, that it was necessary to suppose the earth to be absolutely at rest, he altered the system, but kept as near to it as possible. And he further objected to the earth's motion, because it did not, as he conceived, affect the motions of

comets observed in opposition, as it ought; whereas, if he had made observations on some of them, he would have found that their motions could not otherwise have been accounted for. In his system the earth is supposed to be immoveable in the center of the orbits of the sun and moon, without any rotation about an axis; but he made the sun the center of the orbits of the other planets, which therefore revolved with the sun about the earth. By this system, the different motions and phases of the planets may be solved, the latter of which could not be by the Ptolemaic System; and he was not obliged to retain the epicyclods, in order to account for their retrograde motions and stationary appearances. One obvious objection to this system is, the want of that simplicity by which all the apparent motions may be solved, and the necessity that all the heavenly bodies should revolve about the earth every day; also, it is physically impossible that a large body, as the sun, should revolve in a circle about a small body, as the earth, at rest in its center; if one body be much larger than another, the center about which they revolve must be very near the large body; an argument which holds also against the Ptolemaic System. It appears also from observation, that the plane in which the sun must, upon this supposition, diurnally move, passes through the earth only twice in a year. It cannot therefore be any force in the earth which can retain the sun in it's orbit; for it would move in a spiral, continually changing it's plane. In short, the complex manner in which all the motions are accounted for, and the physical impossibility of such motions being performed, is a sufficient reason for rejecting this system; especially when we consider, in how simple a manner all these motions may be accounted for, and demonstrated from the common principles of motion. Some of *Tycho's* followers, seeing the absurdity of supposing all the heavenly bodies daily to revolve about the earth, allowed a rotatory motion to the earth, in order to account for their

diurnal motion; and this was called the *Semi-Tychonic System*; but the objections to this system are in other respects, the same.

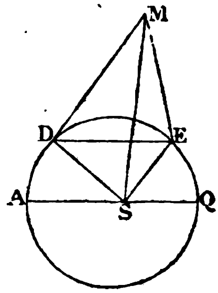
(159.) The system which is now universally received is called the *Copernican*. It was formerly taught by *Pythagoras*, who lived about 500 years before J. C. and *Philolaus*, his disciple, maintained the same; but it was afterwards rejected, till revived by *Copernicus*. Here the Sun is placed in the center of the system, about which the other bodies revolve in the following order; Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian Planet, which was lately discovered by Dr. Herschel; beyond which, at immense distances, are placed the fixed stars; the moon revolves about the earth, and the earth revolves about an axis. This disposition of the planets solves all the phænomena, and in the most simple manner. For from inferior to superior conjunction, Venus and Mercury appear, first horned, then dichotomized, and next gibbous; and the contrary, from superior to inferior conjunction; they are always retrograde in the inferior, and direct in their superior conjunction. Mars and Jupiter appear gibbous about their quadratures; but in Saturn and the Georgian this is not sensible, on account of their great distances. The motions of the superior planets are observed to be direct in their conjunction, and retrograde in their opposition. All these circumstances are such as ought to take place in the Copernican System. The motions also of the planets are such as should take place upon physical principles. We may also further observe, that the supposition of the earth's motion is necessary, in order to account for a small apparent motion which every fixed star is found to have, and which cannot otherwise be accounted for. The harmony of the whole is as satisfactory a proof of the truth of this system, as the most direct demonstration could be; we shall therefore assume this system.

# Chap. IX.

## ON KEPLER'S DISCOVERIES.

(160.) Kepler was the first who discovered the figures of the orbits of the planets to be ellipses, having the sun in one of the foci; this he determined in the following manner.

(161.) Let $S$ be the sun, $M$ Mars, $D$, $E$, two places of the earth when Mars is in the same point $M$ of it's orbit. When the earth was at $D$, he observed the




difference between the longitudes of the sun and Mars, or the angle $MDS$; in like manner, he observed the angle $MES$. Now the places $D$, $E$, of the earth in it's orbit being known, the distances $DS$, $ES$, and the angle $DSE$, will be known; hence, in the triangle $DSE$, we know $DS$, $SE$, and the angle $DSE$, to find $DE$, and the angles $SDE$, $SED$; hence, we know the angles $MDE$, $MED$, and $DE$, to find $MD$; and lastly, in the triangle $MDS$, we know $MD$, $DS$, and the angle $MDS$, to find $MS$, the distance of Mars from the sun. He also found the angle $MSD$, the difference of the heliocentric longitudes of Mars and the earth. By this method, *Kepler*, from observations made on Mars when in aphelion and perihelion

(for he had determined the position of the line of the apsides, by a method which we shall afterwards explain, independent of the form of the orbit, determined the former distance from the sun to be 166780, and the latter 138500, the mean distance of the earth from the sun being 100000; hence, the mean distance of Mars was 152640, and the excentricity of it's orbit 14140. He then determined, in like manner, three other distances, and found them to be 147750, 163100, 166255. He next calculated the same three distances, upon supposition that the orbit was a circle, and found them to be 148539, 163883, 166605; the errors therefore of the circular hypothesis were 789, 783, 350. But he had too good an opinion of *Tycho's* observations (upon which he founded all these calculations) to suppose that these differences might arise from their inaccuracy; and as the distance between the aphelion and perihelion was too great, upon supposition that the orbit was a circle, he knew that the form of the orbit must be an oval; *Itaque planè hoc est: Orbita planetæ non est circulus, sed ingrediens ad latera utraque paulatim, iterumque ad circuli amplitudinem in perigæo exiens, cujusmodi figuram itineris ovalem appellitant*, pag. 213*. And as, of all ovals, the ellipse appeared to be the most simple, he first supposed the orbit to be an ellipse, and placed the sun in one of the foci; and upon calculating the above observed distances, he found they agreed together. He did the same for other points of the orbit, and found that they all agreed; and thus he pronounced the orbit of Mars to be an ellipse, having the sun in one of it's foci. Having determined this for the orbit of Mars, he conjectured the same to be true for all the other planets, and upon trial he found it to be so. Hence, he concluded, *That the six primary planets revolve about the sun in ellipses, having the sun in one of the foci.*

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* See his Work, *De Motibus Stellæ Martis.*

The relative *mean* distances of the planets from the sun are as follows: Mercury, 38710; Venus, 72333; Earth, 100000; Mars, 152369; Jupiter, 520279; Saturn, 954072; Georgian, 1918352.

(162.) Having thus discovered the relative mean distances of the planets from the sun, and knowing their periodic times, he next endeavoured to find if there was any relation between them, having had a strong passion for finding analogies in nature. On March 8, 1618, he began to compare the powers of these quantities, and at that time he took the squares of the periodic times, and compared them with the cubes of the mean distances, but, from some error in the calculation, they did not agree. But on May 15, having made the last calculations again, he discovered his error, and found an exact agreement between them. Thus he discovered that famous law, *That the squares of the periodic times of all the planets are as the cubes of their mean distances from the sun.* Sir *I. Newton* afterwards proved that this is a necessary consequence of the motion of a body in an ellipse, revolving about the focus. *Prin. Phil. Lib.* I. *Sec.* 2. *Pr.* 15.

(163.) *Kepler* also discovered, from observation, that the velocities of the planets, when in their apsides, are inversely as their distances from the sun; whence it followed, that they describe, in these points, equal areas about the sun in equal times. And although he could not prove, from observation, that the same was true in every point of the orbit, yet he had no doubt but that it was so. He therefore applied this principle to find the equation of the orbit (as will be explained in the next Chapter), and finding that his calculations agreed with observations, he concluded that it was true in general, *That the planets describe about the sun, equal areas in equal times.* This discovery was, perhaps, the foundation of the *Principia,* as it might probably suggest to Sir *I. Newton* the idea, that the

proposition was true in general, which he afterwards proved it to be. These important discoveries are the foundation of all Astronomy.

(164.) *Kepler* also speaks of *Gravity* as a power which is mutual between all bodies; and tells us, that the earth and moon would move towards each other, and meet at a point as much nearer to the earth than the moon, as the earth is greater than the moon, if their motions did not hinder it. He further adds, that the tides arise from the gravity of the waters towards the moon.

# CHAP. X.

## ON THE MOTION OF A BODY IN AN ELLIPSE ABOUT THE FOCUS.

(165.) As the orbits which are described by the primary planets revolving about the sun, are ellipses having the sun in one of the foci, and each describes about the sun equal areas in equal times, we next proceed to deduce, from these principles, such consequences as will be found necessary in our enquiries respecting their motions. From the equal description of areas about the sun in equal times, it appears * that

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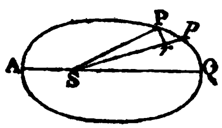
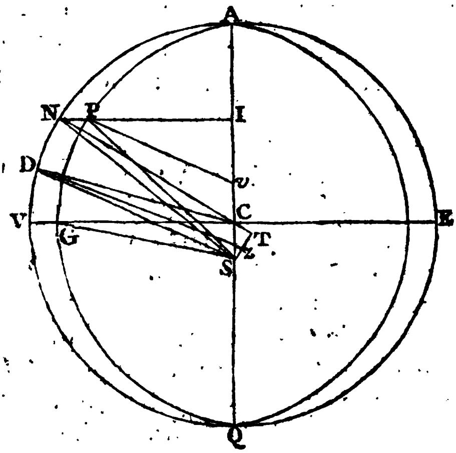
* For if $APQ$ be an ellipse described by a planet about the sun at $S$ in the focus, the indefinitely small area, $PSp$, described in a




given time, will be constant; draw $Pr$ perpendicular to $Sp$; and as the area $SPp$ is constant for the same time, $Pr$ varies as $\frac{1}{Sp}$; but the angle $pSP$ varies as $\frac{Pr}{Sp}$, and therefore it varies as $\frac{1}{Sp^2}$; that is, in the *same* orbit, the angular velocity of a planet varies inversely as the square of it's distance from the sun. For *different* planets, the areas described in the same time are not equal, and therefore $Pr$ varies as $\frac{\text{area } SPp}{Sp}$, consequently the angle $pSP$ varies as $\frac{\text{area } SPp}{Sp^2}$; that is, the angular velocities of *different* planets, are as the areas described in the same time directly, and the squares of their distances from the sun inversely.

the planets move with unequal angular velocities about the sun. The proposition therefore, which we here propose to solve, is, given the periodic time of a planet, the time of it's motion from it's aphelion, and the excentricity of it's orbit, to find it's angular distance from the aphelion, or it's *true* anomaly, and it's distance from the sun. This was first proposed by *Kepler*, and therefore goes by the name of *Kepler's Problem*. He knew no direct method of solving it, and therefore did it by very long and tedious tentative operations.

(116.) Let $AGQ$ be the ellipse described by the body about the sun at $S$ in one of it's foci, $AQ$ the




major, $GC$ the semi-minor axis, $A$ the aphelion, $Q$ the perihelion, $P$ the place of the body, $AVQE$ a circle, $C$ it's center; draw $NPI$ perpendicular to $AQ$, join $PS$, $NS$, and $NC$, on which produced let fall the perpendicular $ST$. Let a body move uniformly in the circle from $A$ to $D$ with the *mean* angular velocity of the body in the ellipse, whilst the body moves in the ellipse from $A$ to $P$; then the angle $ACD$ is the *mean*, and the angle $ASP$ the *true* anomaly; and the difference of these two angles is called the *equation of*

*the planet's center*, or *prosthapheresis*. Let $p$ = the periodic time in the ellipse or circle (the periodic times being equal by supposition), and $t$ = the time of describing $AP$ or $AD$; then, as the bodies in the ellipse and circle describe equal areas in equal times about $S$ and $C$ respectively, we have

area $ADC$ : area of the circle :: $t$ : $p$,
area of the ellipse : area $ASP$ :: $p$ : $t$; also,
area of circ. : area of ellip. :: area $ASN$ : area $ASP$;*

therefore, area $ADC$ : area $ASP$ :: area $ASN$ : area $ASP$; hence, $ADC = ASN$; take away the area $ACN$, which is common to both, and the area $DCN = SNC$; but $DCN = \frac{1}{2} DN \times CN$, and $SNC = \frac{1}{2} ST \times CN$; therefore $ST = DN$. Now if $t$ be given, the arc $AD$ will be given; for as the body in the circle moves uniformly, we have $p : t :: 360° : AD$. Thus we may find the mean anomaly at any given time, knowing the time when the body was in the aphelion; hence, if we can find $ST$, or $ND$, we shall know the angle $NCA$, called the *excentric* anomaly, from whence, by one proportion (167) we shall be able to find the angle $ASP$ the *true* anomaly. The problem is therefore reduced to this; to find a triangle $CST$, such, that the angle $C$ + the degrees of an arc equal to $ST$, may be equal to the given angle $ACD$. This may be expeditiously done by trial in the following manner, given by M. *de la Caille* in his Astronomy. Find what arc of the circumference of the circle $ADQE$ is equal to $CA$, by saying, 355 : 113 :: 180°. : 57°. 17'. 44",8, the number of degrees of an arc equal in length to the radius $CA$; hence, $CA : CS ::$ 57°. 17'. 44",8 : the degrees of an arc equal to $CS$. Assume therefore the angle $SCT$, multiply it's sine into the

* See my *Treatise on the Conic Sections*, *second* edit. prop. 7. of the Ellipse, cor. 3 and 4. And this is the Treatise referred to in the future part of this Work.

degrees in $CS$, and add it to the angle $SCT$, and if it be equal to the given angle $ACD$, the supposition was right; if not, add or subtract the difference to or from the first supposition, according as the result is less or greater than $ACD$, and repeat the operation, and in a very few trials you will get the accurate value of the angle $SCT$. The degrees in $ST$ may be most readily obtained, by adding the logarithm of $CS$ to the logarithm of the sine of the angle $SCT$, and subtracting 10 from the index, and the remainder will be the logarithm of the degrees in $ST$. Having found the value of $AN$, or the angle $ACN$, we proceed next to find the angle $ASP$.

(167.) Let $v$ be the other focus, and put $AC=1$; then by Eucl. B. II. P. 12. $SP^2 - Pv^2 = vS^2 + 2vS \times vI = \overline{vS + 2vI} \times vS = \overline{2Cv + 2vI} \times 2SC = 2CI \times 2SC$; hence, $SP + Pv : 2CI :: 2SC : SP - Pv$, or $2 : 2CI :: 2SC : SP - \overline{2 - SP}$, or $1 : CI :: SC : SP - 1$; hence, $SP = 1 + CS \times CI = 1 + CS \times \cos. \angle ACN$. But (Trig. Art. 94), $\dfrac{1-\cos.ASP}{1+\cos.ASP} = \overline{\tan. \tfrac{1}{2} ASP}^2$; also, (Trig. Art. 125) $SP$, or $1 + CS \times \cos.ACN : \text{rad.} = 1 :: SI$, or $CS + CI$, or $CS + \cos. ANC, : \cos. ASP = \dfrac{CS + \cos.ACN}{1 + CS \times \cos.ACN}$. Hence, $\overline{\tan. \tfrac{1}{2} ASP}^2$; $\left(= \dfrac{1-\cos.ASP}{1+\cos.ASP}\right) = \dfrac{1 + CS \times \cos.ACN - CS - \cos.ACN}{1 + CS \times \cos.ACN + CS + \cos.ACN}$

$$= \frac{1 - CS + \cos.ACN \times \overline{CS - 1}}{1 + CS + \cos.ACN + \overline{CS + 1}} = \frac{SQ - \cos.ACN \times SQ}{SA + \cos.ACN \times SA}$$

$= \dfrac{1 - \cos.ACN}{1 + \cos.ACN} \times \dfrac{SQ}{SA} =$ (Trigonometry, Article 95) $\overline{\tan. \tfrac{1}{2} ACN}^2 \times \dfrac{SQ}{SA}$; therefore $\sqrt{SA} : \sqrt{SQ} ::$ tan. $\tfrac{1}{2} ACN$ : tan. $\tfrac{1}{2} ASP$, consequently we get $ASP$ the *true* anomaly.

**Ex.** Required the true place of *Mercury* on Aug. 26, 1740, at noon, the equation of the center, and it's distance from the sun.

By M. *de la Caille's* Astronomy, Mercury was in it's aphelion on Aug. 9, at $6h.\ 37'$. Hence, on Aug. 26, it had passed it's aphelion $16d.\ 17h.\ 23'$; therefore $87d.\ 23h.\ 15'.\ 23''$ (the time of one revolution) : $16d.\ 17h.\ 23'$ :: $360°$ : $68°.\ 26'.\ 28''$, the arc $AD$, or *mean* anomaly. Now (according to this author) $CA$ : $CS$ :: 1011276 : 211165 (166) :: $57°.71'.44'',8 : 11°.\ 57'.\ 50'' = 43070''$, the value of $CS$ reduced to the arc of a circle, the log. of which is 4,6341749. Also, $68°.\ 26'.\ 28'' = 246388''$. Assume the angle $SCT$ to be $60°. = 216000''$, and the operation (166) to find the angle $ACN$, will stand thus.

4,6341749
9,9375806 log. of . 216000 $=a$

4,5717055 . . . . . . 37300

253300
246388

6912 $=b$

4,6341749
9,9287987 . . . . . . 209088 $=a-b=58°.\ 4'.\ 48''=c$

4,5629736 . . . . . . 36557

245645
246388

743 $=d$

4,6341749
9,9297694 . . . . . . 209831 $=c+d=58°.17'.11''=e$

4,5639443 . . . . . . 36639

246470
246388

82 $=f$

4,6341749
9,9296626 . . . . . . 209749 $=e-f=58°.15'.49''=g$

4,5638375 . . . . . . 36630

246379
246388

9 $=h$; hence, as the difference between the value deduced from the assumption and the true value, is now diminished about nine times every operation, the next difference would be 1″;

if therefore we add $h$ to $g$, and then subtract 1″, we get 58°. 15′. 57″, for the true value of the angle $ACN$, the *excentric* anomaly. Hence, (167) find the *true* anomaly $ASP$, from the proportion there given, by logarithms, thus.

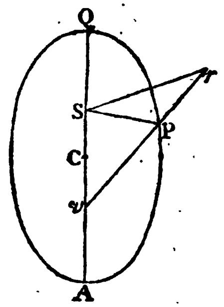
| | |
|---|---|
| Log. tang. 29°. 7′. 58″½ . . . | 9,7461246 |
| ½ Log. $SQ$=800111 . . . . | 2,9515751 |
| | 126976997 |
| ½ Log. $SA$=1222441 . . . . | 3,0436141 |
| Log. tang. 24°.16′.15″ . . . . | 9,6540856 |

Hence, the *true* anomaly is 48°.32′.30″. Now the aphelion $A$ was in 8ˢ. 13°. 54′. 30″; therefore the true place of Mercury was 10ˢ. 2°. 27′. Hence, (166), 68°. 26′. 28″ − 48°. 32′. 30″=19°. 53′. 58″, the *equation of the center*. Also, $SP=1+CS\times\cos.\angle ACN=$ 1,10983 the distance of Mercury from the sun, the radius of the circle, or the mean distance of the planet, being unity. Thus we are able to compute, at any time, the place of a planet in it's orbit, and it's distance from the sun; and this method of computing the *excentric* anomaly appears to be the most simple and easy of application of all others, and capable of any degree of accuracy.

(168.) As the bodies $D$ and $P$ departed from $A$ at the same time, and will coincide again at $Q$, $ADQ$, $APQ$ being described in half the time of a revolution; and as at $A$ the planet moves with it's least angular velocity (by the Note to Art. 165), therefore from $A$ to $Q$, or in the *first* six signs of anomaly, the angle $ACD$ will be greater than $ASP$, or the *mean* will be greater than the *true* anomaly; but from $Q$ to $A$, or in the *last* six signs, as the planet at $Q$ moves with it's greatest angular velocity, the *true* will be greater than the *mean* anomaly. When the equation is greatest in going from $A$ to $Q$, the *mean* place is before the *true* place, by the equation, and from $Q$ to $A$, the *true*

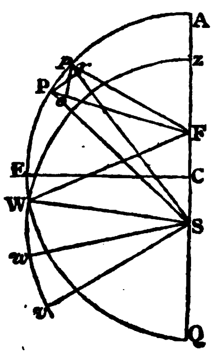
place is before the *mean* place, by the equation; therefore, from the time the equation is greatest till it becomes greatest again, the difference between the true and mean motions, is twice the equation. From apogee to perigee, the true and mean motions are the same.

(169.) The method ascribed by some writers to *Seth Ward*, Professor of Astronomy at Oxford, and published in 1654, although, as M. *de la Lande* observes, it is given both by *Ward* and *Mercator* to *Bullialdus*, is less accurate than these we have already given; yet as it may, in many cases, serve as a useful approximation, it deserves to be mentioned. He




assumed the angular velocity about the other focus $v$ to be uniform*, and therefore made it represent the *mean*

* That this is not true may be thus shown. With the center $S$ and radius $SW = \sqrt{AC \times CE}$ describe the circle $zW$, then the area of this




circle = area of the ellipse (Conic Sections, Ellipse, Prop. 7. Cor. 5): let

anomaly. Produce $vP$ to $r$, and take $Pr=PS$; then in the triangle $Svr$, $rv+vS : rv-vS ::$ tan. $\frac{1}{2} . \angle \overline{vSr+vrS}$ : tan. $\frac{1}{2} . \angle \overline{vSr-vrS}$ (Trig. Art. 135); now $\frac{1}{2} . \overline{rv+vS} = \frac{1}{2} AQ + \frac{1}{2} vS = AS$, and $\frac{1}{2} . \overline{rv-vS} = \frac{1}{2} AQ - \frac{1}{2} vS = SQ$; also, tan. $\frac{1}{2} . \angle \overline{vSr+vrS} =$ tan. $\frac{1}{2} . \angle AvP$, and tan. $\frac{1}{2} . \angle \overline{vSr-vrS} =$ (as $Pr=PS$) tan. $\frac{1}{2} . \angle \overline{vSr-PSr} =$ tan. $\frac{1}{2} . \angle ASP$; hence, *the aphelion distance : perihelion distance :: tan. of $\frac{1}{2}$ the mean anomaly : tan. $\frac{1}{2}$ true anomaly.* This is called the *simple elliptic hypothesis.* In the orbit of the earth, the error is never greater than 17″; in the orbit of the moon, it may be 1′. 35″. By this hypothesis, for 90° from aphelion and perihelion, the computed place is *backwarder* than the true; and for the other part it is *forwarder.*

(170.) The *greatest* equation of the center may be easily found from the Note to Article 169, giving the dimensions of the orbit. For as long as the angular

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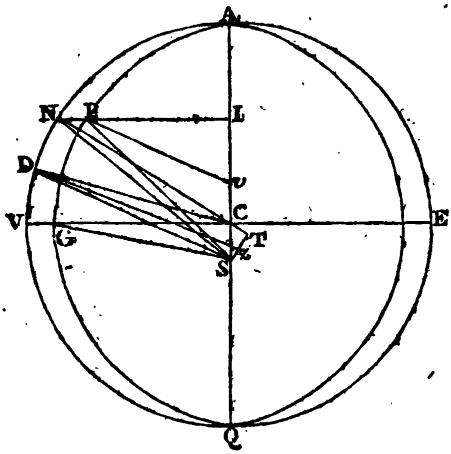
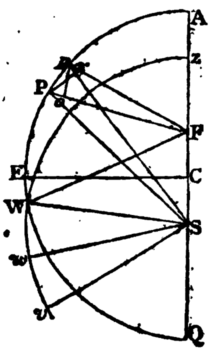
let a body, moving uniformly in it, make one revolution in the same time the body does in the ellipse; and let the bodies set off at the same time from $A$ and $z$, and describe $AP$, $zv$, in the same time; then the angle $zSv$ is the *mean*, and $ASP$ the *true* anomaly. Draw $pS$ indefinitely near to $PS$, and $Pr$, $po$, perpendicular to $Sp$, $FP$; then $Pr=po$. Now the angle $PFp$ varies as $\frac{po}{PF} = \frac{Pr}{PF}$; but, in a given time, the area $PSp$ is given, therefore $Pr$ varies as $\frac{1}{PS}$; hence, the angle $PFp$, described in a given time, varies as $\frac{1}{PF \times PS}$, which is not a constant quantity. Also, $\angle PFp : \angle PSp :: PS : PF :: \frac{1}{PF \times PS} : \frac{1}{PS^2}$. And by the Note to Art. 165, as equal areas are described in equal times in the circle and ellipse about $S$, the angular velocity about $S$ in the circle, becomes $\frac{1}{SW^2}$. Hence, the angular velocity about $S$ is greater or less than the mean angular velocity, according as $PF \times PS$ is less or greater than $SW^2$, or than $AC \times CE$. Also, the angular velocity about $S$ is the same in similar points of the ellipse in respect to the center, or at equal distances from the center.

velocity of the body in the circle is greater than that in the ellipse about $S$, the equation will increase, the




bodies setting out from $A$ and $z$; and when they become equal, the equation must be the greatest; this therefore happens when $\frac{1}{SP^2} = \frac{1}{SW^2} = \frac{1}{AC \times CE}$, or when $AC \times CE = SP^2$; hence, $SP$ is known. Let $SW$ represent this value of $SP$; then as we know




$SW$, $FW$ $(=2AC - SW)$ will be known, and as $SF$ is known, we can find the angle $FSW$ the *true* anomaly. Hence (167) $\sqrt{SQ} : \sqrt{SA} ::$ tan. $\frac{1}{2}$ *true* anom. : tan $\frac{1}{2}$ *excen.* anom. $ACN$, or tan. $\frac{1}{2}$ $SCT$; hence, as we know $SC$, we can find $ST$, or

*ND*; and to convert that into degrees, say, Rad. $= 1 : ND :: 57°. 17'. 44'',8$ : the degrees in *ND*, which added to or subtracted from the angle *ACN*, gives *ACD* the *mean* anomaly, the difference between which and the *true* anomaly is the *greatest* equation. Thus we may find the equation at any other time, having given *SP*.

(171.) The excentricity, and consequently the dimensions of the orbit, may be known from knowing the greatest equation. For (170) the equation is greatest when the distance is a mean between the semi-axes major and minor, and therefore in orbits nearly circular, the body must be nearly at the extremity of the minor axis, and consequently the angle *NCA*, or *SCT* (Fig. p. 103) will be nearly a right angle, therefore *ST* is nearly equal to *SC*; also *NSA* will be very nearly equal to *PSA*. Now the angle $NCA - NSA$ $(PSA) = SNC$, and $DCA - NCA = DCN$; add these together, and $DCA - PSA = DCN + SNC$, which (as *NC* is nearly parallel to *DS*) is nearly equal to 2 *DCN*; that is, the difference between the *true* and *mean* anomaly, or the *equation of the center*, is nearly equal to twice the arc *DN*, or twice *ST*, or very nearly twice *SC*. Hence, $57°. 17'. 44'',8$ : half the greatest equation :: rad. $= 1$ : *SC* the excentricity. But if the orbit be considerably excentric, compute the greatest equation to this excentricity; and then, as the equation varies very nearly as *SC*, say, as the computed equation : excentricity found :: given greatest equation : true excentricity.

Ex. If we suppose, with M. *de la Caille*, that Mercury's greatest equation is $24°. 3'. 5''$; then $57°. 17'. 44'',8 : 12°. 1'. 32'',5 :: 1 : ,209888$ the excentricity very nearly. Now the greatest equation, computed from this excentricity, is $23°. 54'. 28'',5$; hence, $23°. 54'. 28'',5 : 24°. 3'. 5'' :: ,209888 : ,211165$ the true excentricity. M. *de la Lande* makes the greatest equation $23°. 40'$, and the excentricity ,207745.

(172.) The converse of this problem, that is, given

the excentricity and true anomaly, to find the mean, may be very readily and directly solved. The excentricity being given, the ratio of the major to the minor axis is known*, which is the ratio of $NI$ to $PI$; hence, the angle $ASP$ being given, we have $PI : NI ::$ tan. $ASP$ : tan. $ASN$; therefore, in the triangle $NCS$, we know $NC$, $CS$, and the angle $CSN$, to find the angle $SCN$ (Trig. Art. 128), the supplement of which is the angle $ACN$, or $SCT$; hence, in the right-angled triangle $STC$, we know $SC$ and the angle $SCT$, to find $ST$ (Trig Art 128), which is equal to $ND$, the arc measuring the equation, which may be found by saying, radius : $ST$ :: 57°. 17′. 44″,8 : the degrees in $ND$, which added to $ACN$, gives $ACD$ the mean anomaly.

### *To find the hourly Motion of a Planet in it's Orbit, having given the* mean *hourly Motion.*

(173.) The hourly motion of a planet in it's orbit is found immediately from the Note to Art. 169; for it appears from thence, that the angles $PSp$, $WSw$, described by the body at $P$ in the ellipse, and the body $W$ in the circle in the same time, are as $SW^2 : SP^2$, or as (see Fig. p. 103) $AC \times CE : SP^2$; hence, $SPp = WSw \times \frac{AC \times CE}{SP^2}$ the hourly motion of a planet in it's *orbit*, the angle $WSw$ being the *mean* motion of the planet in an hour. For greater accuracy, $SP$ must be taken at the middle of the hour. Thus we may easily compute a table of the hourly motions of the planets in their orbits.

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* For as $AC$, $CS$ are known, we have $GC = \sqrt{GS^2 - SC^2} = \sqrt{AC^2 - SC^2} = \sqrt{\overline{AC+SC} \times \overline{AC-SC}}$, for the computation of which by logarithms, see Trig. Art. 52.

# Chap. XI.

## ON THE OPPOSITIONS AND CONJUNCTIONS OF THE PLANETS.

(174.) The place and time of the opposition of a superior, or conjunction of an inferior planet, are the most important observations for determining the elements of the orbit, because at that time the observed is the same as the true longitude, or that seen from the sun; whereas, if observations be made at any other time, we must reduce the observed to the true longitude, which requires the knowledge of their relative distances, and which, at that time, are supposed not to be known. They also furnish the best means of examining and correcting the tables of the planets' motions, by comparing the computed with the observed places.

(175.) To determine the time of opposition, observe, when the planet comes very near to that situation, the time at which it passed the meridian, and also it's right ascension (111 or 113); take also it's meridian altitude; do the same for the sun, and repeat the observations for several days. From the observed meridian altitudes find the declinations (114), and from the right ascensions and declinations compute (115) the latitudes and longitudes of the planet, and the longitudes of the sun. Then take a day when the difference of their longitudes is nearly 180°, and on that day reduce the sun's longitude, found from observation when it passed the meridian, to the longitude found at the time ($t$) the planet passed, by finding from observation, or computation, at what rate the longitude then increases. Now in opposition the planet is retrograde, and therefore the difference between the

longitudes of the planet and sun increases by the sum of their motions. Hence, the following rule: As the sum *(S)* of their daily motions in longitude: the difference *(D)* between 180° and the difference of their longitudes reduced to the same time ($t$)* (subtracting the sun's longitude from that of the planet to get the difference reckoned from the sun according to the order of the signs) :: $24h.$ : interval between that time ($t$) and the time of opposition. This interval added to or subtracted from the time ($t$), according as the difference of their longitudes at that time was greater or less than 180°, gives the time of opposition. If this be repeated for several days, and the mean of the whole taken, the time will be had more accurately. And if the time of opposition found from observation, be compared with the time by computation from the Tables, the difference will be the error of the Tables, which may serve as means of correcting them.

Ex. On October 24, 1763, M. *de la Lande* observed the difference between the right ascensions of $\beta$ *Aries*, and *Saturn*, which passed the meridian at $12h. 17'. 17''$ apparent time, to be $8°. 5'. 7''$, the star passing first. Now the apparent right ascension of the star at that time was $25°. 24'. 33'',6$; hence, the apparent right ascension of Saturn was $1^s. 3°. 29'. 40'',6$ at $12\ h. 17'. 17''$ apparent time, or $12h. 1'. 37''$ mean time. On the same day he found, from observation of the meridian altitude of Saturn, that it's declination was $10°. 35'. 20''$ N. Hence, from the right ascension and declination of Saturn, it's longitude is found to be $1^s. 4°. 50'. 56''$, and latitude $2°. 43'. 25''$ S. At the same time the sun's longitude was found by calculation to be $7^s. 1°. 19'. 22''$, which subtracted from $1^s. 4°. 50'. 56''$, gives $6^s. 3°. 31'. 34''$; hence, Saturn was $3°. 31'. 34''$

* For this difference shows how far the planet is from opposition; and the proposition is founded on this principle, that the sun approaches the star by spaces in proportion to the times; the spaces *S* and *D* must therefore be as the time $24h$, and the time ($t$) to opposition.

beyond opposition, but being retrograde will afterwards come into opposition. Now, from the observations made on several days at that time, Saturn's longitude was found to decrease 4′. 50″ in 24 hours, and by computation, the sun's longitude increased 59′. 59″ in the same time, the sum of which is 64′. 49″; hence, 64′. 49″ : 3°. 31′. 34″ :: 24*h.* : 78*h.* 20′. 20″, which added to October 24, 12*h.* 1′. 37″, gives 27*d.* 18*h.* 21′. 57″ for the time of opposition. Hence, we may find the longitude of Saturn at the time of opposition, by saying, 24*h.* : 78*h.* 20′. 20″ :: 4′. 50″ : 15′. 47″ the retrograde motion of Saturn in 78*h.* 20′. 20″, which subtracted from 1ˢ. 4°. 50′. 56″, leaves 1ˢ. 4°. 35′. 9″ the longitude of Saturn at the time of opposition. In like manner we may find the sun's longitude at the same time, in order to prove the opposition; for 24*h.* : 78*h.* 20′. 20″ :: 59′. 59″ : 3°. 15′. 47″, which added to 7ˢ. 1°. 19′. 22″, the sun's longitude at the time of observation, gives 7ˢ. 4°. 35′. 9″ for the sun's longitude at the time of opposition, which is exactly opposite to that of Saturn. Hence, we may also find the latitude of Saturn at the same time, by observing, in like manner, the daily variation, or by computation from the Tables after the elements of it's motions are known, and the Tables constructed; by which it appears, that in the interval between the times of observation and opposition, the latitude had increased 6″, and consequently the latitude was 2°. 43′. 31″. Thus we find the times of opposition of all the superior planets.

(176.) The place and time of conjunction of an inferior planet may be found in like manner, when the elongation of the planet from the sun, near the time of conjunction, is sufficient to render it visible; the most favourable time therefore must necessarily be when the geocentric latitude of the planet at the time of conjunction is the greatest. In the year 1689, Venus was in it's inferior conjunction on June 25, and it was observed on 21, 22, and 28; from which observations it's conjunction was found to be at 13*h.* 46′ appa-

rent time at Paris, in longitude ♋ 4°. 53′. 40″, and latitude 3°. 1′. 40″N. The state of the air must be very favourable, that the time and place of the superior conjunction may be thus observed; for as Venus is then about six times as far from the earth as at it's inferior conjunction, it's apparent diameter and the quantity of light which we receive from it, are so small, as to render it difficult to be perceived. But the most accurate method of observing the time of an inferior conjunction both of Venus and Mercury, is from observations made upon them in their transits over the sun's disc.

# CHAP. XII.

## ON THE MEAN MOTIONS OF THE PLANETS.

(177.) THE determination of the *mean* motions of the planets, from their conjunctions and oppositions, would very readily follow, if we knew the place of the aphelia and excentricities of their orbits; for then we could (166) find the equation of the orbit, and reduce the *true* to the *mean* place; and the *mean* places being determined at two points of time, give the mean motion corresponding to the interval between the times. But the place of the aphelion is best found from the mean motion. To determine therefore the mean motion, independent of the place of the aphelion, we must seek for such oppositions or conjunctions, as fall very nearly in the same point of the heavens; for then the planet being very nearly in the same point of it's orbit, the equation will be very nearly the same at each observation, and therefore the comparison between the true places will be nearly a comparison of their mean places. If the equation should differ much in the two observations, it must be considered. Now, by comparing the modern observations, we shall be able to get nearly the time of a revolution; and then, by comparing the modern with the ancient observations, the mean motion may be very accurately determined; for any error, by dividing it amongst a great number of revolutions, will become very small in respect to one revolution. As this will be best explained by an example, we shall give one from M. *Cassini* (*Elem. d'Astron.* p. 362), with the proper explanations as we proceed.

Ex. On September 16, 1701, *Saturn* was in opposition at $2h$. when the place of the sun was ♍ 23°. 21′. 16″, and consequently Saturn in ♓ 23°. 21′. 16″, with 2°. 27′. 45″ south latitude. On September 10, 1730, the opposition was at $12h$. 27′, and Saturn in ♓ 17°. 53′. 57″, with 2°. 19′. 6″ south latitude. On September 23, 1731, the opposition was at $15h$. 51′ in ♈ 0°. 30′. 50″, with 2°. 36′. 55″ south latitude. Now the interval of the two first observations was 29 years (of which seven were bissextiles) wanting $5d$. $13h$. 33′; and the interval of the two last was $1y$. $13d$. $3h$. 24′. Also, the difference of the places of Saturn in the two first observations was 5°. 27′. 19″, and in the two last it was 12°. 36′. 53″. Hence, in $1y$. $13d$. $3h$. 24′, Saturn had moved over 12°. 36′. 53″; therefore 12°. 36′. 53″ : 5°. 27′. 19″ :: $1y$. $13d$. $3h$. 24′ : $163d$. $12h$. 41′, the time of moving over 5°. 27′. 19″ very nearly, because Saturn, being nearly in the same part of it's orbit, will move nearly with the same velocity; this therefore, *added* to the interval between the two first observations (because at the second observation Saturn wanted 5°. 27′. 19″ of being up to the place at the first observation), gives 29 common years $164d$. $23h$. 8′, for the time of one revolution. Hence say $29y$. $164d$. $23h$. 8′ : $365d$. :: 360°. : 12°. 13′. 23″. 50‴ the *mean* annual motion of Saturn in a common year of 365 days, that is, the motion in a year if it had moved uniformly. If we divide this by 365, we shall get 2′. 0″. 28‴ for the mean daily motion of Saturn. The mean motion thus determined will be sufficiently accurate to determine the number of revolutions which the planet must have made, when we compare the modern with the ancient observations, in order to determine the mean motion more accurately.

The most ancient observation which we have of the opposition of Saturn was on March 2, in the year 228, before J. C. at one o'clock in the afternoon, in the meridian of Paris, Saturn being then in ♍ 8°. 23′,

with 2°. 50′ north lat. On February 26, 1714, at 8*h*. 15′, Saturn was found in opposition in ♍ 7°. 56′. 46″, with 2°. 3′ north lat. From this time we must subtract 11 days, in order to reduce it to the same style as at the first observation, and consequently this opposition happened on February 15, at 8*h*. 15′. Hence, the difference between these two places was only 26′. 14″. Also, the opposition in 1715 was on March 11, at 16*h*. 55′, Saturn being then in ♍ 21°. 3′. 14″, with 2°. 25′ north lat. Now between the two first oppositions there were 1942 years (of which 485 were bissextiles) wanting 14*d*. 16*h*. 45′, that is, 1943 common years, and 105*d*. 7*h*. 15′ over. Also, the interval between the times of the two last oppositions was 378*d* 8*h*. 40′, during which time Saturn had moved over 13°. 6′. 28″; hence, 13°. 6′. 28″ : 26′. 14″ :: 378*d*. 8*h*. 40′ : 13*d*. 14*h*. which added to the time of the opposition in 1714, gives the time when the planet had the same longitude as at the opposition in 228 before J. C. This quantity added to 1943 common years 105*d*. 7*h*. 15′, gives 1943*y*. 118*d*. 21*h*. 15′, in which interval of time Saturn must have made a certain complete number of revolutions. Now having found, from the modern observations, that the time of one revolution must be nearly 29 common years 164*d*. 23*h*. 8′, it follows that the number of revolutions in the above interval was 66; dividing therefore that interval by 66, we get 29*y*. 162*d*. 4*h*. 27′ for the time of one revolution. From comparing the oppositions in the years 1714 and 1715, the true movement of Saturn appears to be very nearly equal to the mean movement, which shows that the oppositions have been observed very near the mean distance; consequently the motion of the aphelion cannot have caused any considerable error in the determination of the mean motion. Hence the mean annual motion is 12°. 13′. 35″. 14‴, and the mean daily motion 2′. 0″. 35‴. Dr. *Halley* makes the annual motion to be 12°. 13′. 21″. M. *de Place* makes it 12°. 13′. 36″,8. As the revolu-

tion here determined is in that respect to the *longitude* of the planet, it must be a *tropical* revolution. Hence, to get the sidereal revolution, we must say, 2'. 0". 35''' : 24'. 42". 20''' (the precession in the time of a tropical revolution, Art. 130) :: 1 day : 12*d*. 7*h*. 1'. 57", which added to 29*y*. 162*d*. 4*h*. 27', gives 29*y*. 174*d*. 11*h*. 28'. 57" the length of a sidereal year of Saturn. Thus we find the periodic times of all the superior planets. The periodic times of the inferior are found from their conjunctions.

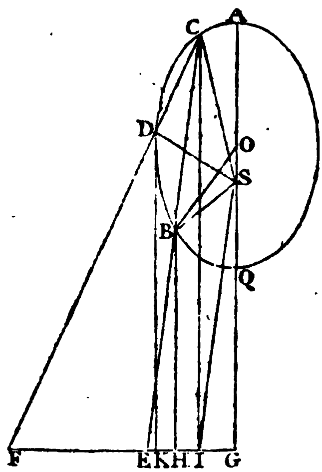
The periodic times of the planets are as follows; Mercury, 87*d*. 23*h*. 15'. 43",6; Venus, 224*d*. 16*h*. 49'. 10",6; Mars, 1*y*. 321*d*. 23*h*. 30'. 35",6; Jupiter, 11*y*. 315*d*. 14*h*. 27'. 10",8; Saturn, 29*y*. 174*d*. 1*h*. 51'. 11",2; the Georgian, 83*y*. 150*d*. 18*h*.

# CHAP. XIII.

## ON THE GREATEST EQUATION, EXCENTRICITY, AND PLACE OF THE APHELIA, OF THE ORBITS OF THE PLANETS.

(178.) HAVING determined the mean motions of the planets, we proceed next to show the method of finding the greatest equation of their orbits, the excentricity, and place of their aphelia. For although, in order to determine the mean motions very accurately, these things were supposed to be known, yet, without them, the mean motions may be so nearly ascertained, that these elements may from thence be very accurately settled. By Art. 161, we may find the distance of a planet from the sun in any point of it's orbit. The problem therefore is, having given in length and position, three lines drawn from the focus of an ellipse, to determine the ellipse.

(179.) Let $SB$, $SC$, $SD$, be the three lines; produce $CB$, $CD$, and take $SB : SC :: EB : EC$, and




$SC : SD :: CF : DF$, then $SC-SB : SC :: BC : EC=\frac{SC\times BC}{SC-SB}$, and $SC-SD : SC :: DC : CF=$

$\frac{SC \times DC}{SC - SD}$. Join $FE$, and draw $DK$, $CI$, $BH$, perpendicular to it. Now, by similar triangles, $IC : HB :: EC : EB ::$ (by const.) $SC : SB$; also, $IC : KD :: CF : DF :: SC : SD$. Hence, the proportion of $IC$, $HB$, $KD$, is the same as $SC$, $SB$, $SD$, consequently $EF$ is the directrix of the ellipse passing through $B$, $C$, $D$, (Con. Sect. p. 31). Through $S$ draw $ASQG$ perpendicular to $FE$; take $GA : AS :: CI : CS$, and $GQ : SQ :: CI : CS$; then $CI + CS : CS :: GS : SQ = \frac{CS \times GS}{CI + CS}$; in like manner we find $AS = \frac{CS \times GS}{CI - CS}$, and $A$, $Q$, will be the vertices of the conic section.

(180.) *Calculation.* In the triangles $SBC$, $SCD$, we know two sides and the included angles, they being the distances of the observed places in the orbit; hence, (Trig. Art. 135) we can find $BC$, $CD$, and the angles $BCS$, $SCD$, and consequently $BCD$. Hence (179) we know $CE$ and $CF$, and the angle $ECF$ being also known, the angle $CEF$ can be found. Therefore in the right-angled triangle $CIE$, $CE$ and the angle $E$ are given; hence, (Trig. Art. 128) $CI$ is known. Join $SI$; then in the triangle $SIC$ we know $CI$, $CS$ and the angle $SCI$ $(= BCS - ECI)$; hence, we know $SI$, and the angles $CIS$, $CSI$, and hence the angle $SIG$ is known; therefore, in the right-angled triangle $SIG$, we know $SI$ and the angle $SIG$, from whence $SG$ is found. Hence, (179) we know $SA$, $SQ$, half the difference of which is the excentricity, and their sum$=AQ$. Lastly, in the triangle $BSO$ ($O$ being the other focus) we know all the sides, to find the angle $BSA$ (Trig. Art. 133), the distance of the aphelion from the observed place $B$.

In the year 1740, on July 17, August 26, September 6, M. *de la Caille* found three distances of Mercury (the mean distance being 10000) as follows: $SB$

$=10351,5$, $IC=11325,5$, $SD=9672,166$, the angle $BSC=3^s.27^\circ.0'.35''$, and $CSD=44^\circ.40'.4''$. Hence, $BSC=29^\circ.55'.5''$, $BC=18941$, $SCD=56^\circ.49'$, $CD=8124,5$, $BCF=86^\circ.44'.5''$, $CE=215004$, $CE=55647$, $CEF=14^\circ.41'.44''$, $CI=54543$, $CSI=124^\circ.47'.45''$, $CIS=9^\circ.49'.4''$, $SI=47281$, $SIG=80^\circ.10'.56''$, $SG=46589$, $SP=8010,5$, $SA=12209$, $SO=4198,5$; hence, the excentricity $=2099,75$, $BSA=71^\circ.37'.23$, or $2^s.11^\circ.37'.23''$, which added to $6^s.2^\circ.13'.51''$, the position of $SB$, gives $8^s.13^\circ.51'.14''$ for the place of the aphelion. Hence, the greatest equation is $24^\circ.3'.5''$.

(181.) Or from the same data, the place of the aphelion and excentricity may be thus found. Put the semi-axis major $=1$, $SB=a$, $SD=b$, $SC=c$, the angle $BSD=v$. $BSC=u$, $BSQ=x$, $OS=e$, half the parameter $=r$. Then, by a well-known property of the ellipse (Conic Sect. Ellipse, Prop. 16), $a=\frac{r}{1+e\cos.x}$,

$b=\frac{r}{1+e.\cos.\overline{v+x}}$, $c=\frac{r}{1+e.\cos.\overline{u+x}}$; hence, $r=a+ae.\cos.x=b+be.\cos.\overline{v+x}=c+ce.\cos.\overline{u+x}$; therefore, $\frac{b-a}{a.\cos.x-b.\cos.\overline{v+x}}=e=\frac{c-a}{a.\cos.x-c.\cos.\overline{u+x}}$, now for $\cos.\overline{v+x}$, and $\cos.\overline{u+x}$, substitute $\cos.v.\cos.x-\sin.v\sin.x$, and $\cos.u.\cos.x-\sin.u.\sin.x$ (Trig. Art. 102,) and we shall have

$$\frac{b-a}{a.\cos.x-b.\cos.v.\cos.x+b.\sin.v.\sin.x}=\frac{c-a}{a.\cos.x-c.\cos.u.\cos.x+c.\sin.u.\sin.x};$$

divide each denominator by $\cos.x$, and we have

$$\frac{b-a}{a-b.\cos.v+b.\sin.v.\tan.x}=\frac{c-a}{a-c.\cos.u+c.\sin.u.\tan.x};$$

hence, $\tan.x=\frac{b.\overline{c-a}.\cos.v-c.\overline{b-a}.\cos.u-a.\overline{c-b}}{b.\overline{c-a}.\sin.v-c.\overline{b-a}.\sin.u}$, which gives the place of the perihelion. Hence, we

know $e = \frac{c-a}{a.\cos.x - c.\cos.\overline{u+x}}$ = the excentricity; consequently $1-e$ and $1+e$, the perihelion and aphelion distances, are known. The *species* of the ellipse being determined, it's major axis may be thus found: Compute the *mean* anomaly corresponding to the angle *CSB*, then say, as that mean anomaly : 360° :: the time of describing the angle *CSB* : the periodic time. The periodic time being known, the major axis is found (162) by *Kepler's* Rule. For other practical methods, see my *Complete System of Astronomy*.

(182.) All the epochs in our Astronomical Tables are reckoned from noon on December 31, in the common years, and from January 1, in the bissextiles.

The places of the aphelia for the beginning of 1750, are, Mercury, $8^s. 13°. 33'. 58''$; Venus, $10^s. 7°. 46'. 42''$; the Earth, $3^s. 8°. 37'. 16''$; Mars, $5^s. 1°. 28'. 14''$; Jupiter, $6^s. 10°. 21'. 4''$; Saturn, $8^s. 28°. 9'. 7''$; the Georgian, $11^s. 16°. 19'. 30''$.

The excentricities of the orbits, the mean distance of the earth from the sun being 100000, are, Mercury, 7955,4; Venus, 498; the Earth, 1681,395; Mars, 14183,7; Jupiter, 25013,3; Saturn, 53640,42; the Georgian, 90804.

The greatest equations are, Mercury, $23°. 40'. 0''$; Venus, $0°. 47'. 20''$; the Earth, $1°. 55'. 36'',5$; Mars, $10°. 40'. 40''$; Jupiter, $5°. 30'. 38'',3$; Saturn, $6°. 26'. 42''$; the Georgian, $5°. 27'. 16''$.

The aphelia of the orbits of the planets have a motion, which may be found, from finding the places of the aphelia of each at two different times. Those motions in longitude in 100 years are, Mercury, $1°. 33'. 45''$; Venus, $1°. 21'. 0''$; the Earth, $1°. 43'. 35''$; Mars, $1°. 51'. 40''$; Jupiter, $1°. 34'. 33''$; Saturn, $1°. 50'. 7''$.

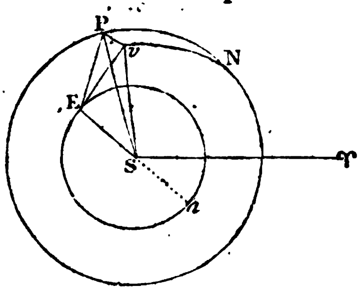
According to the calculation of M. *de la Grange*, the aphelion of the *Georgian* Planet is progressive $3'',17$ in the year, from the action of Jupiter and Saturn; consequently it's motion in longitude is $50'',25 + 3'',17 = 53'',42$.

# Chap. XIV.

## ON THE NODES AND INCLINATIONS OF THE ORBITS OF THE PLANETS TO THE ECLIPTIC.

(183.) From observing the course of the planets for one revolution, their orbits are found to be inclined to the ecliptic, for they appear only twice in a revolution to be in the ecliptic; and as it is frequently requisite to reduce their places in the ecliptic, ascertained from observation, to the corresponding places in their orbits, it is necessary to know the inclinations of their orbits to the ecliptic, and the points of the ecliptic where their orbits intersect it, called the *Nodes*. But, previous to this, we must show the method of reducing the places of the planets seen from the earth to the places seen from the sun, and how to compute the heliocentric latitudes.

(184.) Let $E$ be the place of the earth, $P$ the planet, $S$ the sun, ♈ the first point of aries; draw $Pv$



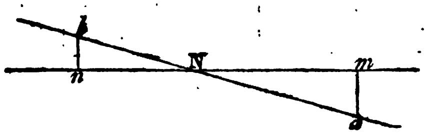

perpendicular to the ecliptic, and produce $Es$ to $a$. Compute, at the time of observation, the longitude of the sun seen at $a$ (115), and you have the longitude of the earth at $E$, or the angle ♈ $SE$; compute also the longitude of the planet, or the angle ♈ $Sv$ (115),

and the difference of these two angles is the angle $ESv$ of *commutation*. Observe the place of the planet in the ecliptic, and the place of the sun being known, we have the angle $vES$ of elongation in respect to longitude; hence, we know the angle $SvE$, which measures the difference of the places of the planets seen from the earth and the sun; therefore, the place of the planet seen from the earth being known, the place seen from the sun will be known. Also,

tan. $PEv$ : rad. :: $vP$ : $Ev$ (Trig. Art. 123)
rad. : tan. $PSv$ :: $vS$ : $vP$

∴ tan. $PEv$ : tan. $PSv$ :: $vS$ : $Ev$ :: sin. $SEv$ : sin. $ESv$; that is, *the sine of elongation in longitude : sin. of the difference of the longitudes of the earth and planet :: tan. of the geocentric latitude : tan. of the heliocentric latitude.* When the latitude is small, $Sv$ : $Ev$ is very nearly as $PS$ : $PE$, which, in opposition, is very nearly as $PS : PS-SE$. Or we may compute (167) the values of $PS$ and $SE$, which we can do with more accuracy than we can compute the angles $SEv$ and $ESv$. The *curtate* distance $Sv$ of the planet from the sun may be found, by saying, rad. : cos. $PSv$ :: $PS$ : $Sv$.

(185.) Now to determine the place of the node, find the planet's heliocentric latitudes just before and after it has passed the node, and let $a$ and $b$ be the places in the orbit, $m$ and $n$ the places reduced to the ecliptic; then the triangles $amN$, $bnN$ (which we




may consider as rectilinear) being similar, we have $am$ : $bn$ :: $Nm$ : $Nn$; therefore, $am+bn$ : $am$ :: $Nm$ $+Nn$ $(mn)$ : $Nm$, or $am+bn$ : $mn$ :: $am$ : $Nm$, that is, *the sum of the two latitudes : the difference of the longitudes :: either latitude: the distance of the node from the longitude corresponding to that latitude.* Or

if we take the two latitudes from the earth, it will be very nearly as accurate when the observations are made in opposition. If the distance of the observations should exceed a degree, this rule will not be sufficiently accurate, in which case we must make our computations for spherical triangles thus. Put $mn = a$, $am = \beta$, $bn = b$, $Nm = x$; then (Trig. Art. 212) $\frac{\sin. a - x}{\tan. b} = \cot. N = \frac{\sin. x}{\tan. \beta}$, radius being unity; but (Trig. Art. 101) $\sin. a - x = \sin. a \times \cos. x - \sin. x \times \cos. a$; hence, $\frac{\sin. a \times \cos. x - \sin. x \times \cos. a}{\tan. b} = \frac{\sin. x}{\tan. \beta}$; therefore, $\frac{\sin. a \times \tan \beta}{\tan. b + \cos. a \times \tan. \beta} = \frac{\sin. x}{\cos. x} = \tan. x$.

Ex. Mr. *Bugge* observed the right ascension and declination of *Saturn*, and thence deduced (114,184) the following heliocentric longitudes and latitudes.

| 1784. | *Apparent Time.* | *Heliocentric Lon.* | *Heliocentric Lat.* |
|---|---|---|---|
| July | 12, at $12^h$. 3′. 1″ | $9^s$. 20°. 37′. 29″ | 0°. 3′. 13″ N. |
| | 20, — 11. 29. 9 | 9. 20. 51. 53 | 0. 2. 41 |
| Aug. | 1, — 10. 38. 25 | 9. 21. 13. 17 | 0. 1. 34 |
| | 8, — 10. 9. 0 | 9. 21. 26. 2 | 0. 0. 56. |
| | 21, — 9. 14. 59 | 9. 21. 49. 27 | 0. 0. 2 |
| | 27, — 8. 50. 19 | 9. 22. 0. 12 | 0. 0. 27 S. |
| | 31, — 8. 33. 47 | 9. 22. 7. 32 | 0. 0. 50 |
| Sept. | 5, — 8. 13. 45 | 9. 22. 16. 28 | 0. 1. 21 |
| | 15, — 7. 33. 45 | 9. 22. 34. 32 | 0. 1. 59 |
| Oct. | 8 — 6. 4. 23 | 9. 23. 16. 15 | 0. 3. 35 |

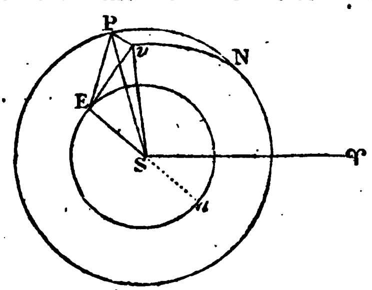
From the observations on August 21 and 27, by considering the triangles as plane, $x = 44'',5$; from those on 21 and 31, $x = 44'',5$; and from those on August 21, and September 5, $x = 40''$; the mean of these is $x = 42''$; Mr. *Bugge* makes $x = 41''$, probably by taking the mean of a greater number, or computing from considering them as spherical triangles; hence, the heliocentric place of the descending node was $9^s$.

21°. 50′. 8″,5. Now on August 21, at 9*h*. 12′. 26″ true time, *Saturn*'s heliocentric longitude was 9ˢ. 21°. 49′. 27″, and on 27, at 8*h*. 49′. 23″ true time, it was 9. 22°. 0′. 12″; therefore, in 5*d*. 23*h*. 36′. 57″ Saturn moved 10′. 45″ in longitude; hence, 10′. 45″ : 41″ :: 5*d*. 23*h*. 36′. 57″ : 9*h*. 7′. 44″ the time of describing 41″ in longitude, which added to August 21, 9*h*. 12′. 26″, gives August 21, 18*h*. 20′. 10″, the time when Saturn was in it's node.

The longitudes of the nodes of the planets for the beginning of 1750, are, Mercury, 1ˢ. 15°. 20′. 43″; Venus, 2ˢ. 14°. 26′. 18″; Mars, 1ˢ. 17°. 38′. 38″; Jupiter, 3ˢ. 7°. 55′. 32″; Saturn, 3ˢ. 21°. 32′. 22″; Georgian, 2ˢ. 12°. 47′.

(186.) To determine the inclination of the orbit, we have $am$ the latitude of the planet, and $mN$ it's distance upon the ecliptic from the node; hence, Trig. Art. 210) sin. $mN$ : tan. $am$ :: rad. : tan. of the angle $N$. But the observations which are near the node must not be used to determine the inclination, as a very small error in the latitude will make a considerable error in the angle. If we take the observation on July 20, it gives the angle 2°. 38′. 15″: if we take that on October 8, it gives the angle 2°. 22′. 13″; the mean of these is 2°. 30′. 14″, the inclination of the orbit to the ecliptic, from these observations. Or the inclination may be found thus.

(187.) Find the angle $PSv$ (184), then the place of the planet and that of it's node being given, we




know $vN$; hence (Trig. Art. 210), sin. $vN$ : tan. $Pv$ :: rad. : tan. $PNv$ the inclination of the orbit.

On March 27, 1694, at 7*h*. 4′. 40″, at Greenwich, Mr. *Flamstead* determined the right ascension of *Mars* to be 115°. 48′. 55″, and it's declination 24°. 10′. 50″ north; hence, (184) the geocentric longitude was ♋ 23°. 26′. 12″, and latitude 2°. 46′. 38″. Let $S$ be the sun, $E$ the Earth, $P$ Mars, $v$ it's place reduced to the ecliptic. Now the true place of Mars (by calculation) seen from the sun was ♌ 28°. 44′. 14″, and the place of the sun was ♈ 7°. 34. 25″; hence, subtracting the place of the sun from the place of Mars seen from the earth, we have the angle $vES$ between the sun and Mars 105°. 51′. 47″; and the place of the earth being ♎ 7°. 34′. 25″, take from it the place of Mars, and we have the angle $ESv$=38°. 50′. 11″; hence, (187) sin. 105°. 51′. 47″ : sin. 38°. 50′. 11″ :: tan. $PEv$=2°.46′. 38″ : tan. $PSv$= 1°. 48′. 36″. Now the place of the node was in ♉ 17°. 15′, which subtracted from ♌ 28°. 44′. 14″, gives 101°. 29′. 14″ for the distance $vN$ of Mars from it's node; hence, sin. $vN$=101°. 29′. 14″ : tan. $Pv$= 1°. 48′. 36″ :: rad. : tan. $PNv$= 1°. 50′. 50″, the inclination of the orbit. Mr. *Bugge* makes the inclination to be 1°. 50′. 56″,56, for Mar. 1788. M. *de la Lande* makes it 1°. 51′ for 1780.

The inclination of the orbits of the planets are, Mercury, 7°. 0′. 0″; Venus, 3°. 23′. 35″; Mars, 1°. 51′. 0″; Jupiter, 1°. 18′. 56″; Saturn, 2°. 29′. 50″; Georgian, 46′. 20″.

(188.) The motion of the nodes is found, by comparing their places at two different times; from whence, that of Mercury in 100 years is found to be 1°. 12′. 10″; Venus, 0°. 51′. 40″; Mars, 0°. 46′. 40″; Jupiter, 0°. 59′. 30″; Saturn, 0°. 55′. 30″. This motion is in respect to the equinox.

The *Georgian* planet has not been discovered long enough to determine the motion of it's nodes from observation. M. *de la Grange* has found the annual motion to be 12″,5 by theory. But if we take the density of Venus according to M. *de la Lande*, it will be 20″.40‴, which he uses in his tables.

Thus we have determined all the elements necessary for computing the place of a planet in it's orbit at any time; but to facilitate the operation, which would be extremely tedious if we had only the elements thus given, astronomers have constructed tables of their motions, by which their places at any time may be very readily computed.

Since the discovery of the *Georgium Sidus*, four other primary planets have been discovered: The first called *Ceres*, was discovered by *M. Piazzi* at *Palermo*, Jan. 1, 1801; the second, called *Pallas*, was discovered by Dr. *Olbers* at *Bremen*, March 28, 1802; the third, called *Juno*, was discovered by *M. Harding* at *Lilienthal*, September 1, 1804; and the fourth, called *Vesta*, was discovered by Dr. *Olbers*, March 29, 1807. The following Table contains the elements of the orbits of the three first; the orbit of the fourth is not yet computed.

| *Elements.* | *Ceres.* | *Pallas.* | *Juno.* |
|---|---|---|---|
| Epoch of Mean Long. 1805 for Merid. of Seeberg | 30°.12′.7″,7 | 18° 13′ 1″,4 | 42. 36. 36 |
| Long. of Aphelion - - | 326. 28. 4,4 | 301. 3. 24,3 | 233. 11. 40 |
| Long. of ascending Node | 81. 0. 41 | 172. 30. 47 | 171. 4. 15 |
| Inclination of the Orbit - | 10. 37. 36 | 34. 37. 43 | 13. 3. 38 |
| Excentricity - - - - | 0,0784699 | 0,2461007 | 0,254236 |
| Log. of Mean Distance - | 0,442004 | 0,4417647 | 0,4256078 |
| Mean Diurn. Trop. Mot. | 771″,0524 | 771″,6802 | 815″,959 |

Dr. *Herschel* makes the diameter of *Pallas* 147 miles; and that of *Ceres* 161,6 miles.

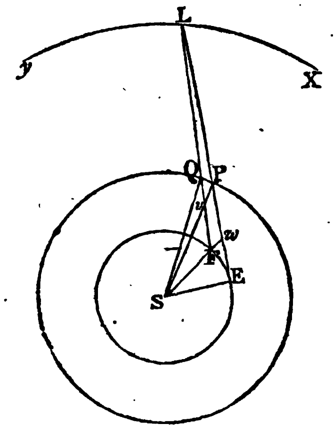
# Chap. XV.

## ON THE APPARENT MOTIONS AND PHASES OF THE PLANETS.

(189.) As all the planets describe orbits about the sun as their center, it is manifest, that to a spectator at the sun they would appear to move in the direction in which they really do move, and shine with full faces. But to a spectator on the earth, which is in motion, they will sometimes appear to move in a direction contrary to their real motion, and sometimes appear stationary; and as the same face which is turned towards the earth, is not turned towards the sun, except in conjunction and opposition, some part of the disc which is towards the earth will not be illuminated. These, with some other appearances and circumstances which are observed to take place among the planets, we shall next proceed to explain; and as they are matters in which great accuracy is never requisite, being of no great practical use, but rather subjects of curiosity, we shall consider the motions of all the planets as performed in circles about the sun in the center, and lying in the place of the ecliptic.

(190.) To find the *position* of a planet when stationary. Let $S$ be the sun, $E$ the earth, $P$ the cotemporary position of the planet, $Xy$ the sphere of the fixed stars, to which we refer the motions of the planets; let $EF$, $PQ$, be two indefinitely small arcs described in the same time, and let $EP$, $FQ$ produced, meet at $L$; then it is manifest, that whilst the earth moves from $E$ to $F$, the planet appears stationary at $L$; and on account of the immense distance of the fixed stars, $EPL$, $FQL$ may be considered as parallel. Draw $SE$, $SFw$, $SvP$, and $SQ$; then, as $EP$ and

$FQ$ are parallel, the angle $QFS - PES = PwS - PES = ESF$, and $SPw - SQF = SvF - SQF = PSQ$; that is, the cotemporary variations of the angles $E$ and $P$ are as $ESF : PSQ$, the cotemporary variations of the angular velocities of the earth and planet, or (because the angular velocities are inversely as the periodic times, or inversely in the sesquiplicate ratio of the distances) as $SP^{\frac{3}{2}} : SE^{\frac{3}{2}}$, or, (if $SP : SE :: a$ 1) as $a^{\frac{3}{2}} : 1^{\frac{3}{2}}$. But sin. $SEP$ : sin. $SPE$ being as $SP : SE$,




or, $a : 1$, the cotemporary variations of these angles will be as their tangents*. Hence, if $x$ and $y$ be the sines of the angles $SEP$ and $SPE$, we have $x : y :: a : 1$, and $\frac{x}{\sqrt{1-x^2}} : \frac{y}{\sqrt{1-y^2}} :: a^{\frac{3}{2}} : 1$, whence $x^2 = \frac{a^3 - a^2}{a^3 - 1} = \frac{a^2}{a^2 + a + 1}$, and $x = \frac{a}{\sqrt{a^2 + a + 1}}$ the sine of the planet's elongation from the sun, when stationary.

Ex. If $P$ be the earth, and $E$ Venus; and we take the distances of the earth and Venus to be 100000 and 72333, we find $x = 0{,}48264$ the sine of $28°.\ 51'.\ 5''$,

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* See the Optics, Art. 421.

the elongation of Venus when stationary, upon the supposition of circular orbits.

For excentric orbits, the points will depend upon the position of the apsides and places of the bodies at the time. We may, however, get a very near approximation thus. Find the time when the planet would be stationary if the orbits were circular, and compute for several days, about that time, the geocentric place of the planet, so that you get two days, on one of which the planet was direct, and on the other retrograde, in which interval it must have been stationary, and the point of time when this happened may be determined by interpolation.

(191.) To find the *time* when a planet is stationary, we must know the time of it's opposition, or inferior conjunction. Let $m$ and $n$ be the daily angular velocities of the earth and planet about the sun, and $v$ the angle $PSE$ when the planet is stationary; then $m-n$, or $n-m$, is the daily *variation* of the angle at the sun between the earth and planet, according as it is a superior or inferior planet; hence, $m-n$, or $n-m$, $: v ::$ 1 day : $\frac{v}{m-n}$, or $\frac{v}{n-m}$, the time from opposition or conjunction to the stationary points both before and after. Hence, the planet must be stationary twice every *synodic** revolution.

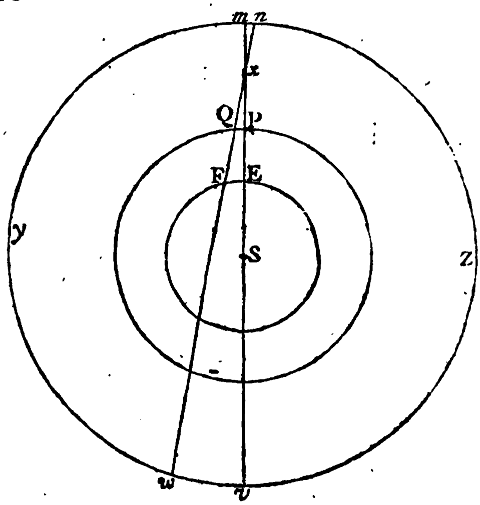
Ex. Let $P$ be the earth, $E$ Venus; then by the Example to Art. 190, the angle $SPE = 28°. 51',5$; therefore, $PSE = 13°$; also, $n-m = 37'$; hence, $37' : 13° ::$ 1 day : 21 days the time between the inferior conjunction and stationary positions.

(192.) If the elongation be observed when stationary, we may find the distance of the planet from the sun, compared with the earth's distance, supposed

* A Synodic revolution is the time between two conjunctions of the same sort, or two oppositions of a planet.

to be unity. For (190) $x^2 = \frac{a^2}{a^2+a+1}$; hence, $a^2 + \frac{x^2}{x^2-1} \times a = -\frac{x^2}{x^2-1} =$ (if $t =$ the tangent of the angle whose sine is $x$) $a^2 - t^2 a = t^2$; consequently $a = \frac{1}{2}t^2 + t\sqrt{1+\frac{t^2}{4}}$, upon the supposition of circular orbits.

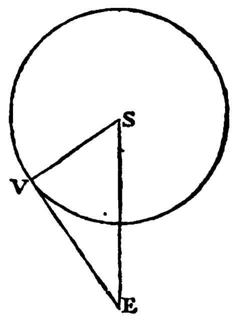
(193.) A superior planet is retrograde in opposition, and an inferior planet is retrograde in it's inferior conjunction; for let $E$ be the earth, $P$ a superior planet in opposition; then, as the velocities are as the inverse square roots of the radii of the orbits, the superior planet moves slowest; hence, if $EF$, $PQ$, be two indefinitely small cotemporary arcs, $PQ$ is less than $EF$, and on account of the immense distance of the sphere $yZ$ of the fixed stars, $FQ$ must cut $EP$ in some point $x$ between $P$ and $m$, consequently, the planet appears retrograde from $m$ to $n$. If $P$ be the




earth, and $E$ an inferior planet in inferior conjunction, it will appear retrograde from $v$ to $w$. These retrograde motions must necessarily continue till the

planets become stationary. Hence, from this and the last Article, a superior planet appears retrograde from it's stationary point before opposition to it's stationary point after; and an inferior planet, from it's stationary point before inferior conjunction to it's stationary point after.

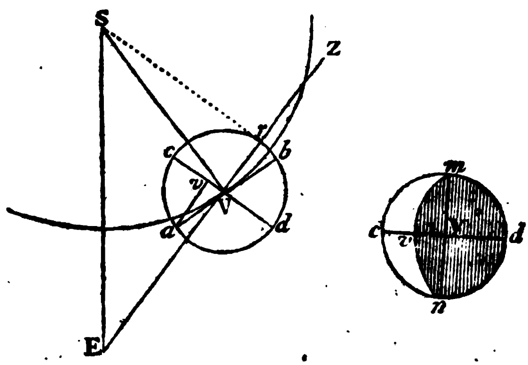
(194.) If $S$ be the sun, $E$ the earth, $V$ Venus or Mercury, and $EV$ a tangent to the orbit of the planet,




then will the angle $SEV$ be the greatest elongation of the planet from the sun; which angle, if the orbits were circles having the sun in their center, would be found by saying, $ES : SV :: \text{rad.} : \sin. SEV$. But the orbits are not circular, in consequence of which the angle $EVS$ will not be a right angle, unless the greatest elongation happens when the planet is at one of it's apsides. The angle $SEV$ is also subject to an alteration from the variation of $SE$ and $SV$. The *greatest* angle $SEV$ happens, when the planet is in it's *aphelion* and the earth in it's *perigee*; and the *least* angle $SEV$, when the planet is in it's *perihelion* and the earth in it's *apogee*. M. *de la Lande* has calculated these greatest elongations, and finds them 47°. 48′. and 44°. 57′ for *Venus*, and 28°. 20′ and 17°. 36′ for *Mercury*. If we take the mean of the greatest elongations of Venus, which is 46°. 22′,5, it gives the angle $VSE$=43°. 37′,5 : and as the difference of the daily *mean* motions of Venus and the earth about the sun is 37′, we have 37′ : 43°. 37′,5 :: 1 day : 70,7

days, the time that would elapse between the greatest elongations and the inferior conjunction, if the motions had been uniform, which will not vary much from the true time.

(195.) To delineate the appearance of a planet at any time. Let $S$ be the sun, $E$ the earth, $V$ Venus,




for example; $aVb$ the plane of illumination perpendicular to $SV$, $cVd$, the plane of vision perpendicular to $EV$, and draw $av$ perpendicular to $cd$; then $ca$ is the breadth of the visible illuminated part, which is projected by the eye into $cv$, the versed sine of $Cva$, or $SVZ$, for $SVc$ is the complement of each. Now the circle terminating the illuminated part of the planet, being seen obliquely, appears to be an ellipse (Con. Sect. p. 37.); therefore, if $cmdn$ represent the projected hemisphere of Venus next to the earth, $mn$, $cd$, two diameters perpendicular to each other, and we take $cv$=the versed sine of $SVZ$, and describe the ellipse $mvn$, then $cv$ is the axis minor, and $mcnvm$ will represent the visible enlightened part, as it appears at the earth; and from the property of the ellipse (Con. Sect. Ell. Prop. 7. Cor. 6.), this area varies as $cv$. Hence, *the visible enlightened part* : *the whole disc* :: *the versed sine of SVZ* : *diameter*.

Hence, *Mercury* and *Venus* will have the same phases from their inferior to their superior conjunction, as the moon has from the new to the full; and

the same from the superior to the inferior conjunction, as the moon has from the full to the new. *Mars* will appear gibbous in quadratures, as the angle $SVZ$ will then differ considerably from two right angles, and consequently the versed sine will sensibly differ from the diameter. For *Jupiter*, *Saturn*, and the *Georgian*, the angle $SVZ$ never differs enough from two right angles to make those planets appear gibbous, so that they always appear full-orbed.

(196.) Let $V$ be the moon; then as $EV$ is very small compared with $VS$, $ES$, these lines will be very nearly parallel, and the angle $SVZ$ very nearly equal to $SEV$; hence, *the visible enlightened part of the moon varies very nearly as the versed sine of it's elongation.*

(197.) Dr. *Halley* proposed the following problem: To find the position of *Venus* when brightest, supposing it's orbit, and that of the earth, to be circles, having the sun in their center. Draw $Sr$ perpendicular to $EVZ$, and put $a=SE$, $b=SV$, $x=EV$, $y=Vr$; then $b-y$ is the versed sine of the angle $SVZ$, which versed sine varies as the illuminated part; and as the intensity of light varies inversely as the square of it's distance, the quantity of light received at the earth varies as $\frac{b-y}{x^2}=\frac{b}{x^2}-\frac{y}{x^2}$; but by Euclid, B. II. P. 12, $a^2=b^2+x^2+2xy$; hence, $y=\frac{a^2-b^2-x^2}{2x}$; substitute this for $y$, and we get the quantity of light to be as $\frac{b}{x^2}-\frac{a^2-b^2-x^2}{2x^3}=\frac{2bx-a^2+b^2+x^2}{2x^3}=$ a maximum; put the fluxion $=0$, and we get $x=\sqrt{3a^2+b^2}-2b$. Now, if $a=1$, $b=,72333$, as in Dr. *Halley's* Tables, then $x=,43036$; hence, the angle $ESV=22°.\ 21'$, but the angle $ESV$, at the time of the planet's greatest elongation, is $43°.\ 40'$; hence, Venus is brightest between it's inferior conjunction and it's greatest elongation; also, the angle $SEV=39°.\ 44'$, the elongation of Venus

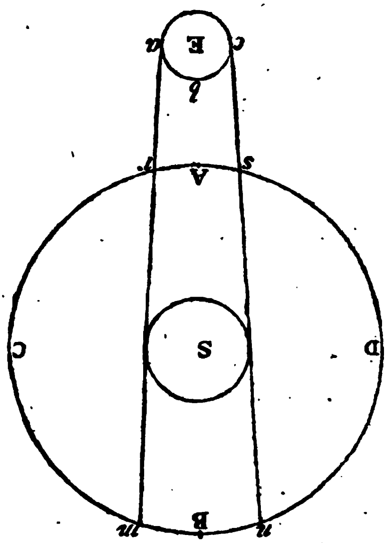
from the sun at the same time, and $\angle SVZ = VSE + VES = 62^\circ.\ 5'$, the versed sine of which is 0,53, radius being unity; hence (195), the visible enlightened part : whole disc. :: 0,53 : 2; Venus therefore appears a little more than one fourth illuminated, and answers to the appearance of the moon when five days old. Her diameter here is about 39″, and therefore the enlightened part is about 10″,25. At this time, Venus is bright enough to cast a shadow at night. This situation happens about 36 days before and after it's inferior conjunction; for, supposing Venus to be in conjunction with the sun, and when seen from the sun to depart from the earth at the rate of 37′ in 1 day, we have 37′ : 22°. 21′ :: 1 day : 36 days nearly, the time from conjugation till Venus is brightest.

(198.) If we apply this to Mercury, $b = ,3171$, and $x = 1,00058$; hence, the angle $ESV = 78^\circ.\ 55'\frac{1}{4}$; but the same angle, at the time of the planet's greatest elongation, is 67°. 13′½. Hence, Mercury is brightest between it's greatest elongation and superior conjunction. Also, the angle $SEV = 22^\circ.\ 18'\frac{1}{4}$, the elongation of Mercury at that time.

(199.) When Venus is brightest, and at the same time is at it's greatest north latitude, it can then be seen with the naked eye at any time of the day, when it is above the horizon; for when it's north latitude is the greatest, it rises highest above the horizon, and therefore is more easily seen, the rays of light having to come through a less part of the atmosphere, the higher the body is. This happens once in about eight years, Venus and the earth returning to the same parts of their orbits after that interval of time.

(200.) *Venus* is a *morning* star from inferior to superior conjunction, and an *evening* star from superior to inferior conjunction. For let *S* be the sun, *E* the earth, *ACBD* the orbit of Venus, *arm*, *csn*, two tangents to the earth, representing the horizon at *a* and *c*. Then the earth, revolving about it's axis according to the order *abc*, when a spectator is at *a*, the part *rCm*

of the orbit of Venus is above the horizon, but the sun is not yet risen; therefore Venus, in going from *r* through *C* to *m*, appears in the morning before sun-rise. When the spectator is carried by the earth's rotation to *c*, the sun is then set, but the part *nDs* of Venus' orbit is still above the horizon; therefore,




Venus, in going from *n* through *D* to *s*, appears in the evening after sun-set.

(201.) If two planets revolve in circular orbits, to find the time from conjunction to conjunction. Let $P$ = the periodic time of a superior planet, $p$ = that of an inferior, $t$ = the time required. Then $P : 1$ day :: $360° : \frac{360°}{P}$ the angle described by the superior planet in 1 day; for the same reason, $\frac{360°}{p}$ is the angle described by the inferior planet in 1 day; hence, $\frac{360°}{p} - \frac{360°}{P}$ is the daily angular velocity of the inferior planet

*from* the superior. Now if they set out from conjunction, they will return into conjunction again after the inferior planet has gained 360°; hence, $\frac{360^\circ}{p} - \frac{360^\circ}{P} : 360^\circ :: 1 \text{ day} : t = \frac{Pp}{P-p}$. This will also give the time between two oppositions, or between any two similar situations.

# CHAP. XVI.

## ON THE MOON'S MOTION FROM OBSERVATION AND IT'S PHÆNOMENA.

(202.) THE moon being the nearest, and after the sun, the most remarkable body in our system, and also useful for the division of time, it is no wonder that the ancient astronomers were attentive to discover it's motions; and it is a very fortunate circumstance that their observations have come down to us, as from thence it's mean motion can be more accurately settled, than it could have been by modern observations only; and it moreover gave occasion to Dr. *Halley*, from the observations of some ancient eclipses, to discover an acceleration in it's mean motion. The proper motion of the moon, in it's orbit about the earth, is from west to east; and from comparing it's place with the fixed stars in one revolution, it is found to describe an orbit inclined to the ecliptic; it's motion also appears not to be uniform; and the position of the orbit, and the line of it's apsides are observed to be subject to a continual change. These circumstances, as they are established by observation, we come now to explain.

### *To determine the Place of the Moon's Nodes.*

(203.) The place of the moon's nodes may be determined as in Art. 185, or by the following method.

In a *central* eclipse of the moon, the moon's place at the middle of the eclipse is directly opposite to the sun, and the moon must also then be in the node; calculate therefore the true place of the sun, or, which is more exact, find it's place by observation, and the

opposite point will be the true place of the moon, and consequently the place of it's node.

Ex. M. *Cassini*, in his Astronomy, p. 281, informs us, that on April 16, 1707, a central eclipse was observed at Paris, the middle of which was determined to be at 13*h*. 48′ apparent time. Now the true place of the sun, calculated for that time, was $0^s. 26^\circ. 19' 17''$; hence, the place of the moon's node was $6^s. 26^\circ. 19'. 17''$. The moon passed from north to south latitude, and therefore this was the descending node.

(204.) To determine the mean motion of the nodes, find (203) the place of the nodes at different times, and it will give their motion in the interval; and the greater the interval, the more accurately you will get the mean motion. *Mayer* makes the mean annual motion of the nodes to be $12^\circ. 19'. 43'',1$.

### *On the Inclination of the Orbit of the Moon to the Ecliptic.*

(205.) To determine the inclination of the orbit, observe the moon's right ascension and declination when it is 90° from it's nodes, and thence compute it's latitude (114), which will be the inclination at that time. Repeat the observation for every distance of the sun from the earth, and for every position of the sun in respect to the moon's nodes, and you will get the inclination at those times. From these observations it appears, that the inclination of the orbit to the ecliptic is variable, and that the *least* inclination is about 5°, which is found to happen when the nodes are in quadratures; and the *greatest* is about 5°. 18′, which is observed to happen when the nodes are in syzygies. The inclination is also found to depend upon the sun's distance from the earth.

### *On the mean Motion of the Moon.*

(206.) The mean motion of the moon is found from observing it's place at two different times, and

you get the mean motion in that interval, supposing the moon to have had the same situation in respect to it's apsides at each observation; and if not, if there be a very great interval of the times, it will be sufficiently exact. To determine this, we must compare together the moon's places, first at a small interval of time from each other, in order to get nearly the mean time of a revolution; and then at a greater interval, in order to get it more accurately. The moon's place may be determined directly from observation, or deduced from an eclipse.

(207.) M. *Cassini*, in his Astronomy, p. 294, observes, that on September 9, 1718, the moon was eclipsed, the middle of which eclipse happened at 8*h*. 4′, when the sun's true place was $5^s$. 16°. 40′. This he compared with another eclipse, the middle of which was observed at 8*h*. 32′. on August 29, 1719, when the sun's place was $5^s$. 5°. 47′. In this interval of 354*d*. 28′ the moon made 12 revolutions and 349°. 7′ over; divide therefore 354*d*. 28′ by 12 revolutions + 349°. 7′. part of a revolution, and it gives 27*d*. 7*h*. 6′ for the time of one revolution. From two eclipses in 1699, 1717, the time was found to be 27*d*. 7*h*. 43′. 6″.

(208.) The moon was observed at Paris to be eclipsed on Sept. 20, 1717, the middle of which eclipse was at 6*h*. 2′. Now *Ptolemy* mentions, that a total eclipse of the moon was observed at Babylon on March 19, 720 years before J. C. the middle of which happened at 9*h*. 30′, at that place, which gives 6*h*. 48′ at Paris. The interval of these times was 2437 years (of which 609 were bissextiles) 147 days wanting 46′; divide this by 27*d*. 7*h*. 43′. 6″, and it gives 32585 revolutions and a little above ½. Now the difference of the two places of the sun, and consequently of the moon, at the times of observations, was $6^s$. 6°. 12′. Therefore, in the interval of 2437*y*. 174*d*. wanting 46′, the moon had made 32585 revolutions $6^s$. 6°. 12′, which gives 27*d*. 7*h*. 43′. 5″ for the mean time of a revolution. This determination is very exact, as the

moon was at each time very nearly at the same distance from it's apside. Hence, the mean *diurnal* motion is 13°. 10'. 35", and the mean *hourly* motion 32'. 56". 27"'½. M. *de la Lande* makes the mean *diurnal* motion 13°. 10'. 35",02784394. This is the mean time of a revolution in respect to the equinoxes. The place of the moon at the middle of the eclipse has here been taken the same as that of the sun, which is not accurate, except for a central eclipse; it is sufficiently accurate, however, for this long interval.

(209.) As the precession of the equinoxes is 50",25 in a year, or about 4" in a month, the mean revolution of the moon in respect to the fixed stars must be greater than that in respect to the equinox, by the time the moon is describing 4" with it's mean motion, which is about 7". Hence, the time of a sidereal revolution of the moon is 27*d*. 7*h*. 43'. 12".

(210.) Observe accurately the place of the moon for a whole revolution as often as it can be done, and by comparing the true and mean motions, the greatest difference will be double the equation. If two observations be found, where the difference of the true and mean motions is nothing, the moon must then have been in it's apogee and perigee (168). *Mayer* makes the mean excentricity 0,05503568, and the corresponding greatest equation 6°. 18'. 31"6. It is 6°. 18'. 32" in his last Tables, published by Mr. *Mason*, under the direction of Dr. *Maskelyne*.

(211.) To determine the place of the apogee, from M. *Cassini's* observations, we have the greatest equation $=5°. 1'. 44''5$; therefore (171), $57°. 17'. 48''8 : 2°. 30'. 52''25 :: AC = 100000 : CS = 4388$ for the moon's excentricity at that time*. Now (Fig. p. 101.) let $v$ be the focus in which the earth is situated; then

* The excentricity of the moon's orbit is subject to a variation, it being greatest when the apsides lie in syzygies, and least when in quadratures.

(169) supposing $QSP$ to be the mean anomaly, as $QvP$ is the true anomaly, their difference $SPv$ is the equation of the orbit, which equation is here $37'. 50'',5$; and as $PS = Pr$, the angle $vrS = 18'. 55'',25$; hence, (Trigonometry, Art. 128) $vS = 8776 : vr = 200000 :: \sin. vrS = 18'. 55'',25 : \sin. vSr$, or $QSr, = 7^{\circ}. 12'. 20''$, from which take $vrS = 18'. 55'',25$, and we have $QvP = 6^{\circ}. 53'. 25''$ the distance of the moon from it's apogee; add this to $2^{s}. 19^{\circ}. 40'$, the true place of the moon, and it gives $2^{s}. 26^{\circ}. 33'. 25''$ for the place of the apogee on December 10, 1685, at $10h. 38'. 10''$ mean time at Paris. This therefore may be considered as an *epoch* of the place of the apogee.

*To determine the mean Motion of the Apogee.*

(212.) Find it's place at different times, and compare the difference of the places with the interval of the time between. To do this, we must first compare observations at a small distance from each other, lest we should be deceived in a whole revolution; and then we can compare those at a greater distance. The mean annual motion of the apogee in a year of 365 days is thus found to be $40^{\circ}. 39'. 50''$, according to *Mayer*. *Horrox*, from observing the diameter of the moon, found the apogee subject to an annual equation of $12^{\circ},5$.

(213.) The motion of the moon having been examined for one month, it was immediately discovered that it was subject to an irregularity, which sometimes amounted to $5^{\circ}$ or $6^{\circ}$, but that this irregularity disappeared about every 14 days. And by continuing the observations for different months, it also appeared, that the points where the inequalities were the greatest, were not fixed, but that they moved forwards in the heavens about $3^{\circ}$ in a month, so that the motion of the moon, in respect to it's apogee, was about $\frac{1}{120}$ less than it's absolute motion; thus it appeared that the apogee

had a progressive motion. *Ptolemy* determined this *first* inequality, or equation of the orbit, from three lunar eclipses observed in the years 719 and 720, before J. C. at Babylon by the Chaldeans; from which he found it amounted to 5°. 1′, when at it's greatest. But he soon discovered that this inequality would not account for all the irregularities of the moon. The distance of the moon from the sun, observed both by *Hipparchus* and himself, sometimes agreed with this inequality, and sometimes it did not. He found that when the apsides of the moon's orbit were in quadratures, this *first* inequality would give the moon's place very well; but that when the apsides were in syzygies, he discovered that there was a further inequality of about $2°\frac{2}{3}$, which made the whole inequality about $7°\frac{2}{3}$. This *second* inequality is called the *Evection*, and arises from a change of excentricity of the moon's orbit. The inequality of the moon was therefore found, by *Ptolemy*, to vary from about 5° to $7°\frac{2}{3}$, and hence the mean quantity was 6°. 20′. *Mayer* makes it 6°. 18′. 31″,6. It is very extraordinary, that *Ptolemy* should have determined this to so great a degree of accuracy. We cannot here enter any further into the inequalities of the motion of the moon. They who wish to see more on this subject, may consult my *Complete System of Astronomy*.

(214.) *Times of the Revolutions of the Moon, of it's Apogee and Nodes, as determined by M.* de la Lande.

| | |
|---|---|
| Tropical revolution - - | $27^d.7^h.43'.4'',6795$ |
| Sidereal revolution - - | 27. 7. 43. 11,5259 |
| Synodic revolution - - | 29.12. 44. 2,8283 |
| Anomalistic revolution - | 27.13. 18. 33,9499 |
| Revolution in respect to the node - - | 27. 5. 5. 35,603 |
| Tropical revolution of the apogee - | $8^y$. 311. 8. 34. 57,6177 |
| Sidereal revolution of the apogee - - | 8. 312. 11. 11. 39,4089 |
| Tropical revolution of the node - - | 18. 228. 4. 52. 52,0296 |
| Sidereal revolution of the node - - - | 18. 223. 7. 13. 17,744 |
| Diurnal motion of the moon in respect to the equinox - - - | 13°. 10'. 35''02784394 |
| Diurnal motion of the apogee | 0. 6. 41,069815195 |
| Diurnal motion of the node - | 0. 3. 10,638603696 |

The years here taken are the common years of 365 days.

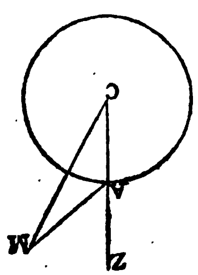
## *On the Diameter of the Moon.*

(215.) The diameter of the moon may be measured, at the time of it's full, by a micrometer; or it may be measured by the time of it's passing over the vertical wire of a transit telescope; but this must be when the moon passes within an hour or two of the time of the full, before the visible disc is sensibly changed from a circle. To find the diameter by the time of it's passage over the meridian, let $d'$ = the horizontal diameter of the moon, $c$ = sec. of it's declination, and $m$ = the length of a lunar day, or the time from the

passage of the moon over the meridian on the day we calculate, to the passage over the meridian the next day. Then (102) $cd'$ is the moon's diameter in right ascension; hence, 360° : $cd'$ :: $m$ : the time ($t$) of passing the meridian; therefore $d' = 360° \times \frac{t}{cm}$. If we observe when the limb of the moon comes to the meridian, we can find the time when the center comes to it, by adding to, or subtracting from, the time when the first or second limb comes to the meridian, half the time of the passage of the moon over the meridian. The time in which the semidiameter of the moon passes the meridian, may be found by two Tables, in the Tables of the moon's motion.

(216.) *Albategnius* made the diameter of the moon to vary from 29'. 30" to 35'. 20", and hence the mean is 32'. 25". *Copernicus* found it from 27'. 34". to 35'. 38", and therefore the mean 31'. 36". *Kepler* made the mean diameter 31'. 22". M. *de la Hire* made it 31'. 30". M. *Cassini* made the diameter from 29'. 30" to 33'. 38". M. *de la Lande*, from his own observations, found the mean diameter to be 31'. 26"; the extremes from 29'. 22" when the moon is in apogee and conjunction, and 33'. 31" when in perigee and opposition. The mean diameter here taken, is the arithmetic mean between the greatest and least diameters; the diameter at the mean distance is 31'. 7".

(217.) When the moon is at different altitudes



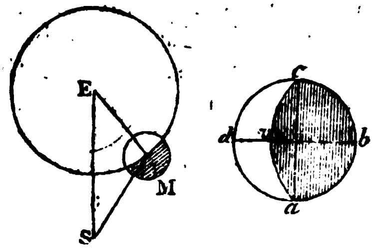

above the horizon, it is at different distances from the

spectator, and therefore there is a change of the apparent diameter. Let $C$ be the center of the earth, $A$ the place of a spectator on it's surface, $Z$ his zenith, $M$ the moon; then (Trig. Art. 128) sin. $CAM$, or $ZAM$ : sin. $ZCM$ :: $CM$ : $AM = \frac{CM \times \text{sin. } ZCM}{\text{sin. } ZAM}$; but the apparent diameter is inversely as it's distance; hence, the apparent diameter varies as $\frac{\text{sin. } ZAM}{\text{sin. } ZCM}$, $CM$ being supposed constant. Now, in the horizon, $\frac{\text{sin. } ZAM}{\text{sin. } ZCM}$ may be considered as equal to unity; hence, 1 : $\frac{\text{sin. } ZAM}{\text{sin. } ZCM}$, or sin. $ZCM$ : sin. $ZAM$, or cos. true alt. ($a$) : cos. apparent alt. ($A$) :: the horizontal diameter : the diameter at the apparent altitude ($A$). Hence, the horizontal diameter : it's increase :: cos. $a$ : cos. $A$ $-$ cos. $a$ = (Trig. Art. 111) 2 sin. $\overline{\tfrac{1}{2}a + \tfrac{1}{2}A}$ $\times$ sin. $\overline{\tfrac{1}{2}a - \tfrac{1}{2}A}$; therefore the increase of the semidiameter = hor. semidiameter $\times \frac{\text{sin } \overline{\frac{1}{2}a + \frac{1}{2}A} \times \text{sin. } \overline{\frac{1}{2}a - \frac{1}{2}A}}{\text{cos. } a}$; from this we may easily construct a table of the increase of the semidiameter for any horizontal semidiameter, and then for any other horizontal semidiameter, the increase will vary in the same proportion.

## *On the Phases of the Moon.*

(218.) By Art. 196, the greatest breadth of the visible illuminated part of the moon's surface, varies as the versed sine of the moon's elongation from the sun, very nearly; and the circle terminating the light and dark part, being seen obliquely, appears an ellipse; hence, the following delineation of the phases. Let $E$ be the earth, $S$ the sun, $M$ the moon; describe the circle $a\,b\,c\,d$, representing the hemisphere of the moon which is towards the earth, projected upon the plane

of vision; $ac$, $db$, two diameters perpendicular to each other; take $dv$ = the versed sine of elongation $SEM$,




and describe the ellipse $avc$, and (195) $adcva$ will represent the visible enlightened part; which will be horned between conjunction and quadratures; a semicircle at quadratures; and gibbous between quadratures and opposition; the versed sine being less than radius in the first case, equal to it in the second, and greater in the third. The visible enlightened part varying as $dv$, we have, the *visible enlightened part : whole :: versed sine of elongation : diameter.*

## *On the Libration of the Moon.*

(219.) Many Astronomers have given maps of the face of the moon; but the most celebrated are those of *Hevelius* in his *Selenographia,* in which he has represented the appearance of the moon in it's different states from the new to the full, and from the full to the new; these figures *Mayer* prefers. *Langrenus* and *Ricciolus* denoted the spots upon the surface by the names of Philosophers, Mathematicians, and other celebrated men, giving the names of the most celebrated characters to the largest spots; *Hevelius* marked them with the geographical names of places upon the earth. The former distinction is now generally followed.

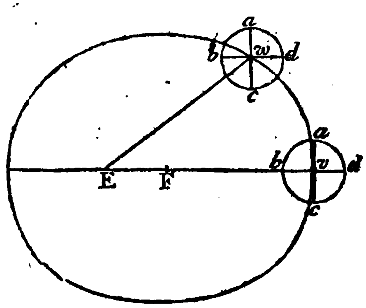
The spots upon the moon are caused by the mountains and vallies upon it's surface; for certain parts are found to project shadows opposite to the sun; and

when the sun becomes vertical to any of them, they are observed to have no shadows; these therefore are mountains; other parts are always dark on that side next to the sun, and illuminated on the opposite side; these therefore are cavities. Hence, the appearance of the face of the moon continually varies, from it's altering it's situation in respect to the sun. The tops of the mountains, on the dark part of the moon, are frequently seen enlightened at a distance from the cosines of the illuminated part. The dark parts have, by some, been thought to be seas, and by others, to be only a great number of caverns and pits, the dark sides of which, next to the sun, would cause those places to appear darker than others. The great irregularity of the line bounding the light and dark part, on every part of the surface, proves that there can be no very large tracts of water, as such a regular surface would necessarily produce a line, terminating the bright part, perfectly free from all irregularity. If there was much water upon it's surface, and an atmosphere, as conjectured by some Astronomers, the clouds and vapours might easily be discovered by the telescopes which we have now in use; but no such phænomena have ever been observed.

(220.) Very nearly the same face of the moon is always turned towards the earth, it being subject only to a small change within certain limits, those spots which lie near to the edge appearing and disappearing by turns; this is called it's *Libration*, and arises from four causes. 1. *Galileo*, who first observed the spots of the moon after the invention of telescopes, discovered this circumstance; he perceived a small daily variation arising from the motion of the spectator about the center of the earth, which, from the rising to the setting of the moon, would cause a little of the western limb of the moon to disappear, and bring into view a little of the eastern limb. 2. He observed likewise, that the north and south poles of the moon appeared and disappeared by turns; this arises from the axis of the

moon not being perpendicular to the plane of it's orbit, and is called a libration in *latitude*. 3. From the unequal angular motion of the moon about the earth, and the uniform motion of the moon about it's axis, a little of the eastern and western parts must gradually appear and disappear by turns, the period of which is a month, and this is called a libration in *longitude*; the cause of this libration was first assigned by *Ricciolus*, but he afterwards gave it up, as he made many observations which this supposition would not satisfy. *Hevelius*, however, found that it would solve all the phænomena of this libration. 4. Another cause of libration arises from the attraction of the earth upon the moon, in consequence of it's spheroidical figure.

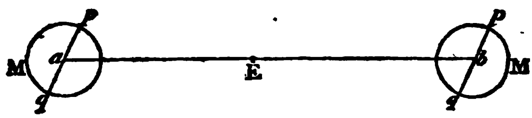
(221.) If the angular velocity of the moon about it's axis were equal to it's angular motion about the earth, the libration in *longitude* would not take place. For if *E* be the earth, *abcd* the moon at *v* and *w*, and *avc*




be perpendicular to *Ebvd*; then *abc* is that hemisphere of the moon at *v* next to the earth. When the moon comes to *w*, if it did not revolve about it's axis, *bwd* would be parallel to *bvd*, and the same face would not lie towards the earth. But if the moon, by revolving about it's axis in the direction *abcd*, had brought *b* into the line *Ew*, the same face would have been turned towards the earth; and the moon would have revolved about it's axis through the angle *bwE*,

which is equal to the alternate angle $wEv$, the angle which the moon has described about the earth.

(222.) When the moon returns to the same point of it's orbit, the same face is observed to lie towards the earth, and therefore (221) the time of the revolution in it's orbit is equal to the time about it's axis. But in the intermediate points it varies, sometimes a little more to the east, and sometimes to the west, becomes visible; and this arises from it's unequal angular motion $wEv$ about the earth, whilst the angular motion about it's axis is equal, in consequence of which these two angles cannot continue equal, and therefore, by the last article, the same face cannot continue towards the earth. Hence, the greatest libration in *longitude* is nearly equal to the equation of the orbit, or about $7°\frac{1}{2}$ at it's maximum, and would be accurately so, if the axis of the moon were perpendicular to it's orbit; for the difference of the moon's mean motion and true motion, or the equation of the orbit, is the same as the difference of the moon's motion about it's axis and it's true motion, which is the libration. The same face will be towards the earth in apogee and perigee, for at those points there is no equation of the orbit. If $E$ be the earth, $M$ the moon, $pq$ it's axis,




not perpendicular to the plane of the orbit $ab$; then at $a$ the pole $p$ will be visible to the earth, and at $b$ the pole $q$ will be visible; as the moon therefore revolves about the earth, the poles must appear and disappear by turns, causing the libration in *latitude*. This is exactly similar to the cause of the variety of our seasons, from the earth's axis not being perpendicular to the plane of it's orbit. Hence, nearly one half of the moon is never visible at the earth. Also, the time of it's rotation about it's axis being a month, the length of the lunar days and nights will be about a fortnight

each, they being subject but to a very small change, on account of the axis of the moon being nearly perpendicular to the ecliptic.

(223.) *Hevelius* (*Selenographia*, p. 245.) observed, that when the moon was at it's greatest north latitude, the libration in latitude was the greatest, the spots which are situated near the northern limb being then nearest to it; and as the moon departed from thence, the spots receded from that limb, and when the moon came to it's greatest south latitude, the spots situated near the southern limb were then nearest to it. This variation he found to be about 1′. 45″, the diameter of the moon being 30′. Hence it follows, that when the moon is at it's greatest latitude, a plane drawn through the earth and moon perpendicular to the plane of the moon's orbit, passes through the axis of the moon; consequently the equator of the moon must intersect the ecliptic in a line parallel to the line of the nodes of the moon's orbit, and therefore, in the heavens, the nodes of the moon's orbit and of it's equator coincide.

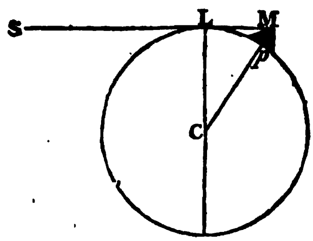
(224.) It is a very extraordinary circumstance, that the time of the moon's revolution about it's axis should be equal to that in it's orbit. Sir *I. Newton*, from the altitude of the tides on the earth, has computed that the altitude of the tides on the moon's surface must be 93 feet, and therefore the diameter of the moon perpendicular to a line drawn from the earth to the moon, ought to be less than the diameter directed to the earth, by 186 feet; hence, says he, the same face must always be towards the earth, except a small oscillation; for if the longest diameter should get a little out of that direction, it would be brought into it again by the attraction of the earth. The supposition of D. *de Mairan* is, that that hemisphere of the moon next the earth is more dense than the opposite one, and hence, the same face would be kept towards the earth, upon the same principle as above.

(225.) When the moon is about three days from the new, the dark part is very visible, by the light re-

flected from the earth, which is moon-light to the Lunarians, considering our earth as a moon to them; and in the most favourable state, some of the principal spots may then be seen. But when the moon gets into quadratures, it's great light prevents the dark part from being visible. According to Dr. *Smith*, the strength of moon-light, at the full moon, is ninety thousand times less than the light of the sun; but, from some experiments of M. *Bouguer*, he concluded it to be three hundred thousand times less. The light of the moon, condensed by the best mirrors, produces no sensible effect upon the thermometer. Our earth, in the course of a month, shows the same phases to the Lunarians, as the moon does to us; the earth is at the full at the time of the new moon, and at the new at the time of the full moon. The surface of the earth being about 13 times greater than that of the moon, it affords 13 times more light to the moon than the moon does to the earth.

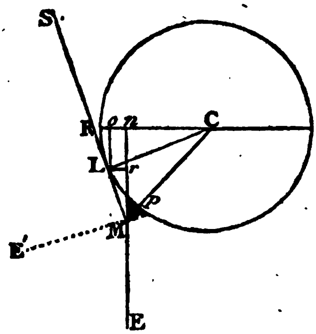
## *On the Altitude of the Lunar Mountains.*

(226.) The method used by *Hevelius*, and others since his time, to determine the height of a lunar mountain is this. Let $SLM$ be a ray of light from




the sun, passing the moon at $L$, and touching the top of the mountain at $M$; then the space between $L$ and $M$ appears dark. With a micrometer, measure $LM$, and compare it with $LC$; then, knowing $LC$, we know $LM$, and by Eucl. B. I. p. 47. $CM = \sqrt{CL^2 + LM^2}$ is known; from which subtract $Cp$,

and we get the height $pM$ of the mountain. But as Dr. *Herschel* observes, in the *Phil. Trans.* 1781, this method is only applicable when the moon is in quadratures; he has therefore given the following general method. Let $E$ be the earth; draw $EMn$ and $Lo$ perpendicular to the moon's radius $RC$, and $Lr$ parallel to $on$, also $ME'$ perpendicular to $SM$. Now $ML$ would measure it's full length when seen from the earth in quadratures at $E'$, but seen from $E$, it only measures the length of $Lr$. As the plane passing through $SM$, $EM$, is perpendicular to a line joining the cusps, the circle $RLp$ may be conceived to be a section of the moon perpendicular to that line. Now it is manifest, that the angle $SLo$ or $LCR$, is very nearly equal to the elongation of the moon from the sun; and the triangles $LrM$, $LCo$, being similar, $Lo$




$: LC :: Lr : LM = \frac{LC \times Lr}{Lo} = \frac{Lr}{\text{sine of elongation}}$, radius being unity. Hence, we find $Mp$ as before.

Ex. On June, 1780, at seven o'clock, Dr. *Herschel* found the angle under which $LM$, or $Lr$ appeared, to be 40″,625, for a mountain in the south-east quadrant; and the sun's distance from the moon was 125°. 8′, whose sine is ,8104; hence, 40″,625 divided by ,8104, gives 50″,13, the angle under which $LM$ would appear, if seen directly. Now the semidiameter of the moon was 16′. 2″,6, and taking its length to be 1090

miles, we have $16'.2'',6 : 50'',13 :: 1090 : LM = 56,73$ miles; hence, $Mp = 1,47$ miles.

(227.) Dr. *Herschel* found the height of a great many more mountains, and thinks he has good reason to believe, that their altitudes are greatly over-rated; and that, a few excepted, they generally do not exceed half a mile. He observes, that it should be examined whether the mountain stands upon level ground, which is necessary, that the measurement may be exact. A low tract of ground between the mountain and the sun will give it higher, and elevated places between will make it lower, than it's true height above the common surface of the moon.

(228.) On April 19, 1787, Dr. *Herschel* discovered three volcanos in the dark part of the moon; two of them seemed to be almost extinct, but the third showed an actual eruption of fire, or luminous matter, resembling a small piece of burning charcoal covered by a very thin coat of white ashes; it had a degree of brightness about as strong as that with which such a coal would be seen to glow in faint day-light. The adjacent parts of the volcanic mountain seemed faintly illuminated by the eruption. A similar eruption appeared on May 4, 1783. *Phil. Trans.* 1787. On March 7, 1794, a few minutes before eight o'clock in the evening, Mr. *Wilkins*, of Norwich, an eminent architect, observed, with the naked eye, a very bright spot upon the dark part of the moon; it was there when he first looked at the moon; the whole time he saw it, it was a fixed, steady light, except the moment before it disappeared, when it's brightness increased; he conjectures that he saw it about five minutes. The same phænomenon was observed by Mr. *T. Stretton*, in St. John's-square, Clerkenwell, London. *Phil. Trans.* 1794. On April 13, 1793, and on February 5, 1794, Mr. *Piazzi*, Astronomer Royal at Palermo, observed a bright spot on the dark part of the moon, near Aristarchus. Several other Astronomers have

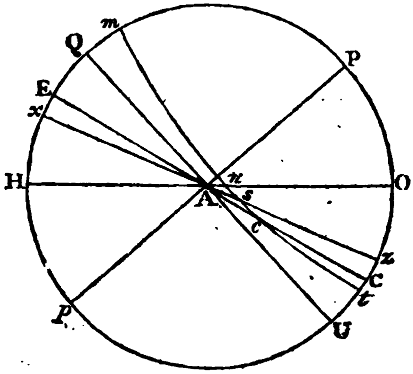
observed the same phænomenon. See the *Memoirs de Berlin*, for 1788.

(229.) It has been a doubt amongst Astronomers, whether the moon has any atmosphere; some suspecting that at an occultation of a fixed star by the moon, the star did not vanish instantly, but lost it's light gradually; whilst others could never observe any such appearance. M. *Schroetar* of Lilianthan, in the duchy of Bremen, has endeavoured to establish the existence of an atmosphere, from the following observations. 1. He observed the moon when two days and an half old, in the evening soon after sun-set, before the dark part was visible, and continued to observe it till it became visible. The two cusps appeared tapering in a very sharp, faint prolongation, each exhibiting it's farthest extremity faintly illuminated by the solar rays, before any part of the dark hemisphere was visible. Soon after, the whole dark limb appeared illuminated. This prolongation of the cusps beyond the simicircle, he thinks, must arise from the refraction of the sun's rays by the moon's atmosphere. He computes also the height of the atmosphere, which refracts light enough into it's dark hemisphere to produce a twilight, more luminous than the light reflected from the earth when the moon is about 32° from the new, to be 1356 Paris feet; and that the greatest height capable of refracting the solar rays is 5376 feet. 2. At an occultation of Jupiter's satellites, the third disappeared, after having been about 1″ or 2″ of time indistinct; the fourth became indiscernible near the limb; this was not observed of the other two. *Phil. Trans.* 1792. If there be no atmosphere of the moon, the heavens, to a Lunarian, must always appear dark like night, and the stars be constantly visible; for it is owing to the reflection and refraction of the sun's light by the atmosphere, that the heavens, in every part, appear bright in the day.

## On the Phænomenon of the Harvest Moon.

(230.) The full moon which happens at, or nearest to, the autumnal equinox, is called the *Harvest* moon; and at that time there is a less difference between the times of it's rising on two successive nights, than at any other full moon in the year; and what we here propose, is to account for this phænomenon.

(231.) Let $P$ be the north pole of the equator $QAU$, $HAO$ the horizon, $EAC$ the ecliptic, $A$ the




first point of Aries; then, in *north* latitudes, $A$ is the ascending node of the ecliptic upon the equator, $AC$ being the order of the sines, and $AQ$ that of the apparent diurnal motion of the heavenly bodies. When Aries rises in north latitudes, the ecliptic makes the least angle with the horizon; and as the moon's orbit makes but a small angle with the ecliptic, let us first suppose $EAC$ to represent the moon's orbit. Let $A$ be the place of the moon at it's rising on one night; now, in mean solar time, the earth makes one revolution in $23h.\ 56'.\ 4''$, and brings the same point $A$ of the equator to the horizon again; but, in that time, let the moon have moved in it's orbit from $A$ to $c$, and draw the parallel of declination $tcm$; then it is manifest that $3'.\ 56''$ before the *same* hour the next night, the moon, in it's diurnal motion, has to describe $cn$

before it rises. Now $cn$ is manifestly the least possible, when the angle $CAn$ is the least, $Ac$ being given. Hence, it rises more nearly at the same hour, when it's orbit makes the least angle with the horizon. Now at the autumnal equinox, when the sun is in the first point of Libra, the moon, at that time at it's full, will be at the first point of Aries, and therefore it rises with the least difference of times, on two successive nights; and it being at the time of it's full, it is more taken notice of; for the same thing happens every month when the moon comes to Aries.

(232.) Hitherto we have supposed the ecliptic to represent the moon's orbit, but as the orbit is inclined to it at an angle of 5°. 9′ at a mean, let $xAsz$ represent the moon's orbit when the ascending node is at $A$, and $As$ the arc described in a day; then the moon's orbit making the least possible angle with the horizon in that position of the nodes, the arc $sn$, and consequently the difference of the times of rising, will be the least possible. As the moon's nodes make a revolution in about 19 years, the least possible difference can only happen once in that time. In the latitude of London the least difference is about 17′.

(233.) The ecliptic makes the greatest angle with the horizon when the first point of Libra rises, consequently, when the moon is in that part of it's orbit, the difference of the times of it's rising will be the greatest; and if the descending node of it's orbit be there at the same time, it will make the difference the greatest possible; and this difference is about 1$h$. 17′ in the latitude of London. This is the case with the vernal full moons. Those signs which make the least angle with the horizon when they rise make the greatest angle when they set, and vice versa; hence, when the difference of the times of rising is the least, the difference of the times of setting is the greatest, and the contrary.

(234.) By increasing the latitude, the angle $zAn$, and consequently $sn$ is diminished; and when the

time of describing *sn*, by the diurnal motion, is 3′. 56″. the moon will then rise at the same solar hour. Let us suppose the latitude to be increased until the angle *sAn* vanishes, then the moon's orbit becomes coincident with the horizon every day, for a moment of time, and consequently the moon rises at the same sidereal hour, or 3′. 56″ sooner, by solar time. Now take a globe, and elevate the north pole to this latitude, and, marking the moon's orbit in this position upon it, turn the globe about, and it will appear, that at the instant after the above coincidence, one half of the moon's orbit, corresponding to Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini, will rise; hence, when the moon is going through that part of it's orbit, or for 13 or 14 days, it rises at the same sidereal hour. Now, taking the angle $xAE = 5°. 9'$, and the angle $EAQ = 23°. 28'$, the angle $QAx$, or $QAH$, when the moon's orbit coincides with the horizon, is $28°. 37'$; hence, (87) the latitude is $61°. 23'$ where these circumstances take place. If the descending node be at *A*, then *x* lying above *E*, $QAx$, or $QAH = 18°. 19'$, and the latitude is $71°. 41'$. In any other situation of the orbit, the latitude will be between these limits. When the angle $QAx$ is greater than the complement of latitude, the moon will rise sooner the next day. As there is a complete revolution of the nodes in about 18 years 8 months, all the varieties of the intervals of the rising and setting of the moon will happen within that time.

## *On the Horizontal Moon.*

(235.) The phænomenon of the horizontal moon is this, that it appears larger in the horizon than in the meridian; whereas, from it's being nearer to us in the latter than in the former case, it subtends a greater angle. *Gassendus* thought that, as the moon was less bright in the horizon, we looked at it there with a greater pupil of the eye, and therefore it appeared

larger. But this is contrary to the principles of Optics, since the magnitude of the image upon the retina does not depend upon the pupil. This opinion was supported by a French *Abbe*, who supposed that the opening of the pupil made the chrystalline humour flatter, and the eye longer, and thereby increased the image. But there is no connection between the muscles of the iris and the other parts of the eye, to produce these effects. *Des Cartes* thought that the moon appeared largest in the horizon, because, when comparing it's distance with the intermediate objects, it appeared then furthest off; and as we judge it's distance greatest in that situation, we of course think it larger, supposing that it subtends the same angle. This opinion was supported by Dr. *Wallis*, in the *Phil. Trans.* N°. 187. Dr. *Berkley* accounts for it thus. Faintness suggests the idea of greater distance; the moon appearing most faint in the horizon, suggests the idea of greater distance, and, supposing the visual angle the same, that must suggest the idea of a greater tangible object. He does not suppose the *visible* extension to be greater, but that the idea of a greater *tangible* extension is suggested, by the alteration of the appearance of the visible extension. He says, 1. That which suggests the idea of greater magnitude, must be something perceived; for what is not perceived can produce no visible effect. 2. It must be something which is variable, because the moon does not always appear of the same magnitude in the horizon. 3. It cannot lie in the intermediate objects, they remaining the same; also, when these objects are excluded from sight, it makes no alteration. 4. It cannot be the visible magnitude, because that is least in the horizon; the cause, therefore, must lie in the visible appearance, which proceeds from the greater paucity of rays coming to the eye, producing *faintness*. Mr. *Rowning* supposes that the moon appears furthest from us in the horizon, because the portion of the sky which we see, appears not an entire *hemisphere*,

but only a portion of one; and in consequence of this, we judge the moon to be furthest from us in the horizon, and therefore to be then largest. Dr. *Smith*, in his *Optics*, gives the same reason. He makes the apparent distance in the horizon to be to that in the zenith as 10 to 3, and therefore the apparent diameters in that ratio. The methods by which he estimated the apparent distances, may be seen in Vol. I. page 65. The same circumstance also takes place in the sun, which appears much larger in the horizon than in the zenith. Also, if we take two stars near each other in the horizon, and two other stars near the zenith at the same angular distance from each other, the two former will appear at a much greater distance from each other, than the two latter. Upon this account, people are, in general, very much deceived in estimating the altitudes of the heavenly bodies above the horizon, judging them to be much greater than they are. Dr. *Smith* found, that when a body was about 23° above the horizon, it appeared to be half way between the zenith and horizon, and therefore at that real altitude it would be estimated to be 45° high. The lower part of a rainbow also appears broader than the upper part. And this may be considered as an argument that the phænomenon cannot depend entirely upon the greater degree of faintness in the object when in the horizon, because the lower part of the bow frequently appears brighter than the upper part, at the same time that it appears broader. Also, this cause could have no effect upon the distance of the stars; and as the difference of the apparent distance of the two stars, whose angular distance is the same, in the horizon and zenith, seems to be fully sufficient to account for the apparent variation of the moon's diameter in these situations, it may be doubtful, whether the faintness of the object enters into any part of the cause.

# Chap. XVII.

## ON THE ROTATION OF THE SUN AND PLANETS.

(236.) The times of rotation of the sun, and planets, and the position of their axes, are determined from the spots which are observed upon their surfaces. The position of the same spot, observed at three different times, will give the position of the axis; for three points of any small circle will determine it's situation, and hence we know the axis of the sphere which is perpendicular to it. The time of rotation may be found, either from observing the arc of the small circle described by a spot in any time, or by observing the return of a spot to the same position in respect to the earth.

### *On the Rotation of the Sun.*

(237.) It is doubtful by whom the spots on the sun were first discovered. *Scheiner*, Professor of Mathematics in Ingolstadt, observed them in May, 1611, and published an account of them in 1612, in a work entitled, *Rosa Ursina*. *Galileo*, in the Preface to a work entitled, *Istoria, Dimostrazioni, intorno alle Macchie Solari, Roma*, 1613, says, that being at Rome in 1611, he then showed the spots of the sun to several persons, and that he had spoken of them, some months before, to his friends at Florence. He imagined them to adhere to the sun. *Kepler*, in his Ephemeris, says, that they were observed by the son of *David Fabricius*, who published an account of them in 1611. In the papers of *Harriot*, not yet printed, it is said, that spots upon the sun were observed on December 8, 1610. As telescopes were in

use at that time, it is probable that each might make the discovery. Admitting these spots to adhere to the sun's body, the reasons for which we shall afterwards give, we proceed to show how the time of it's rotation may be found.

(238.) M. *Cassini* determined the time of rotation, from observing the time in which a spot returns to the same situation upon the disc, or to the circle of latitude passing through the earth. Let $t$ be that interval of time, and let $m$ be equal to the *true* motion of the earth in that time, and $n$ equal to it's *mean* motion; then $360^\circ+m : 360^\circ+n :: t$ : the time of return if the motion had been uniform, and this, from a great number of observations, he determines to be $27d.\ 12h.\ 20'$; now the mean motion of the earth in that time is $27^\circ.\ 7'.\ 8''$; hence, $360^\circ+27^\circ.\ 7'.\ 8'' : 360^\circ :: 27d.\ 12h.\ 29' : 25d.\ 14h.\ 8'$, the time of rotation. *Elem. d'Astron.* p. 104.

(239.) When the earth is in the nodes of the sun's equator, and consequently in it's plane, the spots appear to describe straight lines: this happens about the beginning of June and December. As the earth recedes from the nodes, the path of a spot grows more and more elliptical, till the earth gets $90^\circ$ from the nodes, which happens about the beginning of September and March, at which time the ellipse has it's minor axis the greatest, and is then to the major axis, as the sine of the inclination of the solar equator to radius.

(240.) There has been a great difference of opinions respecting the nature of the solar spots. *Scheiner* supposed them to be solid bodies revolving about the sun, very near to it; but as they are as long visible as they are invisible, this cannot be the case. Moreover, we have a physical argument against this hypothesis, which is, that most of them do not revolve about the sun in a plane passing through it's center, which they necessarily must, if they revolved, like the planets, about the sun. *Galileo* confuted *Scheiner*'s opinion,

by observing that the spots were not permanent; that they varied their figure; that they increased, and sometimes disappeared. He compared them to smoak and clouds. *Hevelius* appears to have been of the same opinion; for in his *Cometographia*, p. 360, speaking of the solar spots, he says, *Hæc materia nunc ea ipsa est evaporatio et exhalatio* (*quia aliunde minime oriri potest*) *quæ ex ipso corpore solis, ut supra ostensum est, expiratur et exhalatur.* But the permanency of most of the spots is an argument against this hypothesis. M. *de la Hire* supposed them to be solid, opaque bodies, which swim upon the liquid matter of the sun, and which are sometimes entirely immersed. M. *de la Lande* supposes that the sun is an opaque body, covered with a liquid fire, and that the spots arise from the opaque parts, like rocks, which, by the alternate flux and reflux of the liquid igneous matter of the sun, are sometimes raised above the surface. The spots are frequently dark in the middle, with an umbra about them; and M. *de la Lande* supposes that the part of the rock which stands above the surface, forms the dark part in the center, and those parts which are but just covered by the igneous matter, form the umbra. Dr. *Wilson*, Professor of Astronomy at Glasgow, opposes this hypothesis of M. *de la Lande*, by this argument. Generally speaking, the umbra immediately contiguous to the dark central part, or nucleous, instead of being very dark, as it ought to be, from our seeing the immersed parts of the opaque rock through a thin stratum of the igneous matter, is, on the contrary, very nearly of the same splendour as the external surface, and the umbra grows darker the further it recedes from the nucleus; this, it must be acknowledged, is a strong argument against the hypothesis of M. *de la Lande*. Dr. *Wilson* further observes, that M. *de la Lande* produces no optical arguments in support of the rock standing above the surface of the sun. The opinion of Dr. *Wilson* is, that the spots are excavations in the luminous matter

of the sun, the bottom of which forms the umbra. They who wish to see the arguments by which this is supported, must consult the *Phil. Trans.* 1774 and 1783. Dr. *Halley* conjectured that the spots are formed in the atmosphere of the sun. Dr. *Herschel* supposes the sun to be an opaque body, and that it has an atmosphere; and if some of the fluids which enter into it's composition should be of a shining brilliancy, whilst others are merely transparent, any temporary cause which may remove the lucid fluid will permit us to see the body of the sun through the transparent ones. See the *Phil. Trans.* 1795. Dr. *Herschel,* on April 19, 1779, saw a spot which measured 1′. 8″,06 in diameter, which is equal in length to more than 31000 miles; this was visible to the naked eye. Besides the dark spots upon the sun, there are also parts of the sun, called *Faculæ, Lucili,* &c. which are brighter than the general surface; these always abound most in the neighbourhood of the spots themselves, or where spots recently have been. Most of the spots appear within the compass of a zone lying 30° on each side of the equator; but on July 5, 1780, M. *de la Lande* observed a spot 40° from the equator. Spots which have disappeared, have been observed to break out again. The spots appear so frequently, that Astronomers very seldom examine the sun with their telescopes, but they see some; *Scheiner* saw fifty at once. The following phænomena of the spots are described by *Scheiner* and *Hevelius.*

I. Every spot which has a nucleus, has also an umbra surrounding it.

II. The boundary between the nucleus and umbra is always well defined.

III. The increase of a spot is gradual, the breadth of the nucleus and umbra dilating at the same time.

IV. The decrease of a spot is gradual, the breadth of the nucleus and umbra contracting at the same time.

V. The exterior boundary of the umbra never consists of sharp angles, but is always curvilinear, however irregular the outline of the nucleus may be.

VI. The nucleus, when on the decrease, in many cases changes it's figure, by the umbra encroaching irregularly upon it.

VII. It often happens, by these encroachments, that the nucleus is divided into two or more nuclei.

VIII. The nucleus vanishes sooner than the umbra.

IX. Small umbræ are often seen without nuclei.

X. An umbra of any considerable size is seldom seen without a nucleus.

XI. When a spot, consisting of a nucleus and umbra, is about to disappear, if it be not succeeded by a facula, or more fulgid appearance, the place it occupied is, soon after, not distinguishable from any other part of the sun's surface.

## *On the Rotation of the Planets.*

(241.) The *Georgian* is at so great a distance, that Astronomers, with their best telescopes, have not been able to discover whether it has any revolution about it's axis.

(242.) *Saturn* was suspected by *Cassini* and *Fato,* in 1683, to have a revolution about it's axis; for they one day saw a bright streak, which disappeared the next, when another came into view near the edge of it's disc; these streaks are called *Belts.* In 1719, when the ring disappeared, *Cassini* saw it's shadow upon the body of the planet, and a belt on each side parallel to the shadow. When the ring was visible, he perceived the curvature of the belts was such as agreed with the elevation of the eye above the plane of the ring. He considered them as similar to our clouds floating in the air; and having a curvature similar to the exterior circumference of the ring, he concluded that they ought to be nearly at the same

distance from the planet, and that consequently the atmosphere of Saturn extended to the ring. Dr. *Herschel* found that the arrangement of the belts always followed the direction of the ring; thus, as the ring opened, the belts began to show an incurvature answering to it. And during his observations on June 19, 20, and 21, 1780, he saw the same spot in three different situations. He conjectured, therefore, that Saturn revolved about an axis perpendicular to the plane of it's ring. Another argument in support of this, is, that the planet is an oblate spheriod, having the diameter in the direction of the ring to the diameter perpendicular to it, as about 11 : 10, according to Dr. *Herschel*; the measures were taken with a wire micrometer prefixed to his 20 feet reflector. The truth of his conjecture he has now verified, having determined that Saturn revolves about it's axis in 10*h*. 16′. 0″,4. *Phil. Trans.* 1794. The rotation is according to the order of the signs.

(243.) *Jupiter* is observed to have belts, and also spots, by which the time of it's rotation can be very accurately ascertained. M. *Cassini* found the time of rotation to be 9*h*. 56′, from a remarkable spot which he observed in 1665. In October 1691, he observed two bright spots almost as broad as the belts; and at the end of the month he saw two more, and found them to revolve in 9*h*. 51′; he also observed some other spots near Jupiter's equator, which revolved in 9*h*. 50′; and, in general, he found that the nearer the spots were to the equator, the quicker they revolved. It is probable, therefore, that the spots are not upon Jupiter's surface, but in it's atmosphere; and for this reason also, that several spots which appeared round at first, grew oblong by degrees in a direction parallel to the belts, and divided themselves into two or three spots. M. *Maraldi*, from a great many observations of the spot observed by *Cassini* in 1665, found the time of rotation to be 9*h*. 56′; and concluded that the spots had a dependence upon the contiguous belt, as

the spot had never appeared without the belt, though the belt had without the spot. It continued to appear and disappear till 1694, and was not seen any more till 1708; hence, he concluded, that the spot was some effusion from the belt upon a fixed place of Jupiter's body, for it always appeared in the same place. Dr. *Herschel* found the time of rotation of different spots to vary; and that the time of rotation of the same spot diminished; for the spot observed in 1788 revolved as follows. From February 25 to March 2, in 9*h*. 55′. 20″; from March 2 to the 14th, in 9*h*. 54′. 58″; from April 7 to the 12th, in 9*h*. 51′. 35″. Also, from a spot observed in 1799, it's rotation was, from April 14 to the 19th, in 9*h*. 51′. 45″; from April 19 to the 23d, in 9*h*. 50′. 48″. This, he observes, is agreeable to the theory of equinoctial winds, as it may be some time before the spot can acquire the velocity of the wind; and if Jupiter's spots should be observed in different parts of it's revolution to be accelerated and retarded, it would amount almost to a demonstration of it's monsoons, and their periodical changes. M. *Schroeter* makes the time of rotation 9*h*. 55′. 36″,6; he observed the same variations as Dr. *Herschel*. The rotation is according to the order of the signs. This planet is observed to be flat at it's poles. Dr. *Pound* measured the polar and equatorial diameters, and found them as 12 : 13. Mr. *Short* made them as 13 : 14. Dr. *Bradley* made them as 12,5 : 13,5. Sir *I. Newton* makes the ratio $9\frac{1}{3} : 10\frac{1}{3}$ by theory. The belts of Jupiter are generally parallel to it's equator, which is very nearly parallel to the ecliptic; they are subject to very great variations, both in respect to their number and figure; sometimes eight have been seen at once, and at other times only one; sometimes they continue for three months without any variation, and sometimes a new belt has been formed in an hour or two. From their being subject to such changes, it is very probable that they

do not adhere to the body of Jupiter, but exist in it's atmosphere.

(244.) *Galileo* discovered the phases of *Mars*; after which, some Italians in 1636, had an imperfect view of a spot. But in 1666, Dr. *Hook* and M. *Cassini* discovered some well-defined spots; and the latter determined the time of the rotation to be 24*h*. 40′. Soon after, M. *Maraldi* observed some spots, and determined the time of rotation to be 24*h*. 39′. He also observed a very bright part near the southern pole, appearing like a polar zone; this, he says, has been observed for 60 years; it is not of equal brightness, more than half of it being brighter than the rest; and that part which is least bright, is subject to great changes, and sometimes disappears. Something like this has been seen about the north pole. The rotation is according to the order of the signs. Dr. *Herschel* makes the time of a sidereal rotation to be 24*h*. 39′. 21″67, without the probability of a greater error than 2″34. He proposes to find the time of a sidereal rotation, in order to discover, by future observations, whether there is any alteration in the time of the revolution of the earth, or of the planets, about their axes; for a change of either would thus be discovered. He chose Mars, because it's spots are permanent. See the *Phil. Trans.* 1781. From further observations upon Mars, which he published in *Phil. Trans.* 1784, he makes it's axis to be inclined to the ecliptic 59°. 42′, and 61°. 18′ to it's orbit; and the north pole to be directed to 17°. 47′ of Pisces upon the ecliptic, and 19°. 28′ on it's orbit. He makes the ratio of the diameters of Mars to be as 16 : 15. Dr. *Maskelyne* has carefully observed Mars at the time of opposition, but could not perceive any difference in it's diameters. Dr. *Herschel* observes, that Mars has a considerable atmosphere.

(245.) *Galileo* first discovered the phases of *Venus* in 1611, and sent the discovery to *William de' Medici*,

to communicate it to *Kepler*. He sent it in this cypher, *Hæc immaturæ a me frustra leguntur, o, y*; which put in order, is, *Cynthiæ figuras æmulatur mater amorum*, that is, *Venus emulates the phases of the moon.* He afterwards wrote a letter to him, giving an account of the discovery, and explaining the cypher. In 1666, M. *Cassini*, at a time when Venus was dichotomized, discovered a bright spot upon it at the straight edge, like some of the bright spots upon the moon's surface; and by observing it's motion, which was upon the edge, he found the sidereal time of rotation to be 23*h*. 16′. In the year 1726, *Bianchini* made some observations upon the spots of Venus, and asserted the time of rotation to be 24⅓ days; that the north pole answered to the 20th degree of Aquarius, and was elevated from 15° to 20° above it's orbit; and that the axis continued parallel to itself. The small angle which the axis of Venus makes with it's orbit is a singular circumstance, and must cause a very great variety in the seasons. M. *Cassini*, the Son, has vindicated his Father, and shown, from *Bianchini*'s observations being interrupted, that he might easily mistake different spots for the same: and he concludes, that if we suppose the periodic time to be 23*h*. 20′, it agrees equally with their observations; but if we take it 24⅓ days, it will not at all agree with his Father's observations. M. *Schroeter* has endeavoured to show that Venus has an atmosphere, from observing that the illuminated limb, when horned, exceeds a semicircle; this he supposes to arise from the refraction of the sun's rays through the atmosphere of Venus at the cusps, by which they appear prolonged. The cusps appeared sometimes to run 15°. 19′ into the dark hemisphere; from which he computes, that the height of the atmosphere, to refract such a quantity of light, must be 15156 Paris feet. But this must depend on the nature and density of the atmosphere, of which we are ignorant. *Phil. Trans.* 1792. He makes the time of rotation to be 23*h*. 21′, and concludes, from his ob-

servations, that there are considerable mountains upon this planet, *Phil. Trans.* 1795. Dr. *Herschel* agrees with M. *Schroeter*, that Venus has a considerable atmosphere; but he has not made any observations, by which he can determine, either the time of rotation, or the position of the axis. *Phil. Trans.* 1793.

(246.) The phases of *Mercury* are easily distinguished to be like those of *Venus*; but no spots have yet been discovered, by which we can ascertain whether it has any rotation.

(247.) The fifth satellite of *Saturn* was observed by M. *Cassini* for several years, as it went through the eastern part of it's orbit, to appear less and less, till it became invisible; and in the western part to increase again. These phænomena can hardly be accounted for, but by supposing some parts of the surface to be incapable of reflecting light, and therefore, when such parts are turned towards the earth, they appear to grow less, or to disappear. As the same appearances returned again when the satellite came to the same part of it's orbit, it affords an argument that the time of the rotation about it's axis is equal to the time of it's revolution about it's primary, a circumstance similar to the case of the moon and earth. See Dr. *Herschel's* account of this in the *Phil. Trans.* 1792. The appearance of this satellite of Saturn is not always the same, and therefore it is probable that the dark parts are not permanent. Dr. *Herschel* has discovered that all the satellites of *Jupiter* have a rotatory motion about their axes, of the same duration with their respective periodic times about their primaries. *Phil. Trans.* 1797.

# CHAP. XVIII.

## ON THE SATELLITES.

(248.) On January 8, 1610, *Galileo* discovered the four satellites of *Jupiter*, and called them *Medicea Sidera*, or *Medicean Stars*, in honour of the family of the *Medici*, his patrons. This was a discovery, very important in it's consequences, as it furnished a ready method of finding the longitudes of places, by means of their eclipses; the eclipses led M. *Roemer* to the discovery of the progressive motion of light; and hence Dr. *Bradley* was enabled to solve an apparent motion in the fixed stars, which could not otherwise have been accounted for.

(249.) The satellites of *Jupiter*, in going from the west to the east, are eclipsed by the shadow of Jupiter, and as they go from east to west, they are observed to pass over it's disc; hence, they revolve about Jupiter, and in the same direction as Jupiter revolves about the sun. The three first satellites are always eclipsed, when they are in opposition to the sun, and the lengths of the eclipses are found to be different at different times: but sometimes the fourth satellite passes through opposition without being eclipsed. Hence it appears, that the planes of the orbits do not coincide with the plane of Jupiter's orbit, for, in that case they would always pass through the center of Jupiter's shadow, and there would always be an eclipse, and of the same, or very nearly the same, duration, at every opposition to the sun. As the planes of the orbits which they describe sometimes pass through the eye, they will then appear to describe straight lines passing

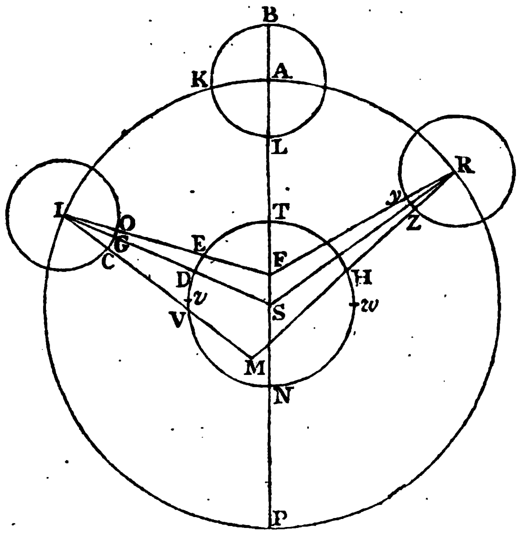
through the center of Jupiter; but at all other times they will appear to describe ellipses, of which Jupiter is the center.

## *On the Periodic Times, and distances of Jupiter's Satellites.*

(250.) To get the mean times of their *synodic* revolutions, or of their revolutions in respect to the sun, observe, when Jupiter is in opposition, the passage of a satellite over the body of Jupiter, and note the time when it appears to be exactly in conjunction with the center of Jupiter, and that will be the time of conjunction with the sun. After a considerable interval of time, repeat the same observation, Jupiter being in opposition, and divide the interval of time by the number of conjunctions with the sun in that interval, and you get the time of a *synodic* revolution of the satellite. This is the revolution which we have occasion principally to consider, it being that on which the eclipses depend. But, owing to the equation of Jupiter's orbit, this will not give the *mean* time of a synodic revolution, unless Jupiter was at the same point of it's orbit at both observations; otherwise, we must proceed thus.

(251.) Let *AIPR* be the orbit of Jupiter, *S* the sun in one focus, and *F* the other focus; and as the excentricity of the orbit is small, the motion about *F* may be considered (169) as uniform. Let Jupiter be in it's aphelion at *A* in opposition to the earth at *T*; and *L* a satellite in conjunction; and let *I* be the place of Jupiter at it's next opposition with the earth at *D*, and the satellite in conjunction at *G*. Then, if the satellite had been at *O*, it would have been in conjunction with *F*, or in mean conjunction; therefore it must describe the angle *FIS* before it comes to the mean conjunction, which angle is (169) the equation of the orbit, according to the *simple elliptic*

*hypothesis*, which may be here used, as the excentricity of the orbit is but small; the angle *FIS* therefore measures the difference between the mean *synodic* revolutions in respect to *F*, and the synodic revolutions in




respect to the sun *S*. If, therefore, *n* be the number of revolutions which the satellite has made in respect to the sun, $n \times 360° - SIF =$ the revolutions in respect to *F*; hence, $n \times 360° - SIF : 360° ::$ the time between the two oppositions : the time of a *mean* synodic revolution about the sun.

(252.) As the satellite is at *O* at the mean conjunction, and at *G* when in conjunction with the sun, it is manifest, that if the angle *FIS* continued the same, the time of a revolution in respect to *S* would be equal to the time in respect to *F*, or to the time of a mean synodic revolution; hence, the difference between the times of any two successive revolutions in respect to *S* and *F* respectively, is as the variation of the angle *FIS*, or variation of the equation of the orbit. When Jupiter is at *A*, the equation vanishes, and the times of the two conjunctions at *F* and *S* coincide. When Jupiter comes to *I*, the mean conjunction at *O* happens

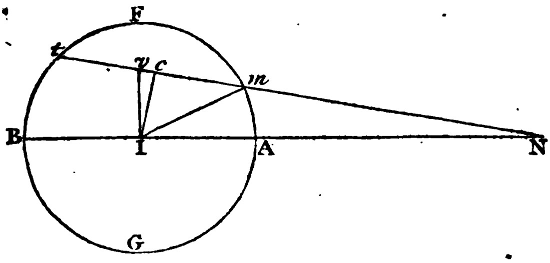
after the true conjunction at *G*, by the time of describing the angle *SIF*, the equation of Jupiter's orbit. This is what Astronomers call the *first* inequality; and by this inequality of the intervals of the times of the true conjunctions, the times of the eclipses of the satellites are affected.

(253.) But as a conjunction of the satellite may not often happen exactly at the time when Jupiter is in opposition, the time of a mean revolution may be found, when he is out of opposition, thus. Let *H* be the earth when the satellite is at *Z* in conjunction with Jupiter at *R*; and let *V* be another position of the earth when the satellite is at *C* in conjunction with Jupiter at *I*; and produce *RH*, *IV*, to meet in *M*; then the motion of Jupiter about the earth, in this interval, is the same as if the earth had been fixed at *M*. Now the difference between the true and mean motions of Jupiter is $RFI - RMI = FIM + FRM$, which shows how much the number of mean revolutions, in respect to *F*, exceeds the same number of apparent revolutions in respect to the earth; hence, $n \times 360° - FIM - FRM : 360°$ :: the time between the observations : the time of a mean synodic revolution of the satellite. If *C* and *Z* lie on the other side of *O* and *Y*, the angles *FIM*, *FRM*, must be added to $n \times 360°$; and if one lie on one side, and the other on the other, one must be added and the other subtracted, according to the circumstances.

(254.) As it is difficult, from the great brightness of Jupiter, to determine accurately the time when the satellite is in conjunction with the center of Jupiter as it passes over it's disc, the time of conjunction is determined by observing it's entrance upon the disc, and it's going off; but as this cannot be determined with so much accuracy as the times of immersion into the shadow of Jupiter, and emersion from it, the time of conjunction can be most accurately determined from the eclipses.

(255.) Let *I* be the center of Jupiter's shadow *FG*,

*Nmt* the orbit of a satellite, *N* the node of the satellite's orbit upon the orbit of Jupiter; draw *Iv* perpen-




dicular to *IN*, and *Ic* to *Nt*; and when the satellite comes to *v*, it is in conjunction* with the sun. Now both the immersion at *m* and emersion at *t* of the second, third, and fourth satellites may sometimes be observed, the middle point of time between which, gives the time of the middle of the eclipse at *c*; and by calculating *cv*, from knowing the angle *N* and *NI*, we get the time of conjunction at *v*. If both the immersion and emersion cannot be observed, take the time of either, and after a very long interval of time, when an eclipse happens as nearly as possible in the same situation in respect to the node, take the time of the same phænomenon, and from the interval of these times you will get the time of a revolution. By these different methods, M. *Cassini* found the times of the mean *synodic* revolutions of the four satellites to be as follows:

| *First.* | *Second.* | *Third.* | *Fourth.* |
|---|---|---|---|
| $1^d.18^h.28'.36''$ | $3^d.13^h.17'.54''$ | $7^d.3^h.59'.36''$ | $16^d.18^h.5'.7''$ |

(256.) Hence it appears, that 247 revolutions of the first satellite are performed in 437*d*. 3*h*. 44′; 123 re-

* A satellite is said to be in conjunction, both when it is between the Sun and Jupiter, and when it is opposite to the Sun; the latter may be called superior, and the former inferior conjunction.

volutions of the second, in 437*d.* 3*h.* 41′; 61 revolutions of the third, in 437*d.* 3*h.* 35′, and 26 revolutions of the fourth, in 435*d.* 14*h.* 13′. Therefore, after an interval of 437 days, the three first satellites return to their relative situations within nine minutes.

(257.) In the return of the satellites to their mean conjunction, they describe a revolution in their orbits, together with the mean angle $a^\circ$ described by Jupiter in that time; therefore, to get the *periodic* time of each, we must say, $360^\circ + a^\circ : 360^\circ ::$ time of a synodic revolution : the time of a periodic revolution; hence, the *periodic* times of each are;

| *First.* | *Second.* | *Third.* | *Fourth.* |
|---|---|---|---|
| $1^d.18^h.27'.33''$ | $3^d.13^h.13'.42''$ | $7^d.3^h.42'.33''$ | $16^d.16^h.32'.8''$ |

(258.) The distances of the satellites from the center of Jupiter may be found at the time of their greatest elongations, by measuring with a micrometer, at that time, their distances from the center of Jupiter, and also the diameter of Jupiter, by which you get their distances in terms of the diameter. Or it may be done thus. When a satellite passes over the middle of the disc of Jupiter, observe the whole time of it's passage, and then, the time of a revolution : the time of it's passage over the disc :: 360° : the arc of it's orbit corresponding to the time of it's passage over the disc; hence, the sine of half that arc : radius :: the semidiameter of Jupiter : the distance of the satellite. Thus M. *Cassini* determined their distances in terms of the semidiameter of Jupiter to be, of the *first*, 5,67; of the *second*, 9; of the *third*, 14,38; and of the *fourth*, 25,3.

(259.) Or, having determined the periodic times, and the distance of one satellite, the distances of the others may be found from the proportion of the squares of the periodic times being as the cubes of their distances. Mr. *Pound*, with a telescope 15 feet long,

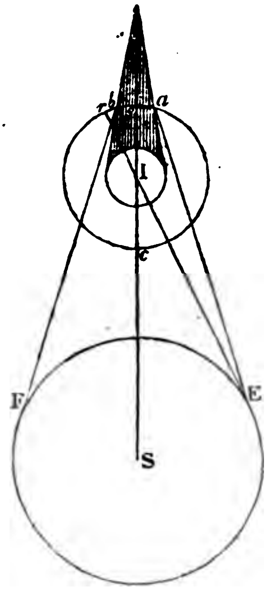
found at the mean distance of Jupiter from the earth, the greatest distance of the *fourth* satellite to be 8′. 16″; and by a telescope 123 feet long, he found the greatest distance of the *third* to be 4′. 42″; hence, the greatest distance of the *second* appears to be 2′. 56″. 47‴, and of the *first*, 1′. 51″. 6‴. Now the diameter of Jupiter, at it's mean distance, was determined, by Sir *I. Newton*, to be 37″¼; hence the distances of the satellites, in terms of the semidiameter of Jupiter, come out 5,965; 9,494; 15,141, and 26,63 respectively. *Prin. Math. Lib. ter. Phœn.*

(260.) Hence, by knowing the greatest elongations of the satellites in minutes and seconds, we get their distances from the center of Jupiter, compared with the mean distance of Jupiter from the earth, by saying, the sine of the greatest elongation of the satellite : radius :: the distance of the satellite from Jupiter : the mean distance of Jupiter from the earth.

## *On the Eclipses of Jupiter's Satellites.*

(261.) Let *S* be the sun, *EF* the orbit of the earth, *I* Jupiter, *abc* the orbit of one of it's satellites. When the earth is at *E* before the opposition of Jupiter, the spectator will see the immersion at *a*; but if it be the first satellite, upon account of it's nearness to Jupiter, the immersion is never visible, the satellite being then always behind the body of Jupiter; the other three satellites *may* have both their immersions and emersions visible; but this will depend upon the position of the earth. When the earth comes to *F* after opposition, we shall then see the emersion of the first, but can then never see the immersion; but we *may* see both the emersion and immersion of the other three. Draw *EIr*; then *sr*, the distance of the center of the shadow from the center of Jupiter, referred to the orbit of the satellite, is measured at Jupiter by *sr*, or the angle *sIr*, or the angle *EIS*. The satellite may be hidden behind the body at *r* without being eclipsed,

which is called an *Occultation.* When the earth is at *E*, the conjunction of the satellite happens *later* at




the earth than at the sun; but when the earth is at *F*, it happens *sooner*.

(262.) The diameter of the shadow of Jupiter, at the distance of any of the satellites, is best found by observing the time of an eclipse when it happens at the node, at which time the satellite passes through the center of the shadow; for the time of a synodic revolution : the time the satellite is passing through the center of the shadow :: 360° : the diameter of the shadow in degrees. But when the first and second satellites are in the nodes, the immersion and emersion cannot both be seen. Astronomers, therefore, compare the immersions some days *before* the opposition of Jupiter with the emersions some days *after*, and then, knowing how many synodic revolutions have been made, they get the time of the transit through the shadow, and thence the corresponding degrees. But on account of the excentricity of some of the orbits,

the times of the central transit must vary; for example, the second satellite is sometimes found to be 2*h*. 50′ in passing through the center of the shadow, and, sometimes 2*h*. 54′; this indicates an excentricity.

(263.) The duration of the eclipses being very unequal, shows that the orbits are inclined to the orbit of Jupiter; sometimes the fourth satellite passes through opposition without suffering an eclipse. The duration of the eclipses must depend upon the situation of the nodes in respect to the sun, just the same as in a lunar eclipse; when the line of the nodes passes through the sun, the satellite will pass through the center of the shadow; but as Jupiter revolves about the sun, the line of the nodes will be carried out of conjunction with the sun, and the time of the eclipse will be shortened, as the satellite will then describe only a chord of a section of the shadow instead of the diameter.

## *On the Rotation of the Satellites of Jupiter.*

(264.) M. *Cassini* suspected that the satellites had a rotation about their axes, as sometimes in their passage over Jupiter's disc they were visible, and at other times not; he conjectured, therefore, that they had spots upon one side and not on the other, and that they were rendered visible in their passage when the spots were next to the earth. At different times also they appear of different magnitudes and of different brightness. The fourth appears generally the smallest, but sometimes the greatest; and the diameter of it's shadow on Jupiter appears sometimes greater than the satellite. The third also appears of a variable magnitude, and the like happens to the other two. Mr. *Pound* also observed, that they appear more luminous at one time than another, and therefore he concluded that they revolve about their axes. Dr. *Herschel* has discovered that they all revolve about their axes, in the

times in which they respectively revolve about Jupiter.

## *On the Satellites of Saturn.*

(265.) In the year 1655, *Huygens* discovered the fourth satellite of *Saturn*; and published a table of it's mean motion in 1659. In 1671, M. *Cassini* discovered the fifth, and the third in 1672; and in 1684, the first and second; and afterwards published Tables of their motions. He called them *Sidera Lodoicea*, in honour of *Louis le Grand*, in whose reign, and observatory, they were discovered. Dr. *Halley* found, by his own observations in 1682, that *Huygens*'s Tables had considerably run out, they being about 15° in 20 years too forward, and therefore he composed new Tables from more correct elements. He also reformed M. *Cassini*'s Tables of the mean motions; and about the year 1720, published them a second time, corrected from Mr. *Pound*'s observations. He observes, that the four innermost satellites describe orbits very nearly in the plane of the ring, which, he says, is, as to the sense, parallel to the equator; and that the orbit of the fifth is a little inclined to them. The following Table contains the periodic times of the five satellites, and their distances in semidiameters of the ring, as determined by Mr. *Pound*, with a micrometer fitted to the telescope given by *Huygens* to the Royal Society. Mr. *Pound* first measured the distance of the fourth, and then deduced the rest from the proportion between the squares of the periodic times and cubes of their distances, and these are found to agree with observations.

| Satellites. | Periodic Times by Pound. | Dist. in semid. of Ring, by Pound. | Dist. in semid. of Saturn by Pound. | Dist. in semid. of Ring by Cassini. | Dist. at the mean dist. of Saturn. |
|---|---|---|---|---|---|
| I | $1^d.21^h.18'.27''$ | 2,097 | 4,893 | $1\frac{1}{15}$ | 0'.43'',5 |
| II | 2. 17. 41. 22 | 2,686 | 6,286 | $2\frac{1}{2}$ | 0. 56 |
| III | 4. 12. 25. 12 | 3,752 | 8,754 | $3\frac{1}{2}$ | 1. 18 |
| IV | 15. 22. 41. 12 | 8,698 | 20,295 | 8 | 3. 0 |
| V | 79. 7. 49. 0 | 25,348 | 59,154 | 23 | 8. 42,5 |

The last column is from *Cassini*; but Dr. *Herschel* makes the distance of the fifth to be 8'. 31'',97, which is probably more exact. In this and the two next Tables, the satellites are numbered from Saturn as they were before the discovery of the other two.

On June 9, 1749, at 10*h.* Mr. *Pound* found the distance of the fourth satellite to be 3'. 7'' with a telescope of 123 feet, and an excellent micrometer fixed to it; and the satellite was at that time very near it's greatest eastern digression. Hence, at the mean distance of the earth from Saturn, that distance becomes 2'. 58'',21; Sir *I. Newton* makes it 3'. 4''.

(266.) The periodic times are found as for the satellites of Jupiter (251). To determine these, M. *Cassini* chose the time when the semi-minor axes of the eclipses which they describe, were the greatest, as Saturn was then 90° from their node, because the place of the satellite in it's orbit is then the same as upon the orbit of Saturn; whereas in every other case, it would be necessary to apply the reduction in order to get the place in it's orbit.

(267.) As it is difficult to see Saturn and the satellites at the same time in the field of view of a telescope, their distances have sometimes been measured, by observing the time of the passage of the body of Saturn

over a wire adjusted as an hour circle in the field of the telescope, and the interval between the times when Saturn and the satellite passed. From comparing the periodic times and distances, M. *Cassini* observed that *Kepler's* Rule (162) agreed very well with observations.

(268.) By comparing the places of the satellites with the ring in different points of their orbits, and the greatest minor axes of the ellipses which they appear to describe, compared with the major axes, the planes of the orbits of the first four are found to be very nearly in the plane of the ring, and therefore are inclined to the orbit of Saturn about 30°; but the orbit of the fifth, according to M. *Cassini* the Son, makes an angle with the ring of about 15 degrees.

(269.) M. *Cassini* places the node of the ring, and consequently the nodes of the four first satellites, in $5^s$. 22° upon the orbit of Saturn, and $5^s$. 21° upon the ecliptic. M. *Huygens* had determined it to be in $5^s$. 20°. 30′. M. *Miraldi*, in 1716, determined the longitude of the node of the ring upon the orbit of Saturn to be $5^s$. 19°. 48′. 30″; and upon the ecliptic to be $5^s$. 16°. 20′. The node of the fifth satellite is placed by M. *Cassini* in $5^s$. 5° upon the orbit of Saturn. M. *de la Lande* makes it $5^s$. 0°. 27′. From the observation of M. *Bernard*, at Marseilles, in 1787, it appears that the node of this satellite is retrograde.

(270.) Dr. *Halley* discovered that the orbit of the fourth satellite was excentric; for, having found it's mean motion, he discovered that it's place by observation was at one time 3° forwarder than by his calculations, and at other observations it was 2°. 30′ behind; this indicated an excentricity; and he placed the line of the apsides in $10^s$. 22°. *Phil. Trans.* N°. 145.

## TABLES *of their* REVOLUTIONS *and* MEAN MOTIONS, *according to* M. de la LANDE.

| *Satel.* | *Diurnal Motion.* | *Motion in 365 Days.* |
|---|---|---|
| I | $6^s.\ 10^\circ.41'.53''$ | $4^s.\ 4^\circ.44'.42''$ |
| II | 4. 11. 32. 6 | 4. 10. 15. 19 |
| III | 2. 19. 41. 25 | 9. 16. 57. 5 |
| IV | 0. 22. 34. 38 | 10. 20. 39. 37 |
| V | 0. 4. 32. 17 | 7. 6. 23. 37. |

| *Satel.* | *Periodic Revolution.* | *Synodic Revolution.* |
|---|---|---|
| I | $1^d.21^h.18'.26'',222$ | $1^d.21^h.18'.54'',778$ |
| II | 2. 17. 44. 51,177 | 2. 17. 45. 51,013 |
| III | 4. 12. 25. 11,100 | 4. 12. 27. 55,239 |
| IV | 15. 22. 41. 16,022 | 15. 23. 15. 23,153 |
| V | 79. 7. 53. 42,772 | 79. 22. 3. 12,883 |

(271.) M. *Cassini* observed, that the fifth satellite disappeared regularly for about half it's revolution, when it was to the east of Saturn; from which he concluded, that it revolved about it's axis; he afterwards, however, doubted of this. But Sir *I. Newton*, in his *Principia*, Lib. III. Prop. 17, concludes from hence, that it revolves about it's axis, and in the same time that it revolves about Saturn; and that the variable appearance arises from some parts of the satellite not reflecting so much light as others. Dr. *Herschel* has confirmed this, by tracing regularly the periodical change of light through more than ten revolutions,

which he found, in all appearances, to be cotemporary with the return of the satellite to the same situation in it's orbit. This is further confirmed by some observations of M. *Bernard*, at Marseilles, in 1787; and is a remarkable instance of analogy among the secondary planets.

(272.) These are all the satellites which were known to revolve about Saturn, till the year 1789, when Dr. *Herschel*, in a Paper in the *Phil. Trans.* for that year, announced the discovery of a *sixth* satellite, interior to all the others, and promised a further account in another Paper. But in the intermediate time he discovered a *seventh* satellite, interior to the sixth; and in a Paper upon Saturn and it's ring, in the *Phil. Trans.* 1790, he has given an account of the discovery; with some of the elements of their motions. He afterwards added Tables of their motions.

(273.) After his observations upon the ring, he says, he cannot quit the subject without mentioning his own surmises, and that of several other Astronomers, of a supposed roughness of the ring, or inequality in the planes and inclinations of it's flat sides. This supposition arose from seeing luminous points on it's boundaries, projecting like the moon's mountains; or from seeing one arm brighter or longer than the other; or even from seeing one arm when the other was invisible. Dr. *Herschel* was of this opinion, till he saw one of these points move off the edge of the ring in the form of a satellite. With his 20 feet telescope he suspected that he saw a sixth satellite; and on August 19, 1787, marked it down as probably being one; and having finished his telescope of 40 feet focal length, he saw six of it's satellites the moment he directed his telescope to the planet. This happened on August 28, 1789. The rotrograde motion of Saturn was then nearly 4'. 30" in a day, which made it very easy to ascertain whether the stars he took to be satellites, were really so; and in about two hours and an half after, he found that the planet had visibly carried

them all away from their places. He continued his observations, and on September 17, he discovered the seventh satellite. These two satellites lie within the orbits of the other five. Their distances from the center of Saturn are 36″,7889, and 28″,6689; and their periodic times are 1*d.* 8*h.* 53′. 8″,9, and 22*h.* 37′. 22″,9. The planes of the orbits of these satellites lie so near to the plane of the ring, that the difference cannot be perceived.

## *On the Satellites of the Georgian.*

(274.) On January 11, 1787, as Dr. *Herschel* was observing the *Georgian*, he perceived, near it's disc, some very small stars, whose places he noted. The next evening, upon examining them, he found that two of them were missing. Suspecting, therefore, that they might be satellites which had disappeared in consequence of having changed their situation, he continued his observations, and in the course of a month discovered them to be satellites, as he had first conjectured. Of this discovery he gave an account in the *Phil. Trans.* 1787.

(275.) In the *Phil. Trans.* 1788, he published a further account of this discovery, containing their periodic times, distances, and positions of their orbits, so far as he was then able to ascertain them. The most convenient method of determining the periodic time of a satellite, is, either from it's eclipses, or from taking it's position in several successive oppositions of the planet; but no eclipses have yet happened since the discovery of these satellites, and it would be waiting a long time to put in practice the other method. Dr. *Herschel*, therefore, took their situations whenever he could ascertain them with some degree of precision, and then reduced them, by computation, to such situations as were necessary for his purpose. In computing the periodic times, he has taken the synodic revolutions, as the positions of their orbits, at the

times when their situations were taken, were not sufficiently known to get very accurate sidereal revolutions. The mean of several results gave the synodic revolution of the first satellite 8*d*. 17*h*. 1′. 19″,3, and of the second 13*d*. 11*h*. 5′. 1″,5. The results, he observes, of which these are a mean, do not much differ among themselves, and therefore the mean is probably tolerably accurate. The epochs from which their situations may, at any time, be computed are, for the *first*, Oct. 19, 1787, at 19*h*. 11′. 28″; and for the *second*, at 17*h*. 22′. 40″; at which times they were 76°. 43′ north, following the planet.

(276.) The next thing to be determined, in the elements of the satellites, was their distances from the planet; to obtain which, he found one distance by observation, and then the other from the periodic times (Article 162). Now, in attempting to discover the distance of the second, the orbit was seemingly elliptical. On March 18, 1787, at 8*h*. 2′. 50″, he found the elongation to be 46″,46, this being the greatest of all the measures he had taken. Hence, at the mean distance of the Georgian from the earth, this elongation will be 44″,23. Admitting, therefore, for the present, says Dr. *Herschel*, that the satellites move in circular orbits, we may take 44″,23 for the true distance, without much error; hence, as the squares of the periodic times are as the cubes of the distances, the distance of the first satellite comes out 33″,09. The synodic revolutions were here used instead of the sidereal, which will make but a small error.

(277.) The last thing to be done, was to determine the inclinations of the orbits, and places of their nodes. And here a difficulty presented itself which could not be got over at the time of his first observation; for it could not then be determined which part of the orbit was inclined *to* the earth, and which *from* it. On the two different suppositions, therefore, Dr. *Herschel* has computed the inclinations of the orbits, and the places of their nodes, and found them as follows. The orbit

of the second satellite is inclined to the ecliptic 99°. 43'. 53",3 or 81°. 6'. 44",4; it's ascending node upon the ecliptic is in $5^s$. 18°, or $8^s$. 6°; and when the planet comes to the ascending node of this satellite, which will happen about the year 1818, the northern half of the orbit will be turned towards the east or west, at the time of it's meridian passage. M. *de la Lambre* makes the ascending node in $5^s$. 21°, or $8^s$. 9°, from Dr. *Herschel's* observations. The situation of the orbit of the first satellite does not materially differ from that of the second. The light of the satellites is extremely faint; the second is the brightest, but the difference is small. The satellites are probably not less than those of *Jupiter*. There will be eclipses of these satellites about the year 1818, when they will appear to ascend through the shadow of the planet, in a direction almost perpendicular to the ecliptic.

Since these discoveries were made, Dr. *Herschel* has discovered four more satellites of the Georgian, and found that their motions are all retrograde. *Phil. Trans.* 1798.

# Chap. XIX.

## ON THE RING OF SATURN.

(278.) Galileo was the first person who observed any thing extraordinary in *Saturn*. The planet appeared to him like a large globe between two small ones. In the year 1610 he announced this discovery. He continued his observations till 1612, when he was surprised to find only the middle globe; but some time after he again discovered the globes on each side, which, in process of time, appeared to change their form; sometimes appearing round, sometimes oblong like an acorn, sometimes semicircular, then with horns towards the globe in the middle, and growing, by degrees, so long and wide as to encompass it, as it were, with an oval ring. Upon this, *Huygens* set about improving the art of grinding object glasses; and made telescopes which magnified two or three times more than any which had been before made, with which he discovered very clearly the ring of Saturn; and having observed it for some time, he published the discovery in 1656. He made the space between the globe and the ring equal to, or rather bigger than the breadth of the ring; and the greater diameter of the ring to that of the globe as 9 to 4. But Mr. *Pound*, with a micrometer applied to *Huygen*'s telescope of 123 feet long, determined the ratio to be as 7 to 3. Mr. *Whiston* in his Memoirs of the Life of Dr. *Clark*, relates, that the Doctor's Father once saw a fixed star between the ring and the body of Saturn. In the year 1675, M. *Cassini* saw the ring, and observed upon it a dark elliptical line, dividing it, as it were, into two rings, the inner of which

appeared brighter than the outer. He also observed a dark belt upon the planet, parallel to the major axis of the ring. Mr. *Hadley* observed that the outer part of the ring seemed narrower than the inner part, and that the dark line was fainter towards it's upper edge; he also saw two belts, and observed the shadow of the ring upon Saturn. In October, 1714, when the plane of the ring very nearly passed through the earth, and was approaching it, M. *Maraldi* observed, that while the arms were decreasing both in length and breadth, the eastern arm appeared a little larger than the other for three or four nights, and yet it vanished first, for, after two nights interruption by clouds, he saw the western arm alone. This inequality of the ring made him suspect that it was not bounded by exactly parallel planes, and that it turned about it's axis. But the best description of this singular phænomenon is that given by Dr. *Herschel*, in the *Phil. Trans.* 1790, who, by his extraordinary telescopes, has discovered many circumstances which had escaped all other observers. We shall here give the substance of his account.

(279.) The black disc, or belt upon the ring of Saturn, is not in the middle of it's breadth; nor is the ring subdivided by many such lines, as has been represented by some Astronomers; but there is one * single, dark, considerable broad line, belt, or zone, which he has constantly found on the north side of the ring. As this dark belt is subject to no change whatever, it is probably owing to some permanent construction of the surface of the ring. This construction cannot be owing to the shadow of a chain of mountains, since it is visible all round on the ring;

* In a Paper in the *Phil. Trans.* 1792, Dr. *Herschel* observes, that, "since the year 1774 to the present time, I can find only four observations where any other black division of the ring is mentioned, than the one which I have constantly observed; these were all in June, 1780."

for at the ends of the ring there could be no shade; and the same argument will hold against any supposed caverns. It is moreover pretty evident, that this dark zone is contained between two concentric circles, as all the phænomena answer to the projection of such a zone. The matter of the ring is undoubtedly no less solid than the planet itself; and it is observed to cast a strong shadow upon the planet. The light of the ring is also generally brighter than that of the planet; for the ring appears sufficiently bright, when the telescope affords scarcely light enough for Saturn. Dr. *Herschel* next takes notice of the extreme thinness of the ring. He frequently saw the first, second, third, fourth, and fifth satellites pass before and behind the ring, in such a manner that they served as an excellent micrometer to measure it's thickness by. It may be proper to mention a few instances, as they serve also to solve some phænomena observed by other Astronomers, without having been accounted for in any manner that could be admitted consistently with other known facts. July 18, 1789, at 16*h*. 41′. 9″ sidereal time, the third satellite seemed to hang upon the following arm, declining a little towards the north, and was seen gradually to advance upon it towards the body of Saturn; but the ring was not so thick as the lucid point. July 23, at 19*h*. 41′. 8″, the fourth satellite was a very little preceding the ring, but the ring appeared to be less than half the thickness of the satellite. July 27, at 20*h*. 15′. 12″, the fourth satellite was about the middle, upon the following arm of the ring, and towards the south; and the second at the farther end, towards the north; but the arm was thinner than either. August 29, at 22*h*. 12′. 25″, the fifth satellite was upon the ring, near the end of the preceding arm, and the thickness of the arm seemed to be about $\frac{1}{3}$ or $\frac{1}{4}$ of the diameter of the satellite, which, in the situation it then was, he took to be less than one-second in diameter. At the same time the first appeared at a little distance following the fifth,

in the shape of a bead upon a thread, projecting on both sides of the same arm; hence, the arm is thinner than the first, which is considerably smaller than the second, and a little less than the third. October 16, he followed the first and second satellites up to the very disc of the planet; and the ring, which was extremely faint, did not obstruct his seeing them gradually approach the disc. These observations are sufficient to show the extreme thinness of the ring. But Dr. *Herschel* further observes, that there may be a refraction through an atmosphere of the ring, by which the satellites may be lifted up and depressed, so as to become visible on both sides of the ring, even though the ring should be equal in thickness to the smallest satellite, which may amount to 1000 miles. From a series of observations upon luminous points of the ring, he has discovered that it has a rotation about it's axis, the time of which is 10*h*. 32′. 15″4.

(280.) The ring is invisible* when it's plane passes through the sun, or the earth, or between them; in the first case, the sun shines only upon it's edge, which is too thin to reflect sufficient light to render it visible; in the second case, the edge only being opposed to us, it is not visible, for the same reason; in the third case, the dark side of the ring is exposed to us, and therefore the edge being the only luminous part which is towards the earth, it is invisible on the same account as before. Observers have differed 10 or 12 days in the time of it's becoming invisible, owing to the difference of the telescopes, and of the state of the atmosphere. Dr. *Herschel* observes that the ring was seen in his telescope, when we were turned towards the unenlightened side; so that he either saw the light reflected from the edge, or else the reflection

* The disappearance of the ring is only with the telescopes in common use among Astronomers; for Dr. *Herschel*, with his large telescopes, has been able to see it in every situation. He thinks the edge of the ring is not flat, but spherical, or spheroidical.

of the light of Saturn upon the dark side of the ring, as we sometimes see the dark part of the moon. He cannot, however, say which of the two might be the case; especially as there are very strong reasons to think, that the edge of the ring is of such a nature as not to reflect much light. M. *de la Lande* thinks that the ring is just visible with the best telescopes in common use, when the sun is elevated 3′ above it's plane, or three days before it's plane passes through the sun; and when the earth is elevated 2′. 20″ above the plane, or one day from the earth's passing it.

(281.) In a paper in the *Phil. Trans.* 1790, Dr. *Herschel* ventured to hint at a suspicion that the ring was divided; this conjecture was strengthened by future observations, after he had an opportunity of seeing both sides of the ring. His reasons are these: 1. The black division, upon the southern side of the ring, is in the same place, of the same breadth, and at the same distance from the outer edge, that it always appeared upon the northern side. 2. With his seven feet reflector and an excellent speculum, he saw the division on the ring, and the open space between the ring and the body, equally dark, and of the same colour with the heavens about the planet. 3. The black division is equally broad on each side of the ring. From these observations, Dr. *Herschel* thinks himself authorised to say, that Saturn has two concentric rings, situated in one plane, which is probably not much inclined to the equator of the planet. The dimensions of the rings are in the following proportions, as nearly as they could be ascertained.

| | *Parts.* |
|---|---|
| Inside diameter of the smaller ring - - | 5900 |
| Outside diameter - - - - - - - | 7510 |
| Inside diameter of the larger ring - | 7740 |
| Outside diameter - - - - - - - | 8300 |
| Breadth of the inner ring - - - - | 805 |
| Breadth of the outer ring - - - - | 280 |
| Breadth of the space between the rings | 115 |

In the *Mem. de l'Acad.* at Paris, 1787, M. *de la Place* supposes that the ring may have many divisions; but Dr. *Herschel* remarks, that no observations will justify this supposition.

(282). From the mean of a great many measures of the diameter of the larger ring, Dr. *Herschel* makes it 46″,677 at the mean distance of Saturn. Hence, it's diameter : the diameter of the earth :: 25,8914 : 1. From the above proportion, therefore, the diameter of the ring must be 204883 miles; and the distance of the two rings 2839 miles.

(283.) The ring being a circle, appears elliptical from it's oblique position; and it appears most open when Saturn is 90° from the nodes of the ring upon the orbit of Saturn, or when Saturn's longitude is about $2^s. 17°$, and $8^s. 17°$. In such a situation, the minor axis is extremely nearly equal to half the major, when the observations are reduced to the sun; consequently the plane of the ring makes an angle of about 30° with the orbit of Saturn.

# Chap. XX.

## ON THE ABERRATION OF LIGHT.

(284.) In the year 1725, Mr. *Molyneux*, assisted by Dr. *Bradley*, fitted up a zenith sector at Kew, in order to discover whether the fixed stars had any sensible parallax *, that is, whether a star would appear to have changed it's place whilst the earth moved from one extremity of the diameter of it's orbit to the other; or, which is the same, to determine whether the diameter of the earth's orbit subtends any sensible angle at the star. The very important discovery arising from their observations is so clearly and fully related by Dr. *Bradley*, in a letter to Dr. *Halley*, that I cannot do better than give it to the reader in his own words. *Phil. Trans.* N°. 406.

(285.) " Mr. *Molyneux*'s apparatus was completed and fitted for observing, about the end of November, 1725, and on the third day of December following, the bright star in the head of *Draco*, marked $\gamma$ by *Bayer*, was for the first time observed as it passed near the zenith, and it's situation carefully taken with the instrument. The like observations were made on the 5th, 11th, and 12th of the same month; and

* Dr. *Hook* was the first inventor of this method, and in the year 1669 he put it in practice at Gresham College, with a telescope 36 feet long. His first observation was July 6, at which time he found the bright star in the head of *Draco*, marked $\gamma$ by *Bayer*, passed about 2′. 12″ northward from the zenith; on July 9, it passed at the same distance; on August 6, it passed 2′. 6″ northward from the zenith; on October 21, it passed 1′. 48″ or 50″ north from the zenith, according to his observations. See his *Cutlerian* Lectures.

there appearing no material difference in the place of the star, a farther repetition of them at this season seemed needless, it being a part of the year wherein no sensible alteration of parallax in this star could soon be expected. It was chiefly, therefore, curiosity that tempted me (being then at Kew, where the instrument was fixed) to prepare for observing the star on December 17, when, having adjusted the instrument as usual, I perceived that it passed a little more southwardly this day than when it was observed before. Not suspecting any other cause of this appearance, we first concluded that it was owing to the uncertainty of the observations, and that either this or the foregoing were not so exact as we had supposed; for which reason we purposed to repeat the observation again, in order to determine from whence this difference proceeded; and upon doing it on December 20, I found that the star passed still more southwardly than in the former observations. This sensible alteration the more surprised us, in that it was the contrary way from what it would have been, had it proceeded from an annual parallax of the star: but being now pretty well satisfied that it could not be entirely owing to the want of exactness in the observations, and having no notion of any thing else that could cause such an apparent motion as this in the star, we began to think that some change of the materials, &c. of the instrument itself might have occasioned it. Under these apprehensions we remained some time; but being at length fully convinced, by several trials, of the great exactness of the instrument, and finding, by the gradual increase of the star's distance from the pole, that there must be some regular cause that produced it, we took care to examine nicely, at the time of each observation, how much it was; and about the beginning of March, 1726, the star was found to be 20″ more southwardly than at the time of the first observation. It now indeed, seemed to have arrived at it's utmost limit southward, because in several trials made about

this time, no sensible difference was observed in it's situation. By the middle of April it appeared to be returning back again towards the north; and about the beginning of June it passed at the same distance from the zenith as it had done in December when it was first observed.

From the quick alteration of the star's declination about this time (it increasing a second in three days) it was concluded that it would now proceed northward, as it before had gone southward of its present situation; and it happened as was conjectured, for the star continued to move northward till September following, when it again became stationary, being then near 20″ more northwardly than in June, and no less than 39″ more northwardly than it was in March. From September the star returned towards the south, till it arrived in December to the same situation it was in at that time twelve months, allowing for the difference of declination on account of the precession of the equinox.

This was a sufficient proof that the instrument had not been the cause of this apparent motion of the star, and to find one adequate to such an effect, seemed a difficulty. A nutation of the earth's axis was one of the first things that offered itself upon this occasion, but it was soon found to be insufficient; for though it might have accounted for the change of declination in $\gamma$ Draconis, yet it would not at the same time agree with the phænomena in other stars, particularly in a small one almost opposite in right ascension to $\gamma$ Draconis, at about the same distance from the north pole of the equator; for though this star seemed to move the same way as a nutation of the earth's axis would have made it, yet it changing it's declination but about half as much as $\gamma$ Draconis in the same time, (as appeared upon comparing the observations of both made upon the same days at different seasons of the year,) this plainly proved that the apparent motion of the stars was not occasioned by a real nutation, since, if that had been the cause, the alteration in both stars would have been nearly equal.

The great regularity of the observations left no room to doubt but that there was some regular cause that produced this unexpected motion, which did not depend on the uncertainty or variety of the seasons of the year. Upon comparing the observations with each other, it was discovered, that in both the fore-mentioned stars, the apparent difference of declination from the maxima was always nearly proportional to the versed sine of the sun's distance from the equinoctial points. This was an inducement to think that the cause, whatever it was, had some relation to the sun's situation with respect to those points. But not being able to frame any hypothesis at that time, sufficient to solve all the phænomena, and being very desirous to search a little farther into this matter, I began to think of erecting an instrument for myself at Wansted; that, having it always at hand, I might with the more ease and certainty enquire into the laws of this new motion. The consideration, likewise, of being able, by another instrument, to confirm the truth of the observations hitherto made with Mr. *Molyneux*'s, was no small inducement to me; but the chief of all was, the opportunity I should thereby have of trying in what manner other stars were affected by the same cause, whatever it was. For Mr. *Molyneux*'s instrument being originally designed for observing $\gamma$ Draconis (in order, as I said before, to try whether it had any sensible parallax) was so contrived as to be capable of but little alteration in it's direction, not above seven or eight minutes of a degree; and there being few stars within half that distance from the zenith of Kew, bright enough to be well observed, he could not with his instrument thoroughly examine how this cause affected stars differently situated with respect to the equinoctial and solstitial points of the ecliptic.

These considerations determined me; and by the contrivance and direction of the very ingenious person Mr. *Graham*, my instrument was fixed up August

19, 1727. As I had no convenient place where I could make use of so long a telescope as Mr. *Molyneux*'s, I contented myself with one of but little more than half the length of his (viz. of about 12½ feet, his being 24½) judging, from the experience which I had already had, that this radius would be long enough to adjust the instrument to a sufficient degree of exactness, and I have had no reason since to change my opinion; for, from all the trials I have yet made, I am well satisfied, that when it is carefully rectified, it's situation may be securely depended upon to half a second. As the place where my instrument was to be hung, in some measure determined it's radius, so did it also the length of the arch or limb on which the divisions were made to adjust it; for the arch could not conveniently be extended farther than to reach to about 6°. 15′ on each side my zenith. This indeed was sufficient, since it gave an opportunity of making choice of several stars very different both in magnitude and situation, there being more than two hundred inserted in the British Catalogue, that may be observed with it. I needed not to have extended the limb so far, but that I was willing to take in Capella, the only star of the first magnitude that comes so near to my zenith.

My instrument being fixed, I immediately began to observe such stars as I judged most proper to give me light into the cause of the motion already mentioned. There was variety enough of small ones, and not less than twelve that I could observe through all the seasons of the year, they being bright enough to be seen in the day-time, when nearest the sun. I had not been long observing, before I perceived that the notion we had before entertained, of the stars being farthest north and south when the sun was about the equinoxes, was only true of those that were near the solstitial colure; and after I had continued my observations a few months, I discovered, what I then apprehended to be a general law, observed by all the stars, viz. that

each of them became stationary, or was farthest north or south when they passed over my zenith at six o'clock either in the morning or evening. I perceived likewise, that whatever situation the stars were in, with respect to the cardinal points of the ecliptic, the apparent motion of every one tended the same way when they passed my instrument about the same hour of the day or night; for they all moved southward while they passed in the day, and northward in the night; so that each was farthest north when it came about six o'clock in the evening, and farthest south when it came about six in the morning.

Though I have since discovered that the maxima in most of these stars do not happen exactly when they come to my instrument at those hours, yet, not being able at that time to prove the contrary, and supposing that they did, I endeavoured to find out what proportion the greatest alterations of declination in different stars bore to each other; it being very evident that they did not all change their declinations equally. I have before taken notice that it appeared from Mr. *Molyneux*'s observations that $\gamma$. Draconis altered it's declination about twice as much as the fore-mentioned small star almost opposite to it; but examining the matter more particularly, I found that the greatest alteration of declination in these stars was as the sine of the latitude of each respectively. This made me suspect, that there might be the like proportion between the maxima of other stars; but finding that the observations of some of them would not perfectly correspond with such an hypothesis, and not knowing whether the small difference I met with might not be owing to the uncertainty and error of the observations, I deferred the farther examination into the truth of this hypothesis, till I should be furnished with a series of observations made in all parts of the year; which might enable me, not only to determine what errors the observations are liable to, or how far they may

safely be depended upon; but to judge whether there had been any sensible change in the parts of the instrument itself.

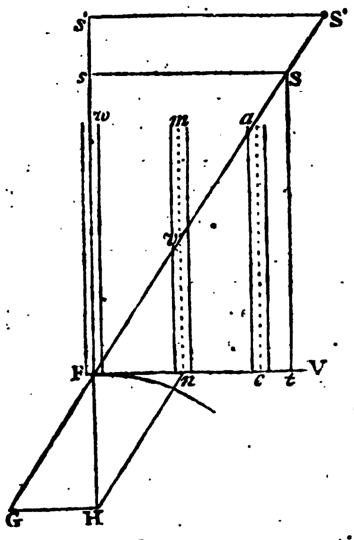
Upon these considerations I laid aside all thoughts at that time about the cause of the fore-mentioned phænomena, hoping that I should the easier discover it, when I was better provided with proper means to determine more precisely what they were.

When the year was completed, I began to examine and compare my observations, and having pretty well satisfied myself as to the general laws of the phænomena, I then endeavoured to find out the cause of them. I was already convinced, that the apparent motion of the stars was not owing to a nutation of the earth's axis. The next thing that offered itself, was an alteration in the direction of the plumb-line with which the instrument was constantly rectified; but this, upon trial, proved insufficient. Then I considered what refraction might do; but here also nothing satisfactory occurred. At last I conjectured, that all the phænomena hitherto mentioned, proceeded from the progressive motion of light and the earth's annual motion in it's orbit. For I perceived, if light was propagated in time, the apparent place of a fixed object would not be the same when the eye is at rest, as when it is moving in any other direction than that of the line passing through the eye and object; and that when the eye is moving in different directions, the apparent place of the object would be different."

This is Dr. *Bradley*'s account of this very important discovery; we shall therefore proceed to show that his principle will solve all the phænomena.

(286.) The situation of any object in the heavens is determined by the position of the axis of the telescope annexed to the instrument with which we measure; for such a position is given to the telescope, that the rays of light from the object may descend down

the axis, and in that situation the index shows the angular distance required. Now if light be progressive, when a ray from any object descends down the axis, the position of the telescope must be different from what it would have been, if light had been instantaneous, and therefore the place to which the telescope is directed, will be different from the true place of the object. For let $S$ be a fixed star, $VF$ the direction of the earth's motion, $SF$ the direction of a particle of light, entering the axis $ac$ of a telescope at $a$, and moving through $aF$ while the earth moves from $c$ to $F$; then, if the telescope continue parallel to itself, the light will descend in the axis. For let the axis, $nm$, $Fw$, continue parallel to $ac$; then, considering each motion * as uniform, the spaces described in the same




time will continue in the same proportion; but $cF$ : $aF$ :: $cn$ : $av$, and by supposition $cF$, $aF$, are described

* The motión of the spectator arising from the rotation of the earth about it's axis is not here taken into consideration, it being so small as not to produce any sensible effect.

in the same time, therefore $cn$, $av$, are described in the same time; hence, when the telescope comes into the situation $nm$, the particle of light will be in the axis at $v$; and this being true for every instant, in this position of the telescope the ray will descend down the axis, and consequently the place of the star, determined by the telescope at $F$, is $s'$, and the angle $SFs'$ is the *aberration*, or *the difference between the true place of the star and the place determined by the instrument*. Hence, if we take any line $FS : Ft$ :: velocity of light : the velocity of the earth, and join $St$, and complete the parallelogram $FtSs$, the aberration will be equal to the angle $FSt$. Also $S$ represents the true place of the star, and $s$ the place determined by the instrument.

(287.) As the place measured by the instrument differs from the true place, let us next consider how the progressive motion of light may effect the place of the star seen by the naked eye. If a ray of light fall upon the eye in motion, it's relative motion, in respect to the eye, will be the same as if equal motions were impressed in the same direction upon each, at the instant of contact; for equal motions in the same direction, impressed upon two bodies, will not affect their relative motions, and therefore the effect of one upon the other will not be altered. Let $VF$ be a tangent to the earth's orbit at $F$, which will represent the direction of the earth's motion at $F$, $S$ the star, join $SF$, and produce it to $G$, and take $FG : Fn$ :: the velocity of light : the velocity of the earth's in it's orbit, and complete the parallelogram $nFGH$, and draw the diagonal $FH$. Now as $FG$, $nF$, represent the motions of light and of the earth in it's orbit, conceive a motion $Fn$ equal, and opposite to $nF$ to be impressed upon the eye at $F$, and upon the ray of light, then the eye will be at rest, and the ray of light, by the two motions $FG$, $Fn$, will describe the diagonal $FH$; this, therefore, is the relative motion of the ray of light in respect to the eye itself. Hence, the object appears

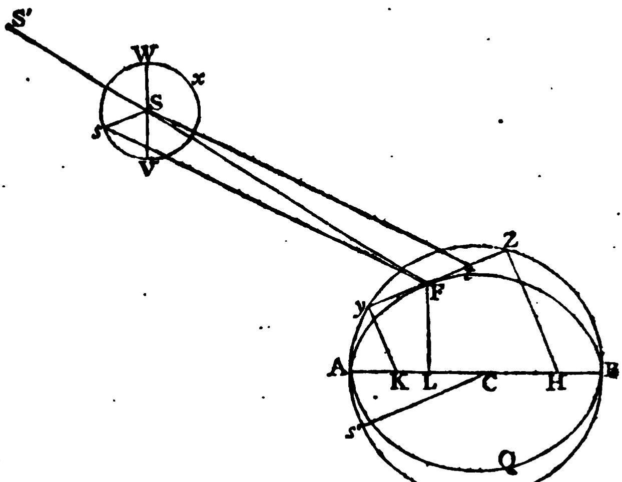
in the direction $HF$, and consequently it's apparent place differs from it's true place by the angle $GFH = FSt$. It appears, therefore, that the apparent place of the object to the naked eye, is the same as the place determined by the instrument. We may therefore call the place, measured by the instrument, the apparent place. Many writers have given the explanation in this article, as the proof of the aberration, not considering that the aberration is the difference between the true place and that determined by the instrument, or the instrumental error; indeed, in this case, the apparent place to the naked eye, coincides with the place determined by the instrument, and therefore no error has been produced by considering it in that point of view; but it introduces a wrong idea of the subject; the correction which we apply, or the aberration, is the correction of the place determined by the instrument, and therefore the investigation ought to proceed upon this principle; how much does the place, determined by the instrument, differ from the true place?

(288.) By Trigonometry, Art. 128, sin. $FSt$ : sin. $FtS$ :: $Ft$ : $FS$ :: velocity of the earth : velocity of light; hence, sine of aberration = sin. $FtS \times \frac{\text{vel. of earth}}{\text{vel. of light}}$; therefore, if we consider the velocity of the earth and of light to be constant, the sine of aberration, or the aberration itself, as it never exceeds 20″, varies as sin. $FtS$, and therefore is greatest when that angle is a right angle; if, therefore, $s$ be put for the sine of $FtS$, we have 1 (rad.) : $s$ :: 20″ : $s \times 20''$ the aberration. Hence, when $Ft$ coincides with $FS$, or the earth is moving directly to or from a star, there is no aberration.

(289.) As (by observation) the angle $FSt = 20''$, when $FtS = 90^\circ$, we have, *the velocity of the earth : velocity of light :: sin.* 20″ : *radius* :: 1 : 10314.

(290.) The aberration $Ss'$ lies, from the true place of the star, in a direction parallel to the direction of the earth's motion, and towards the same part.

(291.) Whilst the earth makes one revolution in it's orbit, the curve, parallel to the ecliptic, described by the apparent place of a fixed star, is a circle. For let $AFBQ$ be the earth's orbit, $K$ the focus in which the sun is, $H$ the other focus; on the major axis $AB$



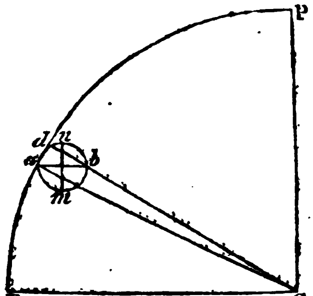

describe a circle in the same plane; draw a tangent $yFZ$ to the point $F$, and $Ky$, $HZ$, perpendicular to it; then (Conic Sect. Ellipse, prop. 5), the points $y$ and $Z$ will be always in the circumference of the circle. Let $S'$ be the true place of the star, any where out of the plane of the ecliptic, which therefore must be conceived as elevated above the plane $AFBQ$, and take $tF : FS$ as the velocity of the earth to the velocity of light, and complete the parallelogram $FtSs$, and $s$ will (286) be the apparent place of the star. Draw $FL$ perpendicular to $AB$, and let $WsVx$ be the curve described by the point $s$, and $WSV$ be parallel to $FL$. Now (from physical principles) the velocity of the earth varies as $\frac{1}{Ky}$, or as $HZ$ (Con. Sec. El. p. 6); but $tF$, or $Ss$ represents the velocity of the earth; hence, $Ss$ varies as $HZ$. Also, as $Ss$, $SV$, are parallel to $Fy$, $FL$, the

angle $sSV$ = the angle $yFL$, which is = the angle $ZHL$, because the angle $LFZ$ added to each makes two right angles, for in the quadrilateral figure $LFZH$, the angles $L$ and $Z$ are right ones. Hence, as $Ss$ varies as $HZ$, and the angle $sSV = ZHA$, the figures described by the points $s$ and $Z$ must be similar; but $Z$ describes a circle in the time of one revolution of the earth in it's orbit; hence, $s$ must describe a circle about $S$ in the same time. And as $Ss$ is always parallel to $tF$ which lies in the plane of the ecliptic, the circle $WsVx$ is parallel to the ecliptic. Also, as $S$ and $H$ are two points similarly situated in $WV$ and $AB$, it appears that the true place of the star divides that diameter, which, although in a different plane, we may consider as perpendicular to the major axis of the earth's orbit, in the same ratio as the focus divides the major axis. But as the earth's orbit is very nearly a circle, we may consider $S$ in the centre of the circle, without any sensible error.

(292.) As we may, for the purposes which we shall here want to consider, conceive the earth's orbit $AFBQ$ to be a circle, and therefore to coincide with $AyZB$, if from the center $C$ we draw $Cs'$ parallel to $Ss$, or $yF$, $s'$ will be the point in that circle corresponding to $s$ in the circle $WsVx$; and as $Fs' = 90°$, the apparent place of the star in the circle of aberration is always 90° *before* the place of the earth in it's orbit, and consequently the apparent angular velocity of the star and earth about their respective centers are always equal. It is further supposed, that the star $S$ is at an indefinitely great distance; for the true place of the star is supposed not to be altered from the motion of the earth, and considering $FS$ as always parallel to itself, it will always be directed to $S$ as a fixed point in the heavens. Hence also, as the apparent place of the sun is opposite to that of the earth, the apparent place of the star, in the circle of aberration, is 90° *behind* that of the sun.

(293). As a small part of the heavens may be con-

ceived as a plane perpendicular to a line joining the star and eye, it follows, from the principles of orthographic projection, that the circle *ambn* parallel to the ecliptic described by the apparent place of the star, projected upon that plane, will be an ellipse; the apparent path of the star in the heavens will therefore be an ellipse, and the major axis will be to the minor, as radius to the sine of the star's latitude. For let *CE* be the plane of the ecliptic, *P* it's pole, *PE* a secondary to it, *PC* perpendicular to *EC*, *C* the place of the eye, and let *ab* be parallel to *CE*, then it will be that diameter of the circle *anbm* of aberration which is seen most obliquely, and consequently that diameter which is projected into the minor axis of the ellipse; let *mn* be perpendicular to *ab*, and it will be seen directly, being perpendicular to a line drawn from it to



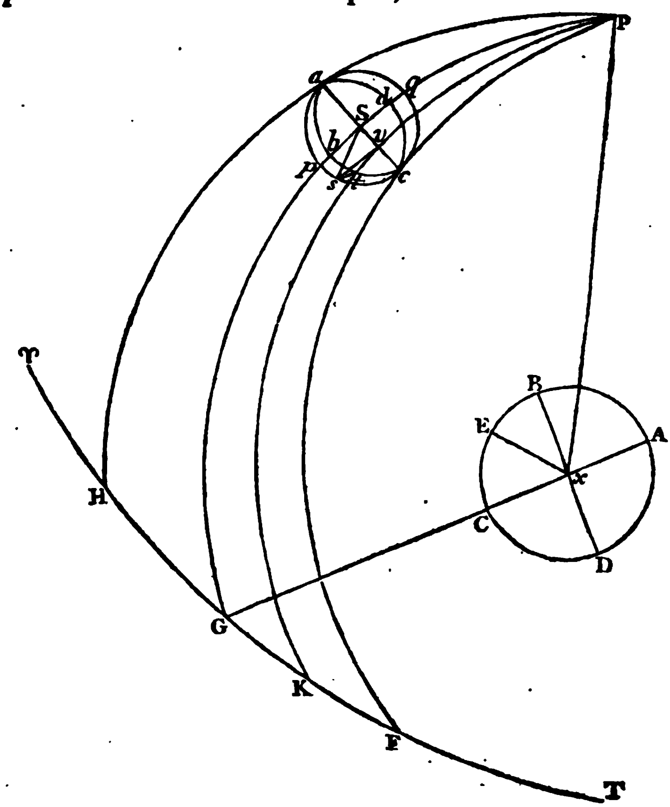

the eye, and therefore will be the major axis; draw *Ca*, *Cbd*, and *ad* is the projection of *ab*; and as *ad* may be considered as a straight line, we have (Trig. Art. 128) *mn* or *ab*, the major axis, : *ad* the minor :: rad. : sin: *abd*, or *ECd* the star's latitude. As the angle *bad* is the complement of *abd*, or of the star's latitude, the circle is projected upon a plane making an angle with it equal to the complement of the star's latitude.

(294.) As the minor axis *da* coincides with a secondary to the ecliptic, it must be perpendicular to it, and the major axis *mn* is parallel to it, it's position not being altered by projection. Hence, in the pole of

the ecliptic, the sine of the star's latitude being radius, the ellipse becomes a circle; and in the plane of the ecliptic, the sine of the star's latitude being $=0$, the minor axis vanishes, and the ellipse becomes a straight line, or rather a very small part of a circular arc.

(295.) To find the aberration in *latitude* and *longitude*. Let $ABCD$ be the earth's orbit, supposed to be a circle with the sun in the center at $x$, and conceive $P$ to be in a line drawn from $x$ perpendicular to $ABCD$, and to represent the pole of the ecliptic; let $S$ be the true place of the star, and conceive $apcq$ to be the circle of aberration parallel to the ecliptic, and $abcd$ the ellipse into which it is projected; let ♈ $T$ be an arc of the ecliptic, and draw the secondary $PSG$ to it, and (293) it will coincide with the minor axis $bd$ into which the diameter $pq$ is projected; draw $GCxA$, and it is parallel to $pq$, and $BxD$ perpendicular to $AC$ must be parallel to the major axis $ac$; then, when the earth is at $A$, the star is in conjunction, and in opposition when the earth is at $C$. Now, as the place of the star in the circle of aberration (292) is always 90° before the earth in it's orbit, when the earth is at $A$, $B$, $C$, $D$, the apparent places of the star in the circle will be at $a$, $p$, $c$, $q$, and in the ellipse at $a$, $b$, $c$, $d$; and to find the place of the star in the circle, when the earth is at any point $E$, take the angle $pSs = ExB$, and $s$ will be the corresponding place of the star in the circle; and to find the projected place in the ellipse, draw $sv$ perpendicular to $Sc$, and $vt$ perpendicular to $Sc$ in the plane of the ellipse, and $t$ will be the apparent place of the star in the ellipse; join $st$, and it will be perpendicular to $vt$, because the projection of the circle into the ellipse is in lines perpendicular to the ellipse; draw the secondary $PvtK$, which will, as to sense, coincide with $vt$, unless the star be very near to the pole of the ecliptic; therefore the rules here given will be sufficiently accurate, except in that case. Now as $cvS$ is parallel to the ecliptic, $S$ and $v$ must have the same latitude; hence, $vt$ is the aberration in

latitude; and as $G$ is the true, and $K$ the apparent place of the star in the ecliptic, $GK$ is the aberration




in longitude. To find these quantities, put $m$ and $n$ for the sine and cosine of the angle $sSc$, or $CxE$, the earth's distance from syzygies, radius being unity; and as (293) the angle $svt$=the complement of the star's latitude, the angle $vst$=the star's latitude, for the sine and cosine of which put $v$ and $w$, and put $r=Sa$, or $Ss$; then in the right-angled triangle $Ssv$ (Trig. Art. 128) $1 : m :: r : sv=rm$; hence, in the triangle $vts$, $1 : v :: rm : tv=rvm$ the aberration in *latitude*. Also, in the triangle $Ssv$, $1 : n :: r : vS=rn$; hence, $w$ (13) : $1 :: rn : GK=\frac{rn}{w}$ the aberration in *longitude*

When the earth is in syzygies, $m=0$, therefore there is no aberration in latitude; and, as $n$ is then greatest, there is the greatest aberration in longitude; if the earth be at *A*, or the star in conjunction, the apparent place of the star is at *a*, and reduced to the ecliptic at *H*; therefore *GH* is the aberration, which diminishes the longitude of the star, the order of the signs being ♈ *GT*; but when the earth is at *C*, or the star in opposition, the apparent place *c* reduced to the ecliptic is at *F*, and the aberration *GF* increases the longitude; hence, the longitude is the greatest when the star is in opposition, and least when in conjunction. When the earth is in quadratures at *D* or *B*, then $n=0$, and $m$ is greatest; therefore there is no aberration in longitude, but the greatest in latitude; when the earth is at *D*, the apparent place of the star is at *d*, and the latitude is there increased; but when the earth is at *B*, the apparent place of the star is at *b*, and the latitude is diminished; hence, the latitude is least in quadrature before opposition, and greatest in quadrature after. From the mean of a great number of observations, Dr. *Bradley* determined the value of $r$ to be $20''$.

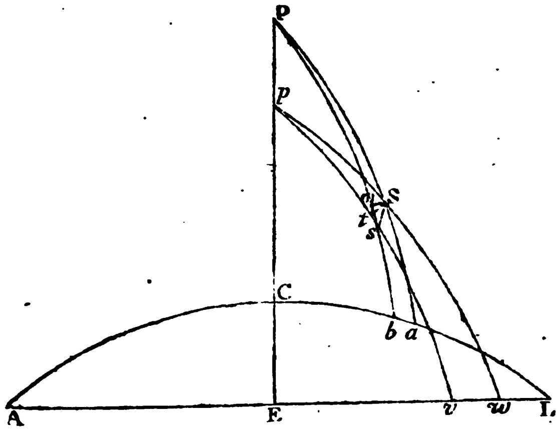
Ex. 1. What is the greatest aberration in latitude and longitude of $\gamma$ *Ursæ minoris*, whose latitude is $75°. 13'$? First, $m=1$, $'v=,9669$ the sine of $75°. 13'$; hence, $20''\times 9669=19'',34$ the greatest aberration in *latitude*. For the greatest aberration in longitude, $n=1$, $w=,2551$; hence, $\frac{20''}{,2551}=78'',4$ the greatest aberration in *longitude*.

Ex. 2. What is the aberration in latitude and longitude of the same star, when the earth is $30°$ from syzygies? Here $m=,5$; hence, $19'',34\times,5=9'',67$ the aberration in *latitude*. If the earth be $30°$ beyond conjunction or before opposition, the latitude is diminished; but if it be $30°$ after opposition or before conjunction, the latitude is increased. Also, $n=,866$; hence, $78'',4\times,866=67'',89$ the aberration in *longi-*

*tude*. If the earth be 30° from conjunction, the longitude is diminished; but if it be 30° from opposition, it is increased.

Ex. 3. For the *Sun*, $m=0$ and $n=1$, $w=1$; hence, it has no aberration in latitude, and the aberration in longitude $=r=20''$ constantly. This quantity $20''$ of aberration of the sun, answers to it's mean motion in $8'.7''.30'''$, which is therefore the time in which the light moves from the sun to the earth at it's mean distance. Hence, the sun always appears $20''$ backwarder than it's true place.

(296.) To find the aberration in *right ascension* and *declination*. Let $AEL$ be the equator, $p$ it's pole;




$ACL$ the ecliptic, $P$ it's pole; $S$ the true place of the star, $s$ the apparent place in the ellipse; draw the great circles, $Psa$, $Psb$, $pSw$, $pSv$, and $Sv$, $st$ perpendicular to $Pb$, $pv$. Now $sv=rvm$ (295); also, $Sv=rn$; hence, (Trig. Art. 123) $rvm\ (vs) : rn\ (Sv) ::$ rad. : tan. $Ssv = \dfrac{n}{vm} = \dfrac{\text{cotan. earth's dist. syzy.}}{\text{sin. star's lat.}} =$ cosec. star's lat. $\times$ cotan. earth's distance from syzygies. Thus we immediately compute the angle $Ssv$; compute also the angle of position $Psp$ from the three sides of that triangle being given (Trig. Art. 239), and we get the

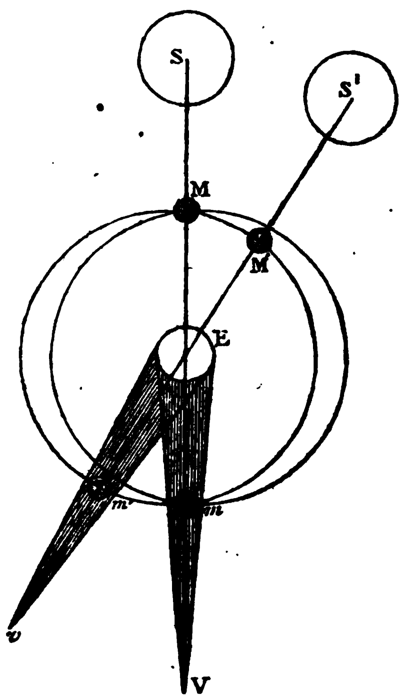
angle $Ssp$, it being the sum or difference of $Ssv$ and $Psp$. Put $a$ and $b$ for the sine and cosine of $Ssv$, $c$ and $d$ for the sine and cosine of $Ssp$, $z$=cosine of the star's declination; then (as $sv$, $st$, are the cosines of $Ssv$, $Sst$, to radius $sS$) $b : d :: sv\ (=rvm) : st = rvm \times \frac{d}{b} = 20'' \times vm \times \frac{d}{b}$ the aberration in *declination*; and (as $Sv$, $St$, are the sines of $Ssv$, $Sst$, to radius $sS$) $a : c :: Sv\ (=rn) : St = \frac{rnc}{a}$; hence (13), $vw \left(= \frac{St}{\text{cos. dec.}}\right) = 20'' \times \frac{nc}{az}$ the aberration in *right ascension*.

# Chap. XXI.

## ON THE ECLIPSES OF THE SUN AND MOON.

(297.) An eclipse of the *Moon* is caused by it's entering into the earth's shadow, and consequently it must happen when the moon is in opposition to the sun, or at the full moon. An eclipse of the *Sun* is caused by the interposition of the moon between the earth and sun, and therefore it must happen when the moon is in conjunction with the sun, or at the new moon. If the plane of the moon's orbit coincided with the plane of the ecliptic, there would be an eclipse at every opposition and conjunction; but the plane of the moon's orbit being inclined to the ecliptic, there can be no eclipse at opposition or conjunction, unless at that time the moon be at, or near to the node. For suppose $MM'mm'$ be the orbit of the moon, and let the other circle represent the plane of the earth's orbit, or that plane in which the sun $S$, appears as seen from the earth $E$, and let these two planes be inclined to each other, so that we may conceive the part $MM'm$ to lie above, and the part $mm'M$ below the plane of the earth's orbit; and $M$, $m$, are the nodes. Now if the moon be at $M$, in conjunction, the three bodies are then in the same plane, and therefore the moon is interposed between the earth and sun, and causes an eclipse of the sun. But if the moon be at $M'$ when the sun comes into conjunction at $S'$, $M'$ is now elevated above the line joining $E$ and $S'$, and $M'$ may be so far from $M$, that the elevation of $M'$ above the line $ES'$ may be so much, that the moon may not be interposed between $E$ and $S'$, in which case there

will be no eclipse of the sun. Whether, therefore, there will be an eclipse of the sun at the conjunction, or not, depends upon the distance of the moon from the node at that time. If the moon be at *m* at the time of opposition, then the three bodies being in the same right line, the shadow *EV* of the earth must fall upon the moon, and the moon must suffer an eclipse. But if the moon be at *m′* at the time of opposition, *m′* may be so far below the shadow *Ev* of the earth, that the moon may not pass through it, in which case there will be no eclipse. Whether, therefore, there




will be a lunar eclipse at the time of opposition, or not, depends upon the distance of the moon from the node at that time. If the two planes coincided, there would evidently be a central interposition every conjunction and opposition, and consequently a total

eclipse. *Meton*, who lived about 430 years before CHRIST, observed, that after 19 years, the new and full moons returned again on the same day of the month. The ancient Astronomers also observed, that at the end of 18 years 10 days, a period of 223 lunations, there was a return of the same eclipses; and hence, they were enabled to foretel when they would happen. This is mentioned by *Pliny* the Naturalist, Lib. II. Ch. 13. and by *Ptolemy*, Lib. IV. Ch. 2. This restitution of eclipses depends upon the return of the following elements to the same state.—1. The sun's place. 2. The moon's place. 3. The place of the moon's apogee. 4. The place of the ascending node of the moon. The exact recurrence of these can never take place; but it so nearly happens in the above time, as to produce eclipses remarkably corresponding. In this manner Dr. *Halley* predicted and published a return of eclipses from 1700 to 1718, many of them corrected from observations; together with the following elements.—1. The apparent time of the middle. 2. The sun's anomaly. 3. The annual argument. 4. The moon's latitude. He says, that in this period of 223 lunations, there are 18 years 10 or 11 days (according as there are five or four leap-years) $7h. 43'\frac{1}{4}$; that if we add this time to the middle of any eclipse observed, we shall have the return of a corresponding one, certainly within $1h. 30'$; and that, by the help of a few equations, we may find the like series for several periods.

## *To explain the Principles of the Calculation of an Eclipse of the Moon.*

(298.) The first thing to be done, is to find the time of the *mean** opposition. To get which, from

* The time of the *mean* opposition is the time when the opposition would have taken place, if the motions of the bodies had been uniform.

the Tables of Epacts*, amongst the Tables of the moon's motion, take out the epact for the year and month, and subtract the sum from 29*d*. 12*h*. 44′. 3″, one synodic revolution of the moon, or two if necessary, so that the remainder may be less than a revolution, and that remainder gives the time of the *mean* conjunction. If to this we add 14*d*. 18*h*. 22′. 1″,5, half a revolution, it gives the time of the next *mean* opposition; or if we subtract, it gives the time of the preceding *mean* opposition. If it be leap-year, in January and February subtract a day from the sum of the epacts, before you make the subtraction. When the day of the *mean* conjunction is 0, it denotes the last day of the preceding month.

Ex. To find the times of the *mean* new and full moons in February, 1795.

| | |
|---|---|
| Epact 1795 - - | $9^d$. $11^h$. 6′. 17″ |
| February - - - | 1. 11. 15. 57 |
| | 10. 22. 22. 24 |
| | 29. 12. 44. 3 |
| Mean new moon - | 18. 14. 21. 49 |
| | 14. 18. 22. 1,5 |
| Mean full moon - | 3. 19. 59. 47,5 |

(299.) To determine whether an eclipse may happen at opposition, find the *mean* longitude of the earth at the time of *mean* opposition, and also the longitude of the moon's node; then, according to M. *Cassini*, if

* The epact for any year is the age of the moon at the beginning of the year from the last *mean* conjunction, that is, from the time when the mean longitudes of the sun and moon were last equal. The epact for any month is the age which the moon would have had at the beginning of the month, if it's age had been nothing at the beginning of the year; therefore, if to the epact for the year, the epact for the month be added, the sum taken from 29*d*. 12*h*. 44′. 3″, or from twice that quantity if the sum exceed it, must give the time of *mean* conjunction.

the difference between the *mean* longitudes of the earth and the moon's node be less than 7°. 30′, there *must* be an eclipse; if it be greater than 14°. 30′, there *cannot* be an eclipse; but between 7°. 30′, and 14°. 30′, there *may*, or may *not*, be an eclipse. M. de *Lambre* makes these limits 7°. 47′, and 13°. 21′*.

Ex. ~~To find~~ whether there will be an eclipse at the full moon on February 3, 1795.

Sun's mean long. at $3^d. 19^h. 59'. 47'',6$. $10^s.13°.27'.20'',8$

| | |
|---|---|
| Mean long. of the earth - - - | 4. 13. 27. 20,8 |
| Long. of the moon's node - - - | 4. 8. 1. 48,5 |
| Difference - - - - - - - - | 0. 5. 25. 32,3 |

Hence, there must be an eclipse.

Examine thus all the new and full moons for a month before and a month after the time at which the sun comes to the place of the nodes of the lunar orbit, and you will be sure not to miss any eclipses. Or, having the eclipses for the last 18 years, if you add to the times of the middle of these eclipses, 18*y*. 10*d*. 7*h*. 43′ ¾, or 18*y*. 11*d*. 7*h*. 43′ ¾, (297) it will give the times when you may expect the eclipses will return.

(300.) To the time of *mean* opposition, compute the true longitudes of the sun and moon, and the moon's true latitude; and find, from the Tables of their motions, the horary motions of the sun and moon in longitude, and the difference ($d$) of their horary motions is the relative horary motion of the moon in respect to the sun, or the motion with which the moon *approaches to*, or *recedes from*, the sun; find also the moon's horary motion in latitude; and suppose, at the time ($t$) of *mean* opposition, the moon is at the distance ($m$) from opposition; then, as we

* This may be found from Art. 306. by finding the true limit, and then applying the greatest difference of the true and mean places.

may suppose the moon to approach the sun, or recede from it, uniformly, $d : m :: 1$ hour : the time ($w$) between $t$ and the opposition, which added to, or subtracted from, the time $t$, according as the moon is not yet got into opposition, or is beyond it, gives the time of the ecliptic opposition.

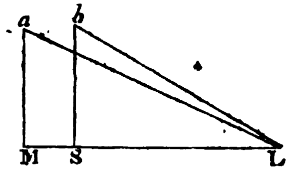
(301.) To find the place of the moon in opposition, let $n$ be the moon's horary motion in longitude; then, 1 hour : $w :: n$ : the increase of the moon's longitude in the time $w$, which applied to the moon's longitude at the time of the mean opposition, gives the true longitude of the moon at the time of the ecliptic opposition. The opposite point to that must be the true longitude of the sun. Find also the moon's true latitude at the time of opposition, by saying, 1 hour : $w$ :: the horary motion in latitude : the motion in latitude in the time $w$, which applied to the moon's latitude at the time of the mean opposition, gives the true latitude at the time of the true opposition*. In like manner you may compute the true time of the ecliptic conjunction, and the places of the sun and moon for that time, when you calculate a solar eclipse.

(302.) With the sun's horary motion in longitude, and the moon's in longitude and latitude, find the inclination of the *relative* orbit, and the horary motion upon it. To do this, let $LM$ be the horary motion of the moon in longitude, $SM$ that of the sun; draw $Ma$ perpendicular to $LM$, and equal to the moon's horary motion in latitude; take $Sb = Ma$, and parallel

---

* For greater certainty, you may compute again, from the Tables, the places of the sun and moon, and if they be not exactly in opposition, which probably may not be the case, as the moon's longitude does not increase uniformly, repeat the operation. This accuracy, however, in eclipses is generally unnecessary; for the best lunar Tables cannot be depended upon to give the moon's longitude nearer than 10″; therefore the probable error from the Tables is vastly greater than that which arises from the motion in longitude not being uniform. Unless, therefore, very great accuracy be required, this operation is unnecessary.

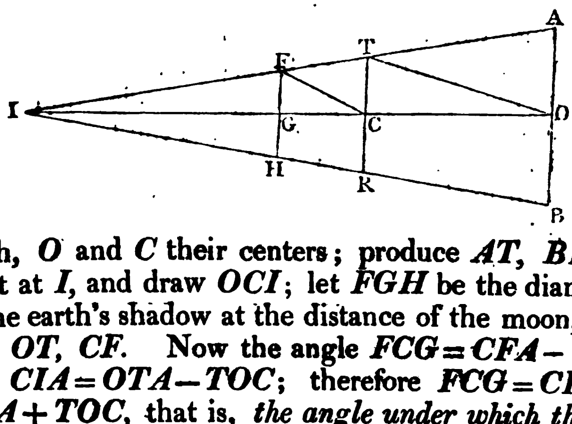
to it, and join *La*, *Lb*; then *La* is the moon's *true* orbit, and *Lb* it's *relative* orbit in respect to the sun.




Hence, *LS* (the difference of the horary motions in longitude) : *Sb* the moon's horary motion in latitude :: radius : tan. *bLS*, the inclination of the relative orbit; and cos. *bLS* : radius :: *LS* : *Lb*, the horary motion in the *relative* orbit.

(303.) At the time of opposition, find, from the Tables, the moon's horizontal parallax, it's semidiameter, and the semidiameter of the sun, the horizontal parallax of which we may here take $=9''$.

(304.) To find the semidiameter of the earth's shadow at the moon, seen from the earth. Let *AB* be the diameter of the sun, *TR* the diameter of the

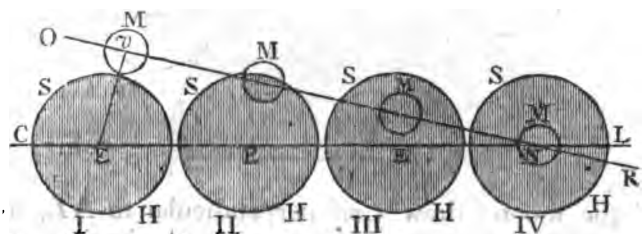



earth, *O* and *C* their centers; produce *AT*, *BR*, to meet at *I*, and draw *OCI*; let *FGH* be the diameter of the earth's shadow at the distance of the moon, and join *OT*, *CF*. Now the angle $FCG=CFA-CIA$, but $CIA=OTA-TOC$; therefore $FCG=CFA-OTA+TOC$, that is, *the angle under which the semidiameter of the earth's shadow, at the moon, appears, is equal to the sum of the horizontal parallaxes of the sun and moon diminished by the apparent semidiameter of the sun.* In eclipses of the moon, the shadow is found to be a little greater than this Rule

gives it, owing to the atmosphere of the earth. This augmentation of the semidiameter is, according to M. *Cassini*, 20″; according to M. *Monnier*, 30″; and according to M. de la *Hire*, 60″. *Mayer* thinks the correction is about $\frac{1}{60}$ of the semidiameter of the shadow, or that you may add as many seconds as the semidiameter contains minutes. Some computers always add 50″; but this must be subject to some uncertainty.

(305.) As the angle $CIT$ $(=QTA-TOC)$ is known, we have sin. $TIC$ : cos. $TIC$ :: $TC$ : $CI$ the length of the earth's shadow. If we take the angle $ATO=16'.\ 3''$ the mean semidiameter of the sun, $TOC=9''$ the horizontal parallax of the sun, we have $CIT=15'.\ 54''$; hence, sin. $15'.\ 54''$ : cos. $15'.\ 54''$, or 1 : 216,2 :: $TC$ : $CI=216{,}2\ TC$.

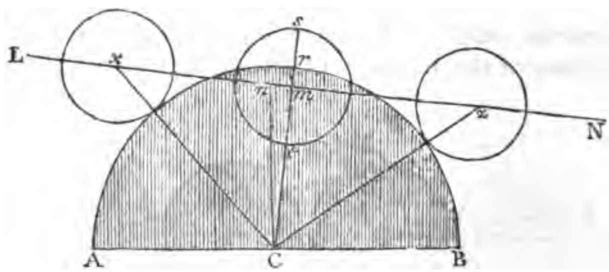
(306.) The different eclipses which may happen of the moon, may be thus explained. Let $CL$ represent the plane of the ecliptic, $OR$ the moon's orbit, cutting




the ecliptic in the node $N$; and let $SH$ represent a section of the earth's shadow at the distance of the moon from the earth, and $M$ the moon at the time when she passes nearest to the center of the earth's shadow. Hence, if the opposition happen as in position I, it is manifest that the moon will just pass by the shadow of the earth without entering it, and there will be no eclipse. In position II, part of the moon will pass through the earth's shadow, and there will be a *partial* eclipse. In position III, the whole of the

moon passes through the earth's shadow, and there is a *total* eclipse. In position IV, the center of the moon passes through the center of the earth's shadow, and there is a *total* and *central* eclipse. It is plain, therefore, that whether there will, or will not, be an eclipse at the time of opposition, depends upon the distance of the moon from the node at that time; or the distance of the earth's shadow, or of the earth, from the node. Now in lunar eclipses we may take the angle at $N=5°. 17'$, and in position I, the value of $Ev$ is about $1°. 3'. 30''$; hence (Trig. Art. 221), sin. $5°. 17'$ : rad. :: sin. $1°. 3'. 30''$ : sin. $EN=11°. 34'$; when, therefore, $EN$ is greater than that quantity at the time of opposition, there can be no eclipse. This quantity $11°. 34'$ is called the *ecliptic limit*.

(307.) Let $Arb$ be that half of the earth's shadow which the moon passes through, $NL$ the relative orbit




of the moon; draw $Cmr$ perpendicular to $NL$, and let $z$ be the center of the moon at the beginning of the eclipse, $m$ at the middle, $x$ at the end; also, let $AB$ be the ecliptic, and $Cn$ perpendicular to it. Now in the right-angled triangle $Cnm$, we know $Cn$ the latitude of the moon at the time of the ecliptic conjunction, and (302) the angle $Cnm$ * the complement of the angle which the relative orbit of the moon makes with the ecliptic; hence (Trig. Art. 125) rad. : cos.

* If the moon at $n$ have north or south latitude increasing, the angle $Cnm$ is to be set off to the right; otherwise, to the left of $Cn$.

*Cnm* :: *Cn* : *nm*, which is called the ***Reduction***; and rad. : sin. *Cnm* :: *Cn* : *Cm*. The horary motion ($h$) of the moon upon it's relative orbit being known, we know the time of describing *mn*, by saying, $h : mn ::$ 1 *hour* : *the time of describing mn*. Hence, knowing the time of the ecliptic conjunction at *n*, we know the time of the middle of the eclipse at *m*. Next, in the right-angled triangle *Cmz*, we know *Cm*, and *Cz* the sum of the semidiameters of the earth's shadow and the moon, to find *mz*, which is done thus by logarithms; as $mz = \sqrt{Cz^2 - Cm^2} = \sqrt{\overline{Cz + Cm} \times \overline{Cz - Cm}}$, the *log. of* $mz = \frac{1}{2} \times log.\ \overline{Cz + Cm} + log.\ \overline{Cz - Cm}$, (Trig. Art. 52). Hence, the horary motion of the moon being known, we know the time of describing *zm*, which *subtracted* from the time at *m* gives the time of the beginning, and *added*, gives the time of the end. The magnitude of the eclipse at the middle is represented by *tr*, which is the greatest distance of the moon within the earth's shadow, and this is measured in terms of the diameter of the moon, conceived to be divided into 12 equal parts, called *Digits*, or *Parts deficient*; to find which, we know *Cm*, the difference between which and *Cr* gives *mr*, which added to *mt*, or if *m* fall *out* of the shadow, take the difference between *mr* and *mt*, and we get *tr*; hence, to find the number of digits eclipsed, say, *mt* : *tr* :: 6 *digits, or* 360′, (it being usual to divide a digit into 60 equal parts, and call them minutes,) : *the digits eclipsed*. If the latitude of the moon be north, we use the *upper* semicircle; if south, we take the *lower*.

(308.) If the earth had no atmosphere, when the moon was totally eclipsed, it would be invisible; but, by the refraction of the atmosphere, some rays will be brought to fall on the moon's surface, upon which account the moon will be visible at that time, and appear of a dusky red colour. M. *Maraldi* (*Mem. de l'Acad.* 1723) has observed, that, in general, the earth's umbra, at a certain distance, is divided by a kind of penumbra,

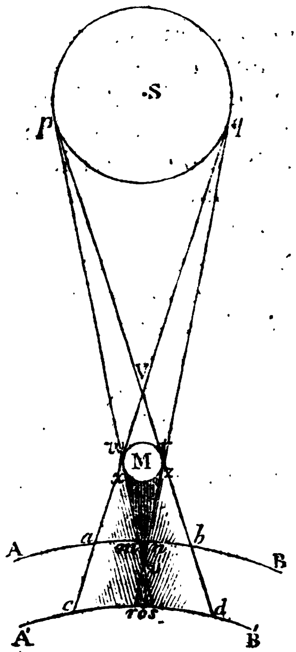
from the refraction of the atmosphere. This will account for the circumstance of the moon being more visible in some total eclipses than in others. It is said, that the moon, in the total eclipses in 1601, 1620, and 1642, entirely disappeared.

(309.) An eclipse of the moon, arising from it's real deprivation of light, must appear to begin at the same instant of time to every place on that hemisphere of the earth which is turned towards the moon. Hence, it affords a very ready method of finding the difference of longitudes of places upon the earth, as will be afterwards explained. The moon enters the penumbra of the earth before it comes to the umbra, and therefore it gradually loses it's light; and the penumbra is so dark just at the umbra, that it is difficult to ascertain the exact time when the moon's limb touches the umbra, or when the eclipse begins. When the moon has entered into the umbra, the shadow upon it's disc is tolerably well defined, and you may determine, to a considerable degree of accuracy, the time when any spot enters into the umbra. Hence, the beginning and end of a lunar eclipse are not so proper to determine the longitude from, as the times at which the umbra touches any of the spots.

## *On Eclipses of the Sun.*

(310.) An eclipse of the sun is caused by the interposition of the moon between the sun and spectator, or by the shadow of the moon falling on the earth at the place of the observer. The different kinds of eclipses will be best explained by a figure. Let $S$ be the sun, $M$ the moon, $AB$ or $A'B'$ the surface of the earth; draw tangents $pxvs$, $qzvr$, from the sun to the *same* side of the moon, and $xvz$ will be the moon's *umbra*, in which no part of the sun can be seen; if tangents $ptbd$, $qwac$, be drawn from the sun to the *opposite* sides of the moon, the space comprehended between the *umbra* and $wac$, $tbd$, is called the

*penumbra*, in which part of the sun only is seen. Now it is manifest, that if *AB* be the surface of the




earth, the space *mn*, where the umbra falls, will suffer a *total* eclipse; the part *am*, *bn*, between the boundaries of the umbra and penumbra, will suffer a *partial* eclipse; but to all the other parts of the earth there will be no eclipse. Now let *A' B'* be the surface of the earth, the earth being, at different times, at different distances from the moon; then the space within *rs* will suffer an *annular* eclipse; for if tangents be drawn from any point *o* within *rs* to the moon, they must evidently fall within the sun, therefore the sun will appear all round about the moon in the form of a ring; the parts *cr*, *sd*, will suffer a *partial* eclipse; and the other parts of the earth will suffer no eclipse. In this case, there can be no total eclipse any where, as the moon's umbra does not reach the earth. Ac-

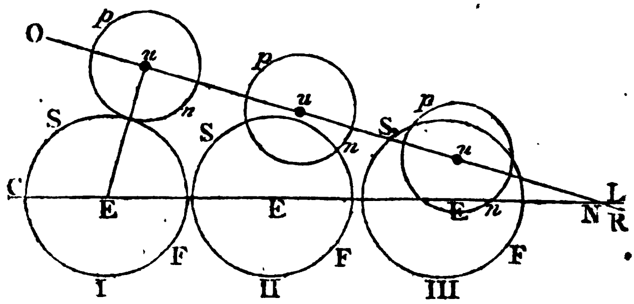
cording to M. du *Sejour*, an eclipse can never be annular longer than 12′. 24″, nor total longer than 7′. 58″.

(311.) The umbra *xvz* is a cone, and the penumbra *wcdt* the frustrum of a cone whose vertex is *V*. Hence, if these be both cut through their common axis perpendicular to it, the section of each will be a circle, having a common center in the line joining the centers of the sun and moon, and the penumbra includes the umbra.

(312.) The moon's mean motion about the center of the earth is at the rate of about 33′ in an hour; but 33′ of the moon's orbit is about 2280 miles, which, therefore, we may consider as the velocity with which the moon's shadow passes over the earth; but this is the velocity upon the surface of the earth where the shadow falls perpendicularly upon it, it being the velocity perpendicular to *Mv*; in every other place, the velocity over the surface will be increased in the proportion of the sine of the angle which *Mv* makes with the surface, in the direction of it's motion, to radius. But the earth having a rotation about it's axis, the *relative* velocity of the moon's shadow over any given point of the surface will be different from this; if the point be moving in the direction of the shadow, the velocity of the shadow, in respect to that point, will be diminished, and consequently the time in which the shadow passes over it will be increased; but if the point be moving in a direction contrary to that of the shadow, as is the case when the shadow falls on the other side of the pole, the time will be diminished. The length of a solar eclipse is therefore affected by the earth's rotation about it's axis.

(313.) The different eclipses of the sun may be thus explained. Let *CL* represent the orbit of the earth, *OR* the line described by the centers of the moon's umbra and penumbra at the earth; *N* the moon's node; *SF* the earth, *E* it's center; *pn* the

moon's penumbra, $u$ the umbra. Then, in position I, the penumbra $pn$ just passes by the earth, without




falling upon it, and therefore there will be no eclipse. In position II, the penumbra $pn$ falls upon the earth, but the umbra $u$ does not; therefore there will be a *partial* eclipse where the penumbra falls, but no total eclipse. In position III, both the penumbra $pn$ and umbra $u$ fall upon the earth; therefore, where the penumbra falls, there will be a *partial* eclipse, and where the umbra falls there will be a *total* eclipse; and to the other parts of the earth there will be no eclipse. Now the *ecliptic limit*, may be thus found. The angle $N$ may be taken $5°. 17'$, and in position I, the value of $Eu$ ($u$ being the center of the umbra) is about $1°. 34'. 27''$; hence (Trig. Art. 221) sin. $5°. 17'$ : rad. :: sin. $1°. 34'. 27''$ : sin. $EN = 17°. 21'. 27''$ the ecliptic limit; if therefore, at the time of conjunction, the earth be within this distance of the node, there will be an eclipse.

(314.) An eclipse of the sun, or rather of the earth, without respect to any particular place, may be calculated exactly in the same manner as an eclipse of the moon, that is, the times when the moon's umbra or penumbra first touches and leaves the earth; but to find the times of the beginning, middle, and end, at any particular place, the *apparent* place of the moon, as seen from thence, must be determined, and conse-

quently it's parallax in latitude and longitude must be computed, which renders the calculation of a solar eclipse extremely long and tedious.

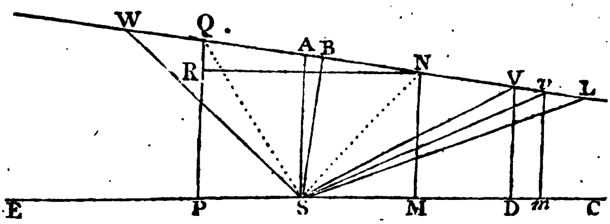
### *To explain the Principles of the Calculation of an Eclipse of the Sun for any particular Place.*

(315.) Having determined (113) that there will be an eclipse somewhere upon the earth, compute, by the Astronomical Tables, the true longitudes of the sun and moon, and the moon's true latitude, at the time of mean conjunction (301); find also the horary motions of the sun and moon in longitude, and the moon's horary motion in latitude; and compute the time of the ecliptic conjunction of the sun and moon, in the same manner (300) as the time of the ecliptic opposition was computed. At the time of the ecliptic conjunction, compute (301) the sun's and moon's longitude, and the moon's latitude; find also the horizontal parallax of the moon from the Tables of the moon's motion, from which subtract the sun's horizontal parallax, and you get the horizontal parallax of the moon from the sun.

(316.) To the latitude of the given place, and the horizontal parallax of the moon from the sun (which we here use instead of the horizontal parallax of the moon, as we want to find what effect the parallax has in altering their apparent relative situations,) at the time of the ecliptic conjunction, compute (144) the moon's parallax in latitude and longitude from the sun; the parallax in latitude applied to the true latitude gives the apparent latitude (*L*) of the moon from the sun; and the parallax in longitude shows the apparent difference (*D*) of the longitudes of the sun and moon.

(317.) Let *S* be the sun, *CE* the ecliptic, according

to the order of the signs; take $SM = D$, draw $MN$ perpendicular to $MS$, and take it $= L$, then $N$ is the




apparent place of the moon, and $SN = \sqrt{D^2 + L^2}$ is the apparent distance of the moon from the sun.

(318.) If the moon be to the *east* of the nonagesimal degree, the parallax *increases* the longitude; if to the *west*, it *diminishes* it (Art. 144); hence, if the *true* longitudes of the sun and moon be equal, in the former case the apparent place will be from $S$ towards $E$, and in the latter, towards $C$. To some time, as an hour, *after* the true conjunction, if the moon be to the *west* of the nonagesimal degree; or *before* the true conjunction, if the moon be to the *east* of the nonagesimal degree, find the sun's and the moon's true longitude, and the moon's true latitude, from their horary motions; and to the same time compute the moon's parallax in latitude and longitude from the sun; apply the parallax in latitude to the true latitude, and it gives the apparent latitude ($l$) of the moon from the sun; take the difference of the sun's and moon's true longitude, and apply the parallax in longitude, and it gives the apparent distance ($d$) of the moon from the sun in longitude. From $S$ set off $SP = d$, and on $EC$ erect the perpendicular $PQ$ equal to $l$, and $Q$ is the apparent place of the moon at one hour from the true conjunction; and $SQ$ $(= \sqrt{d^2 + l^2})$ is the apparent distance of the moon from the sun; draw the straight line $NQ$, and it will represent the relative *apparent* path of the moon, considered as a straight line, in general it being very nearly so; it's value also

represents the relative horary motion of the moon in the apparent orbit, the relative horary motion in longitude being *MP*.

(319.) The difference between the moon's apparent distance in longitude from the sun at the time of the true ecliptic conjunction, and at the interval of an hour, gives the apparent horary motion ($r$) in longitude of the moon from the sun; the difference ($D$) between the true longitude at the ecliptic conjunction, and the moon's apparent longitude, is the apparent distance of the moon from the sun in longitude at the true time of the ecliptic conjunction; hence, $r : D ::$ 1 hour : the time from the true to the apparent conjunction, consequently we know the time of the apparent conjunction. To find whether this time is accurate, we may compute (from the horary motions of the sun and moon) their true longitudes, and the moon's parallax in longitude from the sun, and apply it to the true longitude, and it gives the apparent longitude, and if this be the same as the sun's longitude, the time of the apparent conjunction is truly found; if they be not the same, find from thence the true time, as before. To the true time of the apparent conjunction, find the moon's true latitude from it's horary motion, and compute the parallax in latitude, and you get the apparent latitude at the time of the apparent conjunction. Draw *SA* perpendicular to *CE*, and equal to this apparent latitude; then the point *A* will not probably fall in *NQ*; but suppose it to fall in *QN*, to which draw *SB* perpendicular, and *NR* parallel to *PM*. Then knowing *NR* $(=PM)$, and *QR* $(=QP \sim MN)$ we have

$NR : RQ ::$ rad. : tan. $QNR$, or $ASB$ (Trig. Art. 123)
Sin. $QNR$ : rad. $:: QR : QN$ (Trig. Art. 128)

The time of describing *NQ* in the apparent orbit being equal to the time from *M* to *P* in longitude, *NQ* is the horary motion in the apparent orbit.

Rad. : sin. $ASB$ :: $AS$ : $AB$ (Trig. Art. 125)
Rad. : cos. $ASB$ :: $AS$ : $SB$.

(320.) At the apparent conjunction the moon appears at $A$, which time (319) is known; when the moon appears at $B$, it is at it's nearest distance from the sun, and consequently the time is that of the greatest obscuration, (usually called the time of the middle,) provided there is an eclipse, which will always be the case, when $SB$ is less than the *sum* of the apparent semidiameters of the sun and moon. If, therefore, it appear that there will be an eclipse, we proceed thus to find it's quantity, and the beginning and end. As we may suppose the motion to be uniform, $QN$ : $AB$ :: the time of describing $NQ$ : the time of describing $AB$, which added to or subtracted from the time at $A$, (according as the apparent latitude is decreasing or increasing), gives the time of the greatest obscuration.

(321.) From the sum of the apparent semidiameters of the sun and moon, subtract $BS$, and the remainder shows how much of the sun is covered by the moon; or the parts deficient; hence, semid. ⊙ : parts deficient :: 6 digits : the digits eclipsed. If $SB$ be less than the *difference* of the semidiameters of the sun and moon, and the moon's semidiameter be the *greater*, the eclipse will be *total*; but if it be the *less*, the eclipse will be *annular*, the sun appearing all round the moon; if $B$ and $S$ coincide, the eclipse will be *central*.

(322.) Produce, if necessary, $QN$, and take $SV$, $SW$, equal to the sum of the apparent semidiameters of the sun and moon, at the beginning and end respectively; then $BV=\sqrt{SV^2-SB^2}$, and $BW=\sqrt{SW^2-SB^2}$; and to find the times of describing these, say, as the hourly motion of the moon in the apparent orbit, or $NQ$, : $BV$ :: 1 hour : the time of describing $VB$; and $NQ$ : $BW$ :: 1 hour : the time of describing $BW$,

which times, respectively subtracted from and added to the time of the greatest obscuration, give nearly the times of the beginning and end. But if accuracy be required, a different method must be adopted; for we suppose $VW$ to be a straight line, which supposition will, in general, cause errors, too considerable to be neglected. It may, however, always serve as a rule to assume the time of the beginning and end. Hence it follows, that the time of the greatest obscuration at $B$, is not necessarily equidistant from the beginning and end.

(323.) If the eclipse be total, take $SV$, $SW$, equal to the difference of the semidiameters of the sun and moon, and then $BV = BW = \sqrt{SW^2 - SB^2}$, from whence we may find the times of describing $BV$, $BW$, as before, which we may consider as equal, and which applied to the time of the greatest obscuration at $B$, give the time of the beginning and end of the total darkness.

(324.) To find more accurately the time of the beginning and end of the eclipse, we must proceed thus. At the estimated time of the beginning, find, from the horary motions, and the computed parallaxes, the apparent latitude $VD$ of the moon, and it's apparent longitude $DS$ from the sun, and we have $SV = \sqrt{SD^2 + DV^2}$, and if this be equal to the apparent semid. ☽ + semid. ☉ (which sum call $S$), the estimated time is the time of the beginning; but if $SV$ be not equal to $S$, assume (as the error directs) another time at a small interval from it, *before*, if $SV$ be *less* than $S$, but *after*, if it be *greater*; to that time compute again the moon's apparent latitude $mv$, and apparent longitude $Sm$ from the sun, and find $Sv = \sqrt{Sm^2 + mv^2}$; and if this be not equal to $S$, proceed thus; as the difference of $Sv$ and $SV$ : the difference of $Sv$ and $SL$ $(= S)$ :: the above-assumed interval of time, or time of the motion through $Vv$, : the time through $vL$,

which added to or subtracted from the time at *v*, according as *Sv* is greater or less than *SL*, gives the time of the beginning. The reason of this operation is, that as *Vv*, *vL*, are very small, they will be very nearly proportional to the differences of *SV*, *Sv*, and *Sv*, *SL*. But as the variation of the apparent distance of the sun from the moon is not exactly in proportion to the variation of the differences of the apparent longitudes and latitudes, in cases where the utmost accuracy is required, the time of the beginning thus found (if it appear to be not correct) may be corrected, by assuming it for a third time, and proceeding as before. This correction, however, will never be necessary, except where extreme accuracy is required in order to deduce some consequences from it. But the time thus found is to be considered as accurate, only so far as the Tables of the sun and moon can be depended upon for their accuracy; and the best lunar Tables are subject to an error of $10''$ in latitude. Hence, accurate observations of an eclipse, compared with the computed time, furnish the means of correcting the lunar Tables. In the same manner, the end of the eclipse may be computed.

(325.) As there are not many persons who have an opportunity of seeing a total eclipse of the sun, we shall here give the phænomena which attended that on April 22, 1715. Capt. *Stannyan*, at Bern in Switzerland, says, "the sun was totally dark for four minutes and a half; that a fixed star and planet appeared very bright; and that it's getting out of the eclipse was preceded by a blood-red streak of light, from it's left limb, which continued not longer than six or seven seconds of time; then part of the sun's disc appeared, all on a sudden, as bright as *Venus* was ever seen in the night; nay, brighter, and in that very instant gave a light and shadow to things, as strong as moon-light used to do." The inference drawn from these phænomena is, that the moon has an atmosphere.

*J. C. Facis*, at Geneva, says, "there was seen,

during the whole time of the total immersion, a whiteness, which seemed to break out from behind the moon, and to encompass it on all sides equally; it's breadth was not the twelfth part of the moon's diameter. *Venus*, *Saturn*, and *Mercury* were seen by many; and if the sky had been clear, many more stars might have been seen, and with them *Jupiter* and *Mars*. Some gentlewomen in the country saw more than 16 stars; and many people on the mountains saw the sky starry, in some places where it was not overcast, as during the night at the time of the full moon. The duration of the total darkness was three minutes."

Dr. *J. J. Scheuchzer*, at Zurich, says, "that both planets and fixed stars were seen; the birds went to roost; the bats came out of their holes; and the fishes swam about; we experienced a manifest sense of cold; and the dew fell upon the grass. The total darkness lasted four minutes."

(326.) Dr. *Halley**, who observed this eclipse at London, has thus given the phænomena attending it. "It was *universally* observed, that when the last part of the sun remained on it's east side, it grew very faint, and was easily supportable to the naked eye, even through the telescope, for above a minute of time before the total darkness; whereas, on the contrary, my eye could not endure the splendour of the emerging beams in the telescope from the first moment. To this, per-

* The Doctor begins his account thus. "Though it be certain, from the principles of Astronomy, that there happens necessarily a central eclipse of the sun, in some part or other of the terraqueous globe, about twenty-eight times in each period of eighteen years; and that of these, no less than eight do pass over the parallel of London, three of which eight are total with continuance; yet, from the great variety of the elements, whereof the *calculus* of eclipses consists, it has so happened, that since March 20, 1140, I cannot find that there has been a total eclipse of the sun seen at London, though in the mean time the shade of the moon has often passed over other parts of Great Britain."

haps, two causes concurred; the one that the pupil of the eye did necessarily dilate itself during the darkness, which before had been much contracted by looking on the sun. The other, that the eastern parts of the moon, having been heated with a day near as long as thirty of our's, must of necessity have that part of it's atmosphere replete with vapours, raised by the long-continued action of the sun; and, by consequence, it was more dense near the moon's surface, and more capable of obstructing the lustre of the sun's beams. Whereas at the same time the western edge of the moon had suffered as long a night, during which time there might fall in dews, all the vapours that were raised in the preceding long day; and for this reason, that part of it's atmosphere might be seen much more pure and transparent.

About two minutes before the total immersion, the remaining part of the sun was reduced to a very fine horn, whose extremities seemed to lose their accuteness, and to become round like stars. And for the space of about a quarter of a minute, a small piece of the southern horn of the eclipse seemed to be cut off from the rest by a good interval, and appeared like an oblong star round at both ends; which appearance could proceed from no other cause, but the inequalities of the moon's surface, there being some elevated parts thereof near the moon's southern pole, by which interposition, part of that exceedingly fine filament of light was intercepted.

A few seconds before the sun was totally hid, there discovered itself round the moon a luminous ring, about a digit, or perhaps a tenth part, of the moon's diameter in breadth. It was of a pale whiteness, or rather pearl colour, seeming to me a little tinged with the colours of the *iris*, and to be concentric with the moon; whence I concluded it was the moon's atmosphere. But the great height of it, far exceeding that of our earth's atmosphere; and the observations of some one who found the breadth of the ring to increase

on the west side of the moon, as the emersion approached; together with the contrary sentiments of those, whose judgement I shall always revere, make me less confident, especially as in a matter whereto I gave not all the attention requisite.

Whatever it was, this ring appeared much brighter and whiter near the body of the moon, than at a distance from it; and it's outward circumference, which was ill defined, seemed terminated only by the extreme rarity of the matter it was composed of; and in all respects resembled the appearance of an enlightened atmosphere viewed from far: but whether it belonged to the sun or the moon, I shall not at present undertake to decide.

During the whole time of the total eclipse, I kept my telescope constantly fixed on the moon, in order to observe what might occur in this uncommon appearance, and I saw perpetual flashes or coruscations of light, which seemed for a moment to dart out from behind the moon, now here, now there, on all sides, but more especially on the western side, a little before the emersion; and about two or three seconds before it, on the same western side, where the sun was just coming out, a long and very narrow streak of dusky, but strong red light, seemed to colour the dark edge of the moon, though nothing like it had been seen immediately after the immersion. But this instantly vanished upon the first appearance of the sun, as did also the aforesaid luminous ring.

As to the degree of darkness, it was such, that one might have expected to have seen more stars than were seen in London; the planets *Jupiter*, *Mercury*, and *Venus*, were all that were seen by the gentlemen of the Society from the top of their house, where they had a free horizon; and I do not hear that any one in town saw more than *Capella* and *Aldebaran* of the fixed stars. Nor was the light of the ring round the moon capable of effacing the luster of the stars, for it was vastly inferior to that of the full moon, and so

weak, that I did not observe it cast a shade. But the under-parts of the hemisphere, particularly in the south-east under the sun, had a crepuscular brightness; and all round us, so much of the segment of our atmosphere as was above the horizon, and was without the cone of the moon's shadow, was more or less enlightened by the sun's beams; and it's reflection gave a diffused light, which made the air seem hazy, and hindered the appearance of the stars. And that this was the real cause thereof, is manifest by the darkness being more perfect in those places near which the center of the shade passed, where many more stars were seen, and in some, not less than twenty, though the light of the ring was to all alike.

I forbear to mention the chill and damp, with which the darkness of this eclipse was attended, of which most spectators were sensible, and equally judges; or the concern that appeared in all sorts of animals, birds, beasts, and fishes, upon the extinction of the sun, since ourselves could not behold it without some sense of horror."

(327.) If a conjunction of the sun and moon happen at, or very near, the node, there will be a great solar eclipse; but, in this case, at the preceding opposition, the earth was not got into the lunar ecliptic limits, and at the next opposition it will be got beyond it; hence, at each node there may happen only one solar eclipse, and therefore in a year there *may* happen only two solar eclipses.

There must be one conjunction in the time in which the earth passes through the solar ecliptic limits, and consequently there must be one solar eclipse at each node; hence, there *must* be two solar eclipses at least in a year.

If an opposition happen just before the earth gets into the lunar ecliptic limit, the next opposition may not happen till the earth is got beyond the limit on the other side of the node; consequently there may not be a lunar eclipse at the node; hence, there *may*

*not* be an eclipse of the moon in the course of a year. When, therefore, there are only two eclipses in a year, they must be both of the sun.

If there be an eclipse of the moon as soon as the sun gets within the lunar ecliptic limit, it will be got out of the limit before the next opposition; consequently there can be only one lunar eclipse at the same node. But as the nodes of the moon's orbit move backwards about 19° in a year, the earth may come within the lunar ecliptic limits, at the *same* node, a second time in the course of a year, and therefore there *may* be three lunar eclipses in a year; and there can be no more.

If an eclipse of the moon happen at, or very near to, the node, a conjunction may happen before and after, whilst the earth is within the solar ecliptic limits; hence there may, at each node, happen two eclipses of the sun and one of the moon; and in this case, the eclipses of the sun will be small, and that of the moon large. When, therefore, the eclipses do not happen a second time at either node, there may be six eclipses in a year, four of which will be of the sun, and two of the moon. But if, as in the last case, an eclipse should happen at the return of the earth within the lunar ecliptic limits at the *same* node a second time in the year, there may be six eclipses, three of the sun and three of the moon.

There may be seven eclipses in a year. For twelve lunations are performed in 354 days, or in 11 days less than a common year. If, therefore, an eclipse of the sun should happen before January 11, and there be at that, and at the next node, two solar and one lunar eclipse at each; then the twelfth lunation from the first eclipse will give a new moon within the year, and (on account of the retrograde motion of the moon's nodes) the earth may be got within the solar ecliptic limits, and there may be another solar eclipse. Hence, when there are seven eclipses in a year, five will be of the sun and two of the moon. This is upon sup-

position that the first eclipse is of the sun; but if the first eclipse should be of the moon, there may be three of the sun and four of the moon.

As there are seven eclipses in the year but seldom, the mean number will be about four.

The nodes of the moon move backwards about 19° in a year, which arc the earth describes in about 19 days, consequently the middle of the seasons of the eclipses happens every year about 19 days sooner than in the preceding year.

The ecliptic limits of the sun (313) are greater than those of the moon (306), and hence, there will be more solar than lunar eclipses, in about the same proportion as the limit is greater, that is, as 3 : 2 nearly. But more lunar than solar eclipses are seen at any given place, because a lunar eclipse is visible to a whole hemisphere at once; whereas a solar eclipse is visible only to a part, and therefore there is a greater probability of seeing a lunar than a solar eclipse. Since the moon is as long above the horizon as below, every spectator may expect to see half the number of lunar eclipses which happen.

For the calculation of eclipses, and all the circumstances respecting them, see my *Complete System of Astronomy*.

# Chap. XXII.

## ON THE TRANSITS OF MERCURY AND VENUS OVER THE SUN'S DISC.

(328.) When Dr. *Halley* was at St. Helena, whither he went for the purpose of making a catalogue of the stars in the southern hemisphere, he observed a transit of *Mercury* over the sun's disc; and, by means of a good telescope, it appeared to him that he could determine the time of the ingress and egress, without it's being subject to an error of 1″*; upon which he immediately concluded, that the sun's parallax might be determined by such observations, from the difference of the times of the transit over the sun, at different places upon the earth's surface. But this difference is so small in Mercury, that it would render the conclusion subject to a great degree of inaccuracy; in Venus, however, whose parallax is nearly four times as great as that of the sun, there will be a very considerable difference between the times of the transits seen from different parts of the earth, by which the accuracy of the conclusion will be proportionably increased. The Doctor, therefore, proposed to determine the sun's parallax from the transit of Venus over the sun's disc, observed at different places on the earth; and as it was not probable that he himself should live to observe the

* Hence, Dr. *Halley* concluded, that by a transit of *Venus*, the sun's distance might be determined with certainty to the 500th part of the whole; but the observations upon the transits which happened in 1761 and 1769, showed that the time of contact of the limbs of the Sun and Venus could not be determined to that degree of certainty.

next transits, which happened in 1761 and 1769, he very earnestly recommended the attention of them to the Astronomers who should be alive at that time. Astronomers were therefore sent from England and France to the most proper parts of the earth to observe both those transits, from the result of which, the parallax has been determined to a very great degree of accuracy.

(329.) *Kepler* was the first person who predicted the transits of Venus and Mercury over the sun's disc; he foretold the transit of Mercury in 1631, and the transits of Venus in 1631 and 1761. The first time *Venus* was ever seen upon the sun, was in the year 1639, on November 24, at Hoole, near Liverpool, by our countryman Mr. *Horrox*, who was educated at Emanuel College in this University. He was employed in calculating an Ephemeris from the *Lansberg* Tables, which gave, at the conjunction of Venus with the sun on that day, it's apparent latitude less than the semidiameter of the sun. But as these Tables had so often deceived him, he consulted the Tables constructed by *Kepler*, according to which, the conjunction would be at 8*h*. 1′ A. M. at Manchester, and the planet's latitude 14′. 10″ south; but, from his own corrections, he expected it to happen at 3*h*. 57′ P. M. with 10′ south latitude. He accordingly gave this information to his friend Mr. *Crabtree*, at Manchester, desiring him to observe it; and he himself also prepared to make observations upon it, by transmitting the sun's image through a telescope into a dark chamber. He described a circle of about six inches diameter, and divided the circumference into 360°, and the diameter into 120 equal parts, and caused the sun's image to fill up the circle. He began to observe on the 23d, and repeated his observations on the 24th till one o'clock, when he was unfortunately called away by business; but, returning at 15′ after three o'clock, he had the satisfaction of seeing Venus upon the sun's disc, just wholly entered on the left side, so that the

limbs perfectly coincided. At 35′ after three, he found the distance of Venus from the sun's center to be 13′. 30″; and at 45′ after three, he found it to be 13′; and the sun setting at 50′ after three o'clock, put an end to his observations. From these observations, Mr. *Horrox* endeavoured to correct some of the elements of the orbit of Venus. He found Venus had entered upon the disc at about 62°. 30′ from the vertex towards the right on the image, which, by the telescope, was inverted. He measured the diameter of Venus, and found it to be to that of the sun, as 1,12 : 30, as near as he could measure. Mr. *Crabtree*, on account of the clouds, got only one sight of Venus, which was at 3*h*. 45′. Mr. *Horrox* * wrote a Treatise, entitled *Venus in Sole visa*, but did not live to publish it; it was, however, afterwards published by *Hevelius*. *Gassendus* observed the transit of *Mercury* which happened on November 7, 1631, and this was the first which had ever been observed; he made his observations in the same manner that *Horrox* did after him. Since his time, several transits of Mercury have been observed, as they frequently happen; whereas only two transits of Venus have happened since the time of *Horrox*. If we know the time of the transit at one node, we can determine, in the following manner, when they will probably happen again at the same node.

(330.) The mean time from conjunction to conjunction of Venus or Mercury being known (Art. 201), and the time of one mean conjunction, we shall know the time of all the future mean conjunctions; observe, therefore, those which happen near to the node, and compute the geocentric latitude of the planet at the time of conjunction, and if it be less than the appa-

* The difficulties which this very extraordinary person had to encounter with in his astronomical pursuits, he himself has described, in the *Prolegomena* prefixed to his *Opera Posthuma*, published by Dr. *Wallis*.

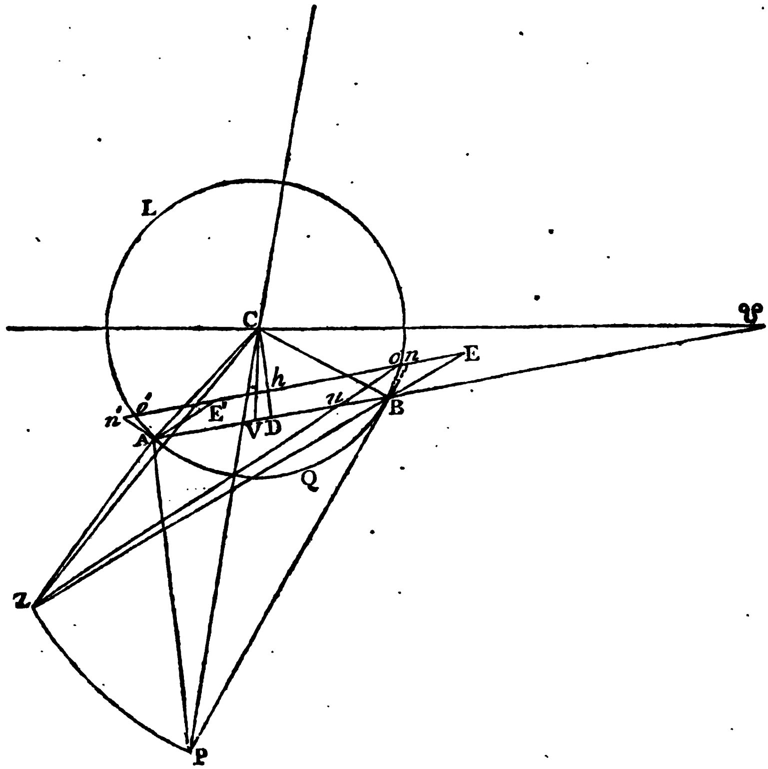
rent semidiameter of the sun, there will be a transit of the planet over the sun's disc; and we may determine the periods when such conjunctions happen, in the following manner. Let $P$ = the periodic time of the earth, $p$ that of Venus or Mercury. Now that a transit may happen again at the same node, the earth must perform a certain number of complete revolutions in the same time that the planet performs a certain number, for then they must come into conjunction again at the same point of the earth's orbit, or nearly in the same position in respect to the node. Let the earth perform $x$ revolutions whilst the planet performs $y$ revolutions; then will $Px = py$, therefore $\frac{x}{y} = \frac{p}{P}$. Now $P = 365,256$, and for *Mercury*, $p = 87,968$; therefore $\frac{x}{y} = \frac{p}{P} = \frac{87,968}{365,256}$ = (by resolving it into it's continued fractions) $\frac{1}{4}, \frac{6}{25}, \frac{7}{29}, \frac{13}{54}, \frac{33}{137}, \frac{46}{191}$, &c. That is, 1, 6, 7, 13, 33, 46, &c. revolutions of the earth are nearly equal to 4, 25, 29, 54, 137, 191, &c. revolutions of Mercury, approaching nearer to a state of equality, the further you go. The first period, or that of one year, is not sufficiently exact; the period of six years will sometimes bring on a return of the transit at the same node; that of seven years more frequently; that of 13 years still more frequently, and so on. Now there was a transit of Mercury at its descending node, in May, 1786; hence, by continually adding 6, 7, 13, 33, 46, &c. to it you get all the years when a transit may be expected to happen at that node. In 1789, there was a transit at the ascending node, and therefore, by adding the same numbers to that year, you will get the years in which the transits may be expected to happen at that node. The next transits at the descending node will happen in 1832, 1845, 1878, 1891; and at the ascending node, in 1815, 1822, 1835, 1848, 1861, 1868, 1881, 1894.

For *Venus*, $p = 224{,}7$; hence, $\frac{x}{y} = \frac{p}{P} = \frac{224{,}7}{365{,}256}$ $= \frac{8}{13}, \frac{235}{382}, \frac{713}{1159}$, &c. Therefore the periods are 8, 235, 713, &c. years. The transits at the same node will therefore, sometimes, return in eight years, but oftener in 235, and still oftener in 713, &c. Now, in 1769, a transit happened at the descending node in June, and the next transits at the same node will be in 2004, 2012, 2247, 2255, 2490, 2498, 2733, 2741; and 2984. In 1639, a transit happened at the ascending node in November; and the next transits at the same node will be in 1874, 1882, 2117, 2125, 2360, 2368, 2603, 2611, 2846, and 2854. These transits are found to happen, by continually adding the periods, and finding the years when they may be expected, and then computing, for each time, the shortest geocentric distance of Venus from the sun's center at the time of conjunction, and if it be less than the semidiameter of the sun, there will be a transit.

### *A new Method of computing the Effect of Parallax, in accelerating or retarding the Time of the Beginning or End of a Transit of* Venus *or* Mercury *over the Sun's Disc. By* Nevil Maskelyne, *D. D. F.R.S. and Astronomer Royal.*

(331.) The scheme which is here given, relates particularly to the transit of *Venus* over the sun which happened in 1769. Let *C* represent the center of the sun *LQ*, *P* the celestial north pole of the equator, *PC* a meridian passing through the sun, *Z* the zenith of the place, *ADB* ☋ the relative path of Venus, ☋ being the relative place of the descending node; *A* the geocentric place of Venus at the ingress, *B* at the egress, and *D* at the nearest approach to the sun's center, as seen from the earth's center, and *o* the apparent place of Venus at the egress to an observer whose

zenith is $Z$; draw $ouZ$, and $u$ is the *true* place of Venus when the *apparent* place is at $o$, and $no$ is the




parallax in altitude of Venus from the sun; and the time of contact will be diminished by the time which Venus takes to describe $uB$; draw $n'o'honE$ parallel to $AB$, meeting $ZB$ produced in $E$, and $Bn$, $An'$, tangents to the circle, and let $ChD$ be perpendicular to $AB$. Now the trapezium $uoEB$, on account of the smallness of it's sides, may be considered as rectilinear, and from the magnitude of $ZB$ compared with $Bu$, $BE$ may be considered as parallel to $uo$, consequently $uoEB$ may be considered as a parallelogram, and therefore $Eo$ may be taken equal to $Bu$. Now $Eo = En \pm no$, according as $E$ falls without or within

the circle $LQ$ of the sun's disc; and (Trig. Art. 128) $En : EB :: \sin. EBn = \cos. CBZ : \sin. BnE = \sin. BCD = \cos. CBD$; hence, $En = \frac{EB \times \cos. CBZ}{\cos. CBD}$; and (by Euclid) $no = \frac{Bn^2}{no'} = \frac{Bn^2}{AB}$ very nearly; but $Bn : BE :: \sin. BEn = \sin. ZBD : \sin. BnE = \cos. CBD$; therefore $Bn^2 = \frac{BE^2 \times \sin. ZBD^2}{\cos. CBD^2}$; hence, $no = \frac{BE^2 \times \sin. ZBD^2}{AB \times \cos. CBD^2}$. Put $h =$ horizontal parallax of Venus from the sun; then (136) $BE = h \times \sin. Zo = h \times \sin. ZB$; hence, $uB = oE = En \pm no = \frac{h \times \sin. ZB \times \cos. CBZ}{\cos. CBD} \pm h^2 \times \sin. ZB^2 \times \frac{\sin. ZBD^2}{AB \times \cos. CBD^2}$ $=$ (Trig. Art. 80) $h \times \sin. ZB \times \cos. CBZ \times \sec. CBD \pm \frac{h^2 \times \sin. ZB^2 \times \sin. ZBD^2 \times \sec. CBD^2}{AB}$. The parallax, therefore, consists of two parts; one part varies as $h$, and the other as $h^2$, the other quantities being the same. Put $t'' =$ the time which Venus takes, by it's geocentric relative motion, to describe the space $h$; to find which, let $m$ be the relative horary motion of Venus; then $m : h :: 1$ hour $= 3600'' : t'' = \frac{h \times 3600''}{m}$. Hence, to find the time of describing $uB$, we have, $h : h \times \sin. ZB \times \cos. CBZ \times \sec. CBD \pm \frac{h^2 \times \sin. ZB^2 \times \sin. ZBD^2 \times \sec. CBD^2}{AB} :: t : t \times \sin. ZB \times \cos. CBZ \times \sec. CBD + \frac{t \times h \times \sin. ZB^2 \times \sin. ZBD^2 \times \sec. CBD^2}{AB}$ the time of describing $uB$, or the effect of parallax in accelerating or retarding the time of contact; the upper sign is to be used when $CBZ$ is acute, and the lower sign when it is obtuse. If $CBZ$ be very nearly a right angle, but obtuse, it may happen that $nE$ may be less than

*no*, in which case, $nE$ is to be taken from *no*, according to the rule. The principal part $nE$ of the effect of parallax will increase or diminish the planet's distance from the sun's center, according as the angle $ZBC$ is acute or obtuse; but the small part *no* of parallax will always increase the planet's distance from the center; take, therefore, the sum or difference of the effects, with the sign of the greater, as to increasing or decreasing the planet's distance from the center of the sun. The second part of the correction will not exceed 9″ or 10″ of time in the transits of Venus in 1761 and 1769, where the nearest approach of Venus to the sun's center was about 10′. In the transit of *Mercury*, the first part alone will be sufficient, except the nearest distance be much greater.

If we suppose the *mean* horizontal parallax of the sun to be 8″,83, then, by calculation from * the above expression, it appears that the total duration at Wardhus was lengthened by parallax 11′. 16″,88, and diminished at Otaheite by 12′. 10″,07; hence, the computed difference of the times is 23′. 26″,95; but the observed difference was 23′. 10″.

(332.) Hence, the correct parallax may be accurately found as follows. Because the observed difference of the total durations at Wardhus and Otaheite is 23′. 10″, and the computed difference, from the assumed mean horizontal parallax of the sun 8″,83, is 23′. 26″,95, the true parallax of the sun is less than that assumed. Let the true parallax be to that assumed as $1-e$ to 1, and (331) the first parts of the computed parallax will be lessened in the ratio of $1-e$ : 1; and the second parts, in the ratio of $\overline{1-e}|^2$ to 1, or of $1-2e$ to 1 nearly. All the first parts, viz. 406″,05; 287″,05; 341″,48; 382″,47, in all = 1417″,05, combine the same way to make the total duration longer at Wardhus than at Otaheite. As to the

* See my *Complete System of Astronomy*, Chap. 25.

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second parts, the effects at Wardhus are $-7'',31$ and $-8'',91$, and at Otaheite are $+1'',63$ and $+4'',49$, in all $=-10'',10$. Therefore $1417'',05\times\overline{1-e}-10'',10\times\overline{1-2e}=1390''$ the excess of the observed total duration at Wardhus above that at Otaheite; or $1417'',05-10'',10-1390''=\overline{1417'',05-20'',20}\times e$; and $e=\frac{16'',95}{1396'',85}=0,0121$. Hence, the mean horizontal parallax of the sun $=8'',83\times\overline{1-0,0121}=8'',72316$; we assume, therefore, the mean horizontal parallax of the sun $=8''\frac{3}{4}$.

Hence, the radius of the earth : the distance of the sun from the earth :: sin. $8''\frac{3}{4}$ : rad. :: 1 : 23575.

(333.) The effect of the parallax being determined, the transit affords a very ready method of finding the difference of the longitudes of two places where the same observations were made. For, compute the effect of parallax in time, and reduce the observations at each place to the time, if seen from the center of the earth, and the difference of the times is the difference of the longitudes. For example, the times at Wardhus and Otaheite, at which the first internal contact would take place at the earth's center, are $9h.\ 40'.\ 44'',6$, and $12h.\ 38'.\ 25'',07$, the difference of which is $12h.\ 2'.\ 19'',53=180°.\ 34'.\ 53''$, the difference of the meridians. From the mean of 63 results from the transits of Mercury, Mr. *Short* found the difference of the meridians of Greenwich and Paris to be $9'.\ 15''$; and from the transit of Venus in 1761, to be $9'.\ 10''$ in time.

(334.) The transit of Venus affords a very accurate method of finding the place of the node. For by the observations made by Mr. *Rittenhouse*, at Norriton in the United States of America, the least distance $CD$ was observed to be $10'.\ 10''$; hence, drawing $CV$ perpendicular to $C$☊, cos.$DCV=8°.\ 28'.\ 54''$ : rad. :: $CD=10'.\ 10''$ : $CV=10'.\ 17''$, the geocentric latitude

of Venus at the time of conjunction; and * 0,72626 : 0,28895 :: 10′. 17″ : 4′. 5″, the heliocentric latitude $CV$ of Venus; hence, considering $C☋V$ as a right lined triangle, tan. $V☋C = 3°. 23′. 35″$ : rad. :: the heliocentric latitude $CV = 4′. 5″$ : $C☋ = 1°. 8′. 52″$, which added to 2ˢ. 13°. 26′. 34″, the place of the sun, gives 2ˢ. 14°. 35′. 26″ for the place of the ascending node of the orbit of Venus.

(335.) The time of the ecliptic conjunction may be thus found. Find, at any time ($t$), the difference ($d$) of longitudes of Venus and the sun's center; find also the apparent geocentric horary motion ($m$) of Venus *from* the sun in longitude, and then say, $m$ : $d$ :: 1 hour : the interval between the time ($t$) and the conjunction, which interval is to be added to or subtracted from $t$, according as the observation was made before or after the conjunction. In the transit in 1761, at 6*h*. 31′. 46″, apparent time at Paris, M. *de la Lande* found $d = 2′. 34″,4$ and $m = 3′. 57″,4$; hence, 3′. 57″,4 : 2′. 34″,4 :: 1 hour : 39′. 1″, which subtracted from 6*h*. 31′. 46″, because at that time the conjunction was past, gives 5*h*. 52′. 45″ for the time of conjunction from this observation. We may also thus find the latitude at conjunction. The horary motion of Venus in latitude was 35″,4; hence, 60′ : 39′,1 :: 35″,4 : 23″, the motion in latitude in 39′,1, which subtracted from 10′. 1″,2, the latitude observed at 6*h*. 31′. 46″, gives 9′. 38″,2 for the latitude at the time of conjunction.

* 0,72626 is the distance of Venus from the sun, her distance from the earth being 0,28895; and the angle subtended by $CV$ is inversely as the distance from $CV$.

# Chap. XXIII.

## ON THE NATURE AND MOTION OF COMETS.

(336.) Comets are solid bodies, revolving in very excentric ellipses about the sun in one of the foci, and are therefore subject to the same laws as the planets, but differ in appearance from them; for as they approach the sun, a tail of light, in some of them, begins to appear, which increases till the comet comes to it's perihelion, and then it decreases again, and vanishes; others have a light encompassing the nucleus, or body of the comet, without any tail. The most ancient philosophers supposed comets to be like planets, performing their revolutions in stated times. *Aristotle*, in his first book of *Meteors*, speaking of comets, says, " But some of the Italians, called Pythagoreans, say, that a *Comet* is one of the *Planets*, but that they do not appear unless after a long time, and are seen but a small time, which happens also to *Mercury*." *Seneca* also, in *Nat. Quest.* Lib. vii. says, " *Apollonius* affirmed, that the *Comets* were, by the Chaldeans, reckoned among the *Planets*, and had their periods like them." *Seneca* himself also, having considered the phænomena of two remarkable comets, believed them to be stars of equal duration with the world, though he was ignorant of the laws that governed them; and foretold, that after-ages would unfold all these mysteries. He recommended it to Astronomers to keep a catalogue of the comets, in order to be able to determine whether they returned at certain periods. Notwithstanding this, most Astronomers, from his time till *Tycho Brahe*, considered them only as meteors, existing in our atmosphere. But that Astronomer,

finding, from his own observations on a comet, that it had no diurnal parallax, placed them above the moon. Afterwards *Kepler* had an opportunity of observing two comets, one of which was very remarkable; and from his observations, which afforded sufficient indications of an annual parallax, he concluded, "that comets moved freely through the planetary orbs, with a motion not much different from a rectilinear one; but of what kind he could not precisely determine." *Hevelius* embraced the hypothesis of a rectilinear motion; but, finding his calculations did not perfectly agree with his observations, he concluded, "that the path of a comet was bent in a curve line, concave towards the sun." He supposed a comet to be generated in the atmosphere of a planet, and to be discharged from it, partly by the rotation of the planet, and then to revolve about the sun in a parabola by the force of projection and it's tendency to the sun, in the same manner as a projectile upon the earth's surface describes a parabola. At length came the famous comet in 1680, which descending nearly in a right line towards the sun, arose again from it in like manner, which proved it's motion in a curve about the sun. *G. S. Doerfell*, Minister at Plaven in Upper Saxony, made observations upon this comet, and found that it's motion might be very well represented by a parabola, having the sun in it's focus. He was ignorant, however, of all the laws by which the motion of a body in a parabola is regulated, and erred considerably in his parabola, making the perihelion distance about twelve times greater than it was. This was published five years before the *Principia*, in which work Sir *I. Newton* having proved that *Kepler*'s law, by which the motions of the planets are regulated, was a necessary consequence of his theory of gravity, it immediately followed, that comets were governed by the same law; and the observations upon them agreed so accurately with his theory, as to leave no doubt of it's truth. That comets describe ellipses, and not parabolas or

hyperbolas, Dr. *Halley* (see his *Synopsis of the Astronomy of Comets*) advances the following reasons.

"Hitherto I have considered the orbits as exactly parabolic, upon which supposition it would follow, that comets, being impelled towards the sun by a centripetal force, would descend as from spaces infinitely distant, and, by their so falling, acquire such a velocity, as that they may again fly off into the remotest parts of the universe, moving upwards with a perpetual tendency, so as never to return again to the sun. But since they appear frequently enough, and since none of them can be found to move with an hyperbolic motion, or a motion swifter than what a comet might acquire by it's gravity to the sun, it is highly probable they rather move in very excentric elliptic orbits, and make their returns after long periods of time; for so their number will be determinate, and, perhaps, not so very great. Besides, the space between the sun and the fixed stars is so immense, that there is room enough for a comet to revolve, though the period of it's revolution be vastly long. Now, the *latus rectum* of an ellipsis is to the *latus rectum* of a parabola, which has the same distance in it's perihelion, as the distance in the aphelion, in the ellipsis, is to the whole axis of the ellipsis. And the velocities are in a subduplicate ratio of the same; wherefore, in very excentric orbits, the ratio comes very near to a ratio of equality; and the very small difference which happens, on account of the greater velocity in the parabola, is easily compensated in determining the situation of the orbit. The principle use, therefore, of the Table of the elements of their motions, and that which indeed induced me to construct it, is, that whenever a new comet shall appear, we may be able to know, by comparing together the elements, whether it be any of those which have appeared before, and consequently to determine it's period, and the axis of it's orbit, and to foretel it's return. And, indeed, there are many things which make me believe that the comet, which

*Apian* discovered in the year 1531, was the same with that which *Kepler* and *Longomontanus* more accurately described in the year 1607; and which I myself have seen return, and observed in the year 1682. All the elements agree, and nothing seems to contradict this my opinion, besides the inequality of the periodic revolutions; which inequality is not so great neither, as that it may not be owing to physical causes. For the motion of Saturn is so disturbed by the rest of the planets, especially Jupiter, that the periodic time of that planet is uncertain for some whole days together. How much more, therefore, will a comet be subject to such like errors, which rises almost four times higher than Saturn, and whose velocity, though increased but a very little, would be sufficient to change it's orbit, from an elliptic to a parabolical one. And I am the more confirmed in my opinion of it's being the same; for in the year 1456, in the summer-time, a comet was seen passing retrograde between the earth and the sun, much after the same manner; which, though nobody made observations upon it, yet, from it's period, and the manner of it's transit, I cannot think different from those I have just now mentioned. And since looking over the histories of comets, I find, at an equal interval of time, a comet to have been seen about Easter in the year 1305, which is another double period of 151 years before the former. Hence, I think, I may venture to foretel that it will return again in the year 1758."

(337.) Dr. *Halley* computed the effect of *Jupiter* upon this comet in 1682, and found that it would increase it's periodic time above a year, in consequence of which he predicted it's return at the end of the year 1758, or the beginning of 1759. He did not make his computations with the utmost accuracy, but, as he himself informs us, *levi calamo*. M. *Clairaut* computed the effects both of Saturn and Jupiter, and found that the former would retard it's return in the last period 100 days, and the latter 511 days; and he deter-

mined the time when the comet would come to it's perihelion to be on April 15, 1759, observing that he might err a month, from neglecting small quantities in the computation. It passed the perihelion on March 13, within 33 days of the time computed. Now if we suppose the time stated by Dr. *Halley* to mean the time of it's passing the perihelion, then if we add to that 100 days, arising from the action of Saturn, which he did not consider, it will bring it very near to the time in which it did pass the perihelion, and prove his computation of the effect of Jupiter to have been very accurate. If he meant the time when it would first appear, his prediction was very accurate, for it was first seen on December 14, 1758, and his computation of the effects of Jupiter will then be more accurate than could have been expected, considering that he made his calculations only by an indirect method, and in a manner confessedly not very accurate. Dr. *Halley*, therefore, had the glory, first to foretel the return of a comet, and the event answered remarkably to his prediction. He further observed, that the action of Jupiter, in the descent of the comet towards it's perihelion in 1682, would tend to increase the inclination of it's orbit; and accordingly the inclination in 1682 was found to be 22′ greater than in 1607. A learned Professor (Dr. *Long*'s Astronomy, p. 562) in Italy to an English gentleman, writes thus:—"Though M. *de la Lande*, and some other French gentlemen, have taken occasion to find fault with the inaccuracies of *Halley*'s calculation, because he himself had said, he only touched upon it slightly; nevertheless they can never rob him of the honour,—First, of finding out that it was one and the same comet which appeared in 1682, 1607, 1531, 1456, and 1305.—Secondly, of having observed, that the planet Jupiter would cause the inclination of the orbit of the comet to be greater, and the period longer.—Thirdly, of having foretold that the return thereof might be retarded till the end of 1758, or the beginning of 1759."

From the observations of M. *Messier* upon a comet in 1770, Mr. *Ædric Prosperin*, Member of the Royal Academies of Stockholm and Upsal, showed, that a parabolic orbit would not answer to it's motions, and he recommended it to Astronomers to seek for the elliptic orbit. This laborious task M. *Lexell* undertook, and has shown that an ellipse, in which the periodic time is about five years and seven months, agrees very well with the observations. See the *Phil. Trans.* 1779. As the ellipses which the comets describe are very excentric, Astronomers, for the ease of calculation, suppose them to move in parabolic orbits, for that part which lies within the reach of observation, by which they can very accurately find the place of the perihelion; it's distance from the sun; the inclination of the plane of it's orbit to the ecliptic, and the place of the node. But it falls not within the plan of this work to enter into an investigation of these matters. For this, I refer the reader to my *Complete System of Astronomy*.

(338.) It is extremely difficult to determine, from computation, the elliptic orbit of a comet, to any degree of accuracy; for when the orbit is very excentric, a very small error in the observation will change the computed orbit into a parabola, or hyperbola. Now from the thickness and inequality of the atmosphere with which the comet is surrounded, it is impossible to determine with any great precision, when either the limb or center of the comet passes the wire at the time of observation. And this uncertainty in the observations will subject the computed orbit to a great error. Hence, it happened, that M. *Bouguer* determined the orbit of the comet to be an hyperbola. M. *Euler* first determined the same for the comet in 1774; but, having received more accurate observations, he found it to be an ellipse. The period of the comet in 1680 appears, from observation, to be 575 years, which Mr. *Euler*, by his computation, determined to be 166½ years. The only safe way to get

the periods of comets, is to compare the elements of all those which have been computed, and where you find they agree very well, you may conclude that they are elements of the *same* comet, it being so extremely improbable that the orbits of two different comets should have the same inclination, the same perihelion distance, and the places of the perihelion and node the same. Thus, knowing the periodic time, we get the major axis of the ellipse; and the perihelion distance being known, the minor axis will be known. When the elements of the orbits agree, the comets may be the same, although the periodic times should vary a little; as that may arise from the attraction of the bodies in our system, and which may also alter all the other elements a little. We have already observed, that the comet which appeared in 1759, had it's periodic time increased considerably by the attraction of *Jupiter* and *Saturn*. This comet was seen in 1682, 1607, and 1531, all the elements agreeing, except a little variation of the periodic time. Dr. *Halley* suspected the comet in 1680, to have been the same which appeared in 1106, 531, and 44 years before CHRIST. He also conjectured, that the comet observed by *Apian*, in 1532, was the same as that observed by *Hevelius*, in 1661; if so, it ought to have returned in 1790, but it has never been observed. But M. *Mechain* having collected all the observations in 1532, and caloulated the orbit again, found it to be sensibly different from that determined by Dr. *Halley*, which renders it very doubtful whether this was the comet which appeared in 1661; and this doubt is increased, by it's not appearing in 1790. The comet in 1770, whose periodic time M. *Lexell* computed to be five years and seven months, has not been observed since. There can be no doubt but that the path of this comet, for the time it was observed, belonged to an orbit whose periodic time was that found by M. *Lexell*, as the computations for such an orbit agreed so very well with the observations. But the revolution was proba-

bly longer before 1770; for as the comet passed very near to Jupiter in 1767, it's periodic time might be sensibly increased by the action of that planet; and as it has not been observed since, we may conjecture, with M. *Lexell,* that having passed in 1772 again into the sphere of sensible attraction of Jupiter, a new disturbing force might probably take place and destroy the effect of the other. According to the above elements, the comet would be in conjunction with Jupiter on August 23, 1779, and it's distance from Jupiter would be only $\frac{1}{419}$ of it's distance from the sun; consequently the sun's action would be only $\frac{1}{224}$ part of that of Jupiter. What a change must this make in the orbit! If the comet returned to it's perihelion in March, 1776, it would then not be visible. See M. *Lexell's* account in the *Phil. Trans.* 1779. The elements of the orbits of the comets, in 1264 and 1556, were so nearly the same, that it is very probable it was the same comet; if so, it ought to appear again about the year 1848.

### *On the Nature and Tails of Comets.*

(339.) Comets are not visible till they come into the planetary regions. They are surrounded with a very dense atmosphere, and from the side opposite to the sun they send forth a tail, which increases as the comet approaches it's perihelion, immediately after which it is longest and most luminous, and then it is generally a little bent and convex towards those parts to which the comet is moving; the tail then decreases, and at last it vanishes. Sometimes the tail is observed to put on this figure ~ towards it's extremity, as that did in 1796. The smallest stars are seen through the tail, notwithstanding it's immense thickness, which proves that it's matter must be extremely rare. The opinion of the ancient philosophers, and of *Aristotle* himself, was, that the tail is a very thin fiery vapour

arising from the comet. *Apian*, *Cardan*, *Tycho*, and others, believed that the sun's rays, being propagated through the transparent head of the comet, were refracted, as in a lens. But the figure of the tail does not answer to this; and, moreover, there should be some reflecting substance to render the rays visible, in like manner as there must be dust or smoke flying about in a dark room, in order that a ray of light entering, it may be seen by a spectator standing sideways from it. *Kepler* supposed, that the rays of the sun carry away some of the gross parts of the comet which reflect the sun's rays, and give the appearance of a tail. *Hevelius* thought that the thinnest parts of the atmosphere of a comet are rarified by the force of the heat, and driven from the fore part and each side of the comet towards the parts turned from the sun. Sir *I. Newton* thinks, that the tail of a comet is a very thin vapour, which the head, or nucleus of the comet, sends out by reason of it's heat. He supposes, that when a comet is descending to it's perihelion, the vapours behind the comet, in respect to the sun, being rarified by the sun's heat, ascend, and take up with them the reflecting particles with which the tail is composed, as air rarefied by heat carries up the particles of smoke in a chimney. But as, beyond the atmosphere of the comet, the ætherial air (*aura ætherea*) is extremely rare, he attributes something to the sun's rays carrying with them the particles of the atmosphere of the comet. And when the tail is thus formed, it, like the nucleus, gravitates towards the sun, and by the projectile force received from the comet, it describes an ellipse about the sun, and accompanies the comet. It conduces also to the ascent of these vapours, that they revolve about the sun, and therefore endeavour to recede from it; whilst the atmosphere of the sun is either at rest, or moves with such a slow motion as it can acquire from the rotation of the sun about it's axis. These are the causes of the ascent of the tails in the neighbourhood of the sun,

where the orbit has a greater curvature, and the comet moves in a denser atmosphere of the sun. The tail of the comet, therefore, being formed from the heat of the sun, will increase till it comes to it's perihelion, and decrease afterwards. The atmosphere of the comet is diminished as the tail increases, and is least immediately after the comet has passed it's perihelion, where it sometimes appears covered with a thick black smoke. As the vapour receives two motions when it leaves the comet, it goes on with the compound motion, and therefore the tail will not be turned directly from the sun, but decline from it towards those parts which are left by the comet; and meeting with a small resistance from the æther, will be a little curved. When the spectator, therefore, is in the plane of the comet's orbit, the curvature will not appear. The vapour, thus rarefied and dilated, may be at last scattered through the heavens, and be gathered up by the planets, to supply the place of those fluids which are spent in vegetation and converted into earth. This is the substance of Sir *I. Newton*'s account of the tails of comets. Against this opinion, Dr. *Hamilton*, in his *Philosophical Essays*, observes, that we have no proof of the existence of a solar atmosphere; and if we had, that when the comet is moving in it's perihelion in a direction at right angles to the direction of it's tail, the vapours which then arise, partaking of the great velocity of the comet, and being also specifically lighter than the medium in which they move, must suffer a much greater resistance than the dense body of the comet does, and therefore ought to be left behind, and would not appear opposite to the sun; and afterwards they ought to appear towards the sun. Also, if the splendour of the tails be owing to the reflection and refraction of the sun's rays, it ought to diminish the lustre of the stars seen through it, which would have their light reflected and refracted in like manner, and consequently their brightness would be diminished. Dr. *Halley*, in his description of the *Aurora Borealis* in

1716, says, "the streams of light so much resembled the long tails of comets, that at first sight they might well be taken for such." And afterwards, "this light seems to have a great affinity to that which the effluvia of electric bodies emit in the dark." *Phil. Trans.* N°. 347. *D. de Mairan* also calls the tail of a comet, the *aurora borealis* of the comet. This opinion Dr. *Hamilton* supports by the following arguments. A spectator, at a distance from the earth, would see the aurora borealis in the form of a tail opposite to the sun, as the tail of a comet lies. The aurora borealis has no effect upon the stars seen through it, nor has the tail of a comet. The atmosphere is known to abound with electric matter, and the appearance of the electric matter in vacuo is exactly like the appearance of the aurora borealis, which, from it's great altitude, may be considered to be in as perfect a vacuum as we can make. The electric matter in vacuo suffers the rays of light to pass through, without being affected by them. The tail of a comet does not expand itself sideways, nor does the electric matter. Hence, he supposes the tails of comets, the aurora borealis, and the electric fluid, to be matter of the same kind. We may add, as a further confirmation of this opinion, that the comet in 1607 appeared to shoot out at the end of it's tail. Le *P. Cysat* remarked the undulations of the tail of the comet in 1618. *Hevelius* observed the same in the tails of the comets in 1652 and 1661. M. *Pingre* took notice of the same appearance in the comet of 1769. These are circumstances exactly similar to the aurora borealis. Dr. *Hamilton* conjectures, that the use of the comets may be to bring the electric matter, which continually escapes from the planets, back into the planetary regions. The arguments are certainly strongly in favour of this hypothesis; and if this be true, we may further add, that the tails are hollow; for if the electric fluid only proceed in it's first direction, and do not diverge sideways, the parts directly behind the comet will not be

filled with it; and this thinness of the tails will account for the appearance of the stars through them.

(340.) In respect to the nature of comets; Sir *I. Newton* observes, that they must be solid bodies like the planets; for if they were nothing but vapours, they must be dissipated when they come near the sun; for the comet in 1680, when it was in it's perihelion, was less distant from the sun than one-sixth of the sun's diameter, consequently the heat of the comet at that time was to the heat of the summer sun as 28000 to 1. But the heat of boiling water is about three times greater than the heat which dry earth acquires from the summer's sun; and the heat of red-hot iron about three or four times greater than the heat of boiling water. Therefore the heat of dry earth at the comet, when in it's perihelion, was about 2000 times greater than red-hot iron. By such heat, all vapours would be immediately dissipated.

(341.) This heat of the comet must be retained a very long time. For a red-hot globe of iron, of an inch diameter, exposed to the open air, scarce loses all it's heat in an hour; but a greater globe would retain it's heat longer, in proportion to it's diameter, because the surface, at which it grows cold, varies in that proportion less than the quantity of hot matter. Therefore a globe of red-hot iron, as large as our earth, would scarcely cool in 50000 years.

(342.) The comet in 1680, coming so near to the sun, must have been considerably retarded by the sun's atmosphere, and therefore, being attracted nearer at every revolution, it will at last fall into the sun, and be a fresh supply of fuel for what the sun loses by it's constant emission of light. In like manner, fixed stars which have been gradually wasted, may be supplied with fresh fuel, and acquire new splendour, and pass for new stars. Of this kind are those fixed stars which appear on a sudden, and shine with a wonderful brightness at first, and afterwards vanish by degrees. Such is the conjecture of Sir *I. Newton.*

(343.) From the beginning of our æra to this time, it is probable, according to the best accounts, that there have appeared about 500 comets. Before that time, about 100 others are recorded to have been seen, but it is probable that not above half of them were comets. And when we consider, that many others may not have been perceived, from being too near the sun—from appearing in moon-light—from being in the other hemisphere—from being too small to be perceived, or which may not have been recorded, we might imagine the whole number to be considerably greater; but it is likely, that of the comets which are recorded to have been seen, the same may have appeared several times, and therefore the number may be less than is here stated. The comet in 1786, which first appeared on August 1, was discovered by Miss *Caroline Herschel*, a sister of Dr. *Herschel*; since that time, she has discovered three others. As the plan of this work does not permit us to give the methods by which the orbits of comets may be computed, and all the opinions respecting them, if the reader wish to see any thing further on the subject, I refer him to my *Complete System of Astronomy*; or to a Treatise, entitled *Cométographie, ou Traité Historique et Théorique des Cometès, par M.* PINGRE', *II. Tom. quarto. Paris*, 1784; or Sir H. ENGLEFIELD's *Determination of the Orbits of Comets*, a very valuable work, in which the ingenious Author has explained, with great clearness and accuracy, the manner of computing the orbits of comets, according to the methods of *Boscovich* and M. *de la Place*.

# Chap. XXIV.

## ON THE FIXED STARS.

(344.) All the heavenly bodies beyond our system, are called *Fixed Stars,* because (except some few) they do not appear to have any proper motion of their own. From their immense distance, they must be bodies of very great magnitude, otherwise they could not be visible; and when we consider the weakness of reflected light, there can be no doubt but that they shine with their own light. They are easily known from the planets, by their twinkling. The number of stars visible at once to the naked eye is about 1000; but Dr. *Herschel,* by his improvements of the reflecting telescope, has discovered that the whole number is great, beyond all conception. In that bright tract of the heavens, called the *Milky Way,* which, when examined by good telescopes, appears to be an immense collection of stars which gives that whitish appearance to the naked eye, he has, in a quarter of an hour, seen 116000 stars pass through the field of view of a telescope of only 15′ aperture. Every improvement of his telescopes has discovered stars not seen before, so that there appears to be no bounds to their number, or to the extent of the universe. These stars, which can be of no use to us, are probably suns to other systems of planets.

(345.) From an attentive examination of the stars with good telescopes, many, which appear only single to the naked eye, are found to consist of two, three, or more stars. Dr. *Maskelyne* had observed *α Herculis* to be a double star: Dr. *Hornsby* had found *π Bootis*

to be double; M. *Cassini*, Mr. *Mayer*, Mr. *Pigott*, and many other Astronomers, had made discoveries of the like kind. But Dr. *Herschel*, by his improved telescopes, has found about 700, of which, not above 42 had been observed before. We shall here give an account of a few of the most remarkable.

*a Herculis*, FLAM. 64, a beautiful double star; the two stars very unequal, the largest is red, and the smallest blue, inclining to green.

*δ Lyræ*, FLAM. 12, double, very unequal, the largest red, and smallest dusky; not easily to be seen with a magnifying power of 227.

*a Geminorum*, FLAM. 66, double, a little unequal, both white; with a power of 146; their distance appears equal to the diameter of the smallest.

*ε Lyræ*, FLAM. 4 and 5, a double-double star; at first sight it appears double at a considerable distance, and, by a little attention, each will appear double; one set are equal, and both white; the other unequal, the largest white, and the smallest inclined to red. The interval of the stars, of the unequal set, is one diameter of the largest, with a power of 227.

*γ Andromedæ*, FLAM. 57, double, very unequal, the largest reddish white, the smallest a fine bright sky-blue, inclining to green. A very beautiful object.

*a Ursæ minoris*, FLAM. 1, double, very unequal, the largest white, the smallest red.

*β Lyræ*, FLAM. 10, quadruple, unequal, white, but three of them a little inclined to red.

*a Leonis*, FLAM. 32, double, very unequal, largest white, smallest dusky.

*ε Bootis*, FLAM. 36, double, very unequal, largest reddish, smallest blue, or rather a faint lilac; very beautiful.

*b Draconis*, FLAM. 39, a very small double star, very unequal, the largest white, smallest inclining to red.

*λ Orionis*, FLAM. 39, quadruple, or rather a double star, and has two more at a small distance, the double

star considerably unequal, the largest white, smallest pale rose colour.

$\xi$ *Libræ*, Flam. *ultima*, double-double, one set very unequal, the largest a very fine white.

$\mu$ *Cygni*, Flam. 78, double, considerably unequal, the largest white, the smallest blueish.

$\mu$ *Herculis*, Flam. 86, double, very unequal; the small star is not visible with a power of 278, but is seen very well with one of 460; the largest is inclined to a pale red, smallest duskish.

$\alpha$ *Capricorni*, Flam. 5, double, very unequal, the largest white, smallest dusky.

$\nu$ *Lyræ*, Flam. 8, treble, very unequal, the largest white, smallest both dusky.

$\alpha$ *Lyræ*, Flam. 3, double, very unequal, the largest a fine brilliant white, the smallest dusky; it appears with a power of 227. Dr. *Herschel* measured the diameter of this fine star, and found it to be 0″,3553.

(346.) These are a few of the principal double, &c. stars mentioned by Dr. *Herschel*, in his catalogues, which he has given us in the *Phil. Trans.* 1782 and 1785. The examination of double stars with a telescope, is a very excellent and ready method of proving it's powers. Dr. *Herschel* recommends the following method. The telescope and the observer having been some time in the open air, adjust the focus of the telescope to some single star of nearly the same magnitude, altitude, and colour of the star to be examined; attend to all the phænomena of the adjusting star as it passes through the field of view, whether it be perfectly round and well defined, or affected with little appendages playing about the edge, or any other circumstances of the like kind. Such deceptions may be detected by turning the object glass a little in it's cell, when these appendages will turn the same way. Thus you will detect the imperfections of the instrument, and therefore will not be deceived when you come to examine the double star.

(347.) Several stars, mentioned by ancient Astrono-

mers, are not now to be found, and several are now observed, which do not appear in their catalogues. The most ancient observation of a new star, is that by *Hipparchus*, about 120 years before J. C. which occasioned his making a catalogue of the fixed stars, in order that future Astronomers might see what alterations had taken place since his time. We have no account where this new star appeared. A new star is also said to have appeared in the year 130; another in 389; another in the ninth century, in 15° of *Scorpio*; a fifth in 945; and a sixth in 1264; but the accounts we have of all these are very imperfect.

(348.) The first new star we have any accurate account of, is that which was discovered by *Cornelius Gemma*, on November 8, 1572, in the *Chair of Cassiopea*. It exceeded *Sirius* in brightness, and was seen at mid-day. It first appeared bigger than *Jupiter*, but it gradually decayed, and after sixteen months it entirely disappeared. It was observed by *Tycho Brahe*, who found that it had no sensible parallax; and he concluded that it was a fixed star. Some have supposed that this is the same which appeared in 945 and 1264, the situation of it's place favouring this opinion.

(349.) On August 13, 1596, *David Fabricius* observed a new star in the *Neck of the Whale*, in 25°. 45′ of Aries, with 15°. 54′ south latitude. It disappeared after October in the same year. *Phocyllides Holwarda* discovered it again in 1637, not knowing that it had ever been seen before; and after having disappeared for nine months, he saw it come into view again. *Bullialdus* determined the periodic time between it's greatest brightness to be 333 days. It's greatest brightness is that of a star of the second magnitude, and it's least, that of a star of the sixth. It's greatest degree of brightness, however, is not always the same, nor are the same phases always at the same interval.

(350.) In the year 1600, *William Jansenius* dis-

covered a changeable star in the *Neck of the Swan.* It was seen by *Kepler*, who wrote a Treatise upon it, and determined it's place to be 16°. 18′ ♒, with 55°. 30′ or 32′ north latitude. *Ricciolus* saw it in 1616, 1621, 1624, and 1629. He is positive that it was invisible in the last years from 1640 to 1650. M. *Cassini* saw it again in 1655; it increased till 1660, and then grew less, and at the end of 1661, it disappeared. In November 1665, it appeared again, and disappeared in 1681. In 1715 it appeared of the sixth magnitude, as it does at present.

(351.) On June 20, 1670, another changeable star was discovered near the *Swan's Head*, by *P. Anthelme.* It disappeared in October, and was seen again on March 17, 1671. On September 11, it disappeared. It appeared again in March 1672, and disappeared in the same month, and has never since been seen. It's longitude was 1°. 52′. 26″ of ♒, and it's latitude 47°. 25′. 22″N. The days are here put down for the new style.

(352.) In 1686, *Kirchius* observed $\chi$ in the *Swan* to be a changeable star; and, from twenty years observations, the period of the return of the same phases was found to be 405 days; the variations of it's magnitude, however, were subject to some irregularity.

(353.) In the year 1604, at the beginning of October, *Kepler* discovered a new star near the heel of the right foot of *Serpentarius*, so very brilliant, that it exceeded every fixed star, and even *Jupiter*, in magnitude. It was observed to be every moment changing into some of the colours of the rainbow, except when it was near the horizon, when it was generally white. It gradually diminished, and disappeared about October 1605, when it came too near the sun to be visible, and was never seen after. It's longitude was 17°. 40′ of ♐, with 1°. 56′ north latitude, and was found to have no parallax.

(354.) *Montanari* discovered two stars in the constellation of the *Ship*, marked $\beta$ and $\gamma$ by *Bayer*, to be

wanting. He saw them in 1664, but lost them in 1668. The star $\theta$ in the tail of the *Serpent*, reckoned by *Tycho* of the third, was found, by him, of the fifth magnitude. The star $\rho$ in *Serpentarius* did not appear, from the time it was observed by him, till 1695. The star $\psi$ in the *Lion*, after disappearing, was seen by him in 1667. He observed also, that $\beta$ in *Medusa*'s *Head* varied in it's magnitude.

(355.) M. *Cassini* discovered *one* new star of the fourth, and *two* of the fifth magnitude in *Cassiopea*; also *five* new stars in the same constellation, of which three have disappeared; *two* new ones in the beginning of the constellation *Eridanus*, of the fourth and fifth magnitude; and *four* new ones of the fifth or sixth magnitude, near the north pole. He further observed, that the star, placed by *Bayer* near $\epsilon$ of the *Little Bear*, is no longer visible; that the star A of *Andromeda*, which had disappeared, had come into view again in 1695; that in the same constellation, instead of one in the *Knee*, marked $\upsilon$, there are two others come more northerly; and that $\xi$ is diminished; that the star placed by *Tycho*, at the end of the *Chain of Andromeda*, as of the fourth magnitude, could then scarcely be seen; and that the star which, in *Tycho*'s catalogue, is the twentieth of *Pisces*, was no longer visible.

(356.) M. *Maraldi* observed, that the star $\kappa$ in the left leg of *Sagittarius*, marked by *Bayer* of the third magnitude, appeared of the sixth, in 1671; in 1676 it was found, by Dr. *Halley*, to be of the third; in 1692 it could hardly be perceived, but in 1693 and 1694 it was of the fourth magnitude. In 1704 he discovered a star in *Hydra* to be periodical; it's position is in a right line with those in the tail marked $\pi$ and $\gamma$. The time between it's greatest lustre, which is of the fourth magnitude, was about two years; in the intermediate time it disappeared. In 1666, *Hevelius* says, he could not find a star of the fourth magnitude in the eastern scale of *Libra*, observed by

*Tycho* and *Bayer*; but *Maraldi*, in 1709, says, that it had then been seen for 15 years, smaller than one of the fourth.

(357.) *J. Goodricke*, Esq. has determined the periodic variation of *Algol*, or $\beta$ *Persei* (observed by *Montanari* to be variable) to be about *2d.* *21h.* It's greatest brightness is of the second magnitude, and least of the fourth. It changes from the second to the fourth in about three hours and a half, and back again in the same time, and retains it's greatest brightness for the other part of the time.

(358.) Mr. *Goodricke* also discovered, that $\beta$ *Lyræ* was subject to a periodic variation. The following is the result of his observations. It completes all it's phases in 12 days 19 hours, during which time, it undergoes the following changes:—1. It is of the third magnitude for about two days. 2. It diminishes in about $1\frac{1}{4}$ days. 3. It is between the fourth and fifth magnitude for less than a day. 4. It increases in about two days. 5. It is of the third magnitude for about three days. 6. It diminishes in about one day. 7. It is something larger than the fourth magnitude for a little less than a day. 8. It increases in about one day and three quarters to the first point, and so completes a whole period. See the *Phil. Trans.* 1785. He has also found, that $\delta$ *Cephei* is subject to a periodic variation of *5d.* *8h.* $37'\frac{1}{2}$; during which time it undergoes the following changes: 1. It is at it's greatest brightness about one day thirteen hours. 2. It's diminution is performed in about one day eighteen hours. 3. It is at it's greatest obscuration about one day twelve hours. 4. It increases in about thirteen hours. It's greatest and least brightness is that between the third and fourth, and between the fourth and fifth magnitudes.

(359.) *E. Pigott*, Esq. has discovered $\eta$ *Antinoi* to be a variable star, with a period of *7d.* *4h.* $38'$. The changes happen as follows: 1. It is at it's greatest brightness 44 $\pm$ hours. 2. It decreases 62 $\pm$ hours.

3. It is at it's least brightness 30 ± hours. 4. It increases 36 ± hours. When most bright, it is of the third or fourth magnitude, and when least, of the fourth or fifth. See the *Phil. Trans.* 1785.

(360.) In the *Phil. Trans.* 1796, Dr. *Herschel* has proposed a method of observing the changes that may happen to the fixed stars; with a catalogue of their comparative brightness, in order to ascertain the permanency of their lustre.

(361.) Dr. *Herschel*, in a Paper of the *Phil. Trans.* 1783, upon the proper motion of the solar system, has given a large collection of stars which were formerly seen, but are now lost; also a catalogue of variable stars, and of new stars; and very justly observes, that it is not easy to prove that a star was never seen before; for though it should not be contained in any catalogue whatever, yet the argument for it's former non-appearance, which is taken from it's not having been observed before, is only so far to be regarded, as it can be made probable, or almost certain, that a star would have been observed, had it been visible.

(362.) There have been various conjectures to account for the appearances of the changeable stars. M. *Maupertuis* supposes, that they may have so quick a motion about their axes, that the centrifugal force may reduce them to flat oblate spheroids, not much unlike a mill-stone; and it's plane may be inclined to the plane of the orbits of it's planets, by whose attraction the position of the body may be altered, so that when it's plane passes through the earth, it may be almost or entirely invisible, and then become again visible as it's broadside is turned towards us. Others have conjectured, that considerable parts of their surfaces are covered with dark spots, so that when, by the rotation of the star, these spots are presented to us, the stars become almost or entirely invisible. Others have supposed, that these stars have very large opaque bodies revolving about and very near to them, so as to obscure them when they come in conjunction with us.

The irregularity of the phases of some of them shows the cause to be variable, and therefore may, perhaps, be best accounted for by supposing that a great part of the body of the star is covered with spots, which appear and disappear like those on the sun's surface. The total disappearance of a star may probably be the destruction of it's system; and the appearance of a new star, the creation of a new system of planets.

(363.) The fixed stars are not all evenly spread through the heavens, but the greater part of them are collected into clusters, of which it requires a large magnifying power, with a great quantity of light, to be able to distinguish the stars separately. With a small magnifying power and quantity of light, they only appear small whitish spots, something like a small light cloud, and thence they were called *Nebulæ*. There are some *nebulæ*, however, which do not receive their light from stars. For in the year 1656, *Huygens* discovered a nebula in the middle of *Orion's Sword*; it contains only seven stars, and the other part is a bright spot upon a dark ground, and appears like an opening into brighter regions beyond. In 1612, *Simon Marius* discovered a nebula in the *Girdle of Andromeda*. Dr. *Halley*, when he was observing the southern stars, discovered one in the *Centaur*, but this is never visible in England. In 1714, he found another in *Hercules*, nearly in a line with $\zeta$ and $\eta$ of *Bayer*. This shows itself to the naked eye, when the sky is clear and the moon absent. M. *Cassini* discovered one between the *Great Dog* and the *Ship*, which he describes as very full of stars, and very beautiful, when viewed with a good telescope. There are two whitish spots near the south pole, called, by the sailors, the *Magellanic Clouds*, which, to the naked eye, resemble the milky way, but, through telescopes, they appear to be composed of stars. M. *de la Caille* in his catalogue of fixed stars observed at the Cape of Good Hope, has remarked 42 nebulæ which he observed, and which he divided into three

classes; 14, in which he could not discover the stars; 14, in which he could see a distinct mass of stars; and 14, in which the stars appeared of the sixth magnitude, or below, accompanied with white spots, and nebulæ of the first and third kind. In the *Connoissance des Temps*, for 1783 and 1784, there is a catalogue of 103 nebulæ, observed by *Messier* and *Mechain*, some of which they could resolve, and others they could not. But Dr. *Herschel* has given us a catalogue of 2000 nebulæ and clusters of stars, which he himself has discovered. Some of them form a round, compact system; others are more irregular, of various forms; and some are long and narrow. The globular systems of stars appear thicker in the middle than they would do if the stars were all at equal distances from each other; they are, therefore, condensed towards the center. That the stars should be thus accidentally disposed, is too improbable a supposition to be admitted; he supposes, therefore, that they are thus brought together by their mutual attractions, and that the gradual condensation towards the center is a proof of a central power of that kind. He further observes, that there are some additional circumstances in the appearance of extended clusters and nebulæ, that very much favour the idea of a power lodged in the brightest part. For although the form of them be not globular, it is plainly to be seen that there is a tendency towards sphericity, by the swell of the dimensions as they draw near the most luminous place, denoting, as it were, a course, or tide of stars, setting towards a center. As the stars in the same nebula must be very nearly all at the same relative distance from us, and they appear nearly of the same size, their real magnitudes must be nearly equal. Granting, therefore, that these nebulæ and clusters of stars are formed by their mutual attraction, Dr. *Herschel* concludes that we may judge of their relative age by the disposition of their component parts, those being the oldest which are most compressed. He supposes the milky way to be a nebula,

of which our sun is one of it's component parts. See the *Phil. Trans.* 1786 and 1789.

(364.) Dr. *Herschel* has discovered other phænomena in the heavens, which he calls *Nebulous Stars*; that is, stars surrounded with a faint luminous atmosphere, of a considerable extent. Cloudy or nebulous stars, he observes, have been mentioned by several Astronomers; but this name ought not to be applied to the objects which they have pointed out as such; for, on examination, they prove to be either clusters of stars, or such appearances as may reasonably be supposed to be occasioned by a multitude of stars at a vast distance. He has given an account of seventeen of these stars, one of which he has thus described. "November 13, 1790. A most singular phænomenon; a star of the eighth magnitude, with a faint luminous atmosphere, of a circular form, and of about 3′ diameter. The star is perfectly in the center, and the atmosphere is so diluted, faint, and equal throughout, that there can be no surmise of it's consisting of stars; nor can there be a doubt of the evident connexion between the atmosphere and the star. Another star not much less in brightness, and in the same field of view with the above, was perfectly free from any such appearance." Hence, he draws the following consequences. Granting the connexion between the star and the surrounding nebulosity, if it consist of stars very remote which give the nebulous appearance, the central star, which is visible, must be immensely greater than the rest; or if the central star be not larger than common, how extremely small and compressed must be those other luminous points which occasion the nebulosity! As, by the former supposition, the luminous central point must far exceed the standard of what we call a star, so, in the latter, the shining matter about the center will be much too small to come under the same denomination; we therefore either have a central body which is not a star, or a star which is involved in a shining fluid, of a nature totally

unknown to us. This last opinion Dr. *Herschel* adopts. The existence of this shining matter, he says, does not seem to be so essentially connected with the central points, that it might not exist without them. The great resemblance there is between the chevelure of these stars, and the diffused nebulosity there is about the constellation *Orion*, which takes up a space of more than 60 square degrees, renders it highly probable that they are of the same nature. If this be admitted, the separate existence of the luminous matter is fully proved. Light reflected from the star could not be seen at this distance. And, besides, the outward parts are nearly as bright as those near the star. In further confirmation of this, he observes, that a cluster of stars will not so completely account for the milkiness, or soft tint of the light of these nebulæ, as a self-luminous fluid. This luminous matter seems more fit to produce a star by it's condensation, than to depend on the star for it's existence. There is a telescopic milky way extending in right ascension from $5h. 15'. 8''$ to $5h. 39'. 1''$, and in polar distance from $87°. 46'$ to $98°. 10'$. This, Dr. *Herschel* thinks, is better accounted for, by a luminous matter, than from a collection of stars. He observes, that perhaps some may account for these nebulous stars, by supposing that the nebulosity may be formed by a collection of stars at an immense distance, and that the central star may be a near star, accidently so placed; the appearance, however, of the luminous part does not, in his opinion, at all favour the supposition that it is produced by a great number of stars; on the other hand, it must be granted that it is extremely difficult to admit the other supposition, when we know that nothing but a solid body is self-luminous, or, at least, that a fixed luminary must immediately depend upon such, as the flame of a candle upon the candle itself. See Dr. *Herschel's* Account, in the *Phil. Trans.* 1791.

## *On the Constellations.*

(365.) As soon as Astronomy began to be studied, it must have been found necessary to divide the heavens into separate parts, and to give some representations to them, in order that Astronomers might describe and speak of the stars, so as to be understood. Accordingly we find that these circumstances took place very early. The ancients divided the heavens into *Constellations*, or collections of stars, and represented them by animals, and other figures, according to the ideas which the dispositions of the stars suggested. We find some of them mentioned by JOB; and although it has been disputed, whether our translation has sometimes given the true meaning to the Hebrew words, yet it is agreed, that they signify constellations. Some of them are mentioned by *Homer* and *Hesiod*, but *Aratus* professedly treats of all the ancient ones, except three which were invented after his time. The number of the ancient constellations was 48, but the present number upon a globe is about 70; by rectifying which, and setting it to correspond with the stars in the heavens, you may, by comparing them, very easily get a knowledge of the different constellations and stars. Those stars which do not come into any of the constellations, are called *Unformed Stars*. The stars visible to the naked eye are divided into six classes, according to their magnitudes; the largest are called of the first magnitude, the next of the second, and so on. Those which cannot be seen without telescopes, are called *Telescopic Stars*. The stars are now generally marked upon maps and globes with *Bayer*'s letters; the 1st letter in the Greek alphabet being put to the greatest star of each constellation; the 2d letter to the next greatest, and so on; and when any more letters are wanted, the Italic

letters are generally used; this serves as a name to the star, by which it may be pointed out. Twelve of these constellations lie upon the ecliptic, including a space about 15° broad, called the *Zodiac*, within which all the planets move. The constellation *Aries*, or the *Ram*, about 2000 years since, lay in the *first* sign of the ecliptic; but, on account of the precession of the equinox, it now lies in the *second*. The following are the names of the constellations, and the number of the stars observed in them by different Astronomers. *Antinous* was made out of the unformed stars near *Aquila*; and *Coma Berenices* out of the unformed stars near the *Lion's Tail*. They are both mentioned by *Ptolemy*, but as unformed stars. The constellations as far as the *Triangle*, with *Coma Berenices*, are *northern*; those after *Pisces*, are *southern*.

| THE ANCIENT CONSTELLATIONS. | | *Ptolemy.* | *Tycho.* | *Hevelius.* | *Flamstead.* |
|---|---|---|---|---|---|
| Ursa Minor | The Little Bear | 8 | 7 | 12 | 24 |
| Ursa Major | The Great Bear | 35 | 29 | 73 | 87 |
| Draco | The Dragon | 31 | 32 | 40 | 80 |
| Cæpheus | Cæpheus | 13 | 4 | 51 | 35 |
| Bootes | Bootes | 23 | 18 | 52 | 54 |
| Corona Borealis | The Northern Crown | 8 | 8 | 8 | 21 |
| Hercules | Hercules kneeling | 29 | 28 | 45 | 113 |
| Lyra | The Harp | 10 | 11 | 17 | 21 |
| Cygnus | The Swan | 19 | 18 | 47 | 81 |
| Cassiopea | The Lady in her Chair | 13 | 26 | 37 | 55 |
| Perseus | Perseus | 29 | 29 | 46 | 59 |
| Auriga | The Waggoner | 14 | 9 | 40 | 66 |
| Serpentarius | Serpentarius | 29 | 15 | 40 | 74 |
| Serpens | The Serpent | 18 | 13 | 22 | 64 |
| Sagitta | The Arrow | 5 | 5 | 5 | 18 |
| Aquila | The Eagle | 15 | 12 | 23 | 71 |
| Antinous | Antinous | | 3 | 19 | |
| Delphinus | The Dolphin | 10 | 10 | 14 | 18 |

THE ANCIENT CONSTELLATIONS CONTINUED.

| | | *Ptolemy.* | *Tycho.* | *Hevelius.* | *Flamstead.* |
|---|---|---|---|---|---|
| Equulus | The Horse's Head | 4 | 4 | 6 | 10 |
| Pegasus | The Flying Horse | 20 | 19 | 38 | 89 |
| Andromeda | Andromeda | 23 | 23 | 47 | 66 |
| Triangulum | The Triangle | 4 | 4 | 12 | 16 |
| Aries | The Ram | 18 | 21 | 27 | 66 |
| Taurus | The Bull | 44 | 43 | 51 | 141 |
| Gemini | The Twins | 25 | 25 | 38 | 85 |
| Cancer | The Crab | 23 | 15 | 29 | 83 |
| Leo | The Lion | 35 | 30 | 49 | 95 |
| Coma Berenices | Berenice's Hair | | 14 | 21 | 43 |
| Virgo | The Virgin | 32 | 33 | 50 | 110 |
| Libra | The Scales | 17 | 10 | 20 | 51 |
| Scorpius | The Scorpion | 24 | 10 | 20 | 44 |
| Sagittarius | The Archer | 31 | 14 | 22 | 69 |
| Capricornus | The Goat | 28 | 28 | 29 | 51 |
| Aquarius | The Water-bearer | 45 | 41 | 47 | 108 |
| Pisces | The Fishes | 38 | 36 | 39 | 113 |
| Cetus | The Whale | 22 | 21 | 45 | 97 |
| Orion | Orion | 38 | 42 | 62 | 78 |
| Eridanus | Eridanus | 34 | 10 | 27 | 84 |
| Lepus | The Hare | 12 | 13 | 16 | 19 |
| Canis Major | The Great Dog | 29 | 13 | 21 | 31 |
| Canis Minor | The Little Dog | 2 | 2 | 13 | 14 |
| Argo | The Ship | 45 | 3 | 4 | 64 |
| Hydra | The Hydra | 27 | 19 | 31 | 60 |
| Crater | The Cup | 7 | 3 | 10 | 31 |
| Corvus | The Crow | 7 | 4 | 0 | 9 |
| Centaurus | The Centaur | 37 | 0 | 0 | 35 |
| Lupus | The Wolf | 19 | 0 | 0 | 24 |
| Ara | The Altar | 7 | 0 | 0 | 9 |
| Corona Australis | The Southern Crown | 13 | 0 | 0 | 12 |
| Pisces Australis | The Southern Fish | 18 | 0 | 0 | 24 |

THE NEW SOUTHERN CONSTELLATIONS.

| | | |
|---|---|---|
| Columba Noachi | Noah's Dove - - - - - | 10 |
| Robur Carolinum | The Royal Oak - - - | 12 |
| Grus | The Crane - - - - - - | 13 |
| Phœnix | The Phœnix - - - - - | 13 |
| Indus | The Indian - - - - | 12 |
| Pavo | The Peacock - - - - | 14 |
| Apus, *Avis Indica* | The Bird of Paradise - - | 11 |
| Apis, *Musca* | The Bee, or Fly - - - | 4 |
| Chamæleon | The Chameleon - - - | 10 |
| Triangulum Australe | The South Triangle - - | 5 |
| Piscis volans, *Passer* | The Fly in gFish - - - | 8 |
| Dorado, *Xiphias* | The Sword Fish - - - | 6 |
| Toucan | The American Goose - | 9 |
| Hydrus | The Water Snake - - | 10 |

*HEVELIUS's CONSTELLATIONS,*

MADE OUT OF THE UNFORMED STARS.

| | | | |
|---|---|---|---|
| Lynx | The Lynx | 19 | 44 |
| Leo Minor | The Little Lion | | 53 |
| Asteron and Chara | The Greyhounds | 23 | 25 |
| Cerberus | Cerberus | 4 | |
| Vulpecula and Anser | The Fox and Goose | 27 | 35 |
| Scutum Sobieski | Sobieski's Shield | 7 | |
| Lacerta | The Lizard | | 16 |
| Camelopardalis | The Camelopard | 32 | 58 |
| Monoceros | The Unicorn | 19 | 31 |
| Sextans | The Sextant | 11 | 41 |

Besides the letters which are prefixed to the stars, many of them have names, as *Regulus, Sirius, Arcturus*, &c.

(366.) *Kepler*, who was afterwards, in this conjecture, followed by Dr. *Halley*, has made a very ingenious observation upon the magnitudes and distances

of the fixed stars. He observes, that there can be only 13 points upon the surface of a sphere as far distant from each other as from the center; and supposing the nearest fixed stars to be as far from each other as from the sun, he concludes that there can be only thirteen stars of the first magnitude. Hence, at twice that distance from the sun, there may be placed four times as many, or 52; at three times that distance, nine times as many, or 117; and so on. These numbers will give, pretty nearly, the number of stars of the first, second, third, &c. magnitudes. Dr. *Halley* further remarks, that if the number of stars be finite, and occupy only a part of space, the outward stars would be continually attracted towards those which are within, and, in process of time, they would coalesce and unite into one. But if the number be infinite, and they occupy an infinite space, all the parts would be nearly in equilibrio, and consequently, each fixed star being drawn in opposite directions, would keep it's place, or move on till it had found an equilibrium. *Phil Trans.* N°. 364.

## *On the Catalogues of the Fixed Stars.*

(367.) At the time of *Hipparchus* of Rhodes, about 120 years before J. C. a new star appeared, upon which he set about numbering the fixed stars, and reducing them to a *Catalogue*, that posterity might know whether any changes had taken place in the heavens. *Ptolemy*, however, mentions that *Tymocharis* and *Arystillus* left several observations made 180 years before. The catalogue of *Hipparchus* contained 1022 stars, with their latitudes and longitudes, which *Ptolemy* published, with the addition of four more. These Astronomers made their observations with an armillary sphere, placing the armilla, or hoop representing the ecliptic, to coincide with the ecliptic in the heavens by means of the sun in the day-time, and

then they determined the place of the moon in respect of the sun by a moveable circle of latitude. The next night by the help of the moon (whose place before found they corrected by allowing for it's motion in the interval of time) they placed the hoop in such a situation as was agreeable to the present moment of time, and then compared, in like manner, the places of the stars with the moon. Thus they found the latitudes and longitudes of the stars; it could not, however, be done with such an instrument to any very great degree of accuracy. *Ptolemy* adapted his catalogue to the year 137 after J.C.; but supposing, with *Hipparchus*, who made the discovery, the precession of the equinoxes to be 1° in 100 years, instead of about 72 years, he only added 2°. 40′. to the numbers in *Hipparchus* for 265 years (the difference of the epochs) instead of 3°. 42′. 22″, according to Dr. *Maskelyne*'s Tables. To compare his Tables, therefore, with the present, we must first increase his numbers by 1°. 2′. 22″, and then allow for the precession from that time to this. The next Astronomer who observed the fixed stars a-new, was *Ulugh Beigh*, the grandson of *Tamerlane* the Great; he made a catalogue of 1022 stars, reduced to the year 1437. *William*, the most illustrious Landgrave of Hesse, made a catalogue of 400 stars which he observed; he computed their latitudes and longitudes from their observed right ascensions and declinations. In the year 1610, *Tycho Brahe*'s catalogue of 777 stars was published from his own observations, made with great care and diligence. It was afterwards, in 1627, copied into the *Rudolphine Tables*, and increased by 223 stars, from other observations of *Tycho*. Instead of a *zodiacal* armilla, *Tycho* substituted the *equatorial* armilla, by which he observed the difference of right ascensions, and the declinations, out of the meridian, the meridian altitude being always made use of to confirm the others. From thence he computed the latitudes and longitudes. *Tycho* compared *Venus* with the sun, and then the

other stars with Venus, allowing for it's parallax and refraction; and having thus ascertained the places of a few stars, he settled the rest from them; and although his instrument was very large, and constructed with great accuracy, yet, not having pendulum clocks to measure his time, his observations cannot be very accurate. The next catalogue was that of *R. P. Ricciolus*, which was taken from *Tycho*'s, except 101 stars which he himself had observed. *Hevelius* of Dantzick, in 1690, published a catalogue of 1930 stars, of which 950 were known to the ancients; 603 he calls his own, because they had not been accurately observed by any one before himself; and 377 of Dr. *Halley*, which were invisible to his hemisphere. Their places were fixed for the year 1660. The *British Catalogue*, which was published by Mr. *Flamstead*, contains 3000 stars, rectified for the year 1689. They are distinguished into seven degrees of magnitude (of which the seventh degree is telescopic) in their proper constellations. This catalogue is more correct than any of the others, the observations having been made with better instruments. He also published an *Atlas Cœlestis*, or maps of the stars, in which each star is laid down in it's true place, and delineated of it's own magnitude. Each star is marked with a letter, beginning with the first letter *α* of the Greek alphabet for the largest star of each constellation, and so on, according to their magnitudes, following, in this respect, the charts of the same kind which were published by *J. Bayer*, a German, 1603. In the year 1757, M. *de la Caille* published his *Fundamenta Astronomiæ*, in which there is a catalogue of 397 stars; and in 1763, he published a catalogue of 1942 southern stars, from the tropic of Capricorn to the south pole, with their right ascensions and declinations for 1750. He also published a catalogue of zodiacal stars in the *Ephemerides* from 1765 to 1774. Mr. *Mayer* also published a catalogue of 600 zodiacal stars. In the *Nautical Almanac* for 1773, there is published a catalogue of

380 stars observed by Dr. *Bradley*, with their longitudes and latitudes. In the year 1782, *J. E. Bode*, Astronomer at Berlin, published a set of *Celestial Charts*, containing a greater number of stars than in those of Mr. *Flamstead*, with many of the double stars and nebulæ. He also published, in the same work, a catalogue of stars, that of *Flamstead* being the foundation, omitting some stars, whose positions were left incomplete, and altering the numbers; to which he has added stars from *Hevelius*, M. *de la Caille*, *Mayer*, and others. In the year 1776, there was published at Berlin, a work entitled *Recueil de Tables Astronomiques*, in which is contained a very large catalogue of stars from *Hevelius*, *Flamstead*, M. *de la Caille*, and Dr. *Bradley*, with their latitudes and longitudes for the beginning of 1800; with a catalogue of the southern stars of M. *de la Caille*; of double stars; of changeable stars, and of nebulous stars. This is a very useful work for the practical Astronomer. But the most complete catalogue is that published by the Rev. Mr. *Wollaston*, F. R. S. in 1789, entitled, *A Specimen of a General Astronomical Catalogue, arranged in Zones of North Polar Distance, and adapted to January* 1, 1790; *containing a Comparative View of the Mean Positions of Stars, as they come out upon Calculation from the Tables of several principal Observers.*

### *On the* Proper Motions *of the Fixed Stars.*

(368.) Dr. *Maskelyne*, in the explanation and use of his Tables, which he published with the first volume of his *Observations*, observes, that many, if not all the fixed stars, have small motions among themselves, which are called their *Proper Motions*; the cause and laws of which are hid, for the present, in almost equal obscurity. From comparing his own observations at that time, with those of Dr. *Bradley*, Mr. *Flamstead*,

and Mr. *Roemer*, he then found the annual proper motion of the following stars, in right ascension, to be, of *Sirius* – 0″,63, of *Castor* – 0″,28, of *Procyon* – 0″,8, of *Pollux* – 0″,93, of *Regulus* – 0″,41, of *Arcturus* – 1″,4, and of *a Aquilæ* + 0″,57; and of *Sirius* in north polar distance 1″,20, and of *Arcturus* 2″,01, both southwards. But since that time he had continued his observations, and from a catalogue of right ascensions of 36 principal stars (which he communicated to Mr. *Wollaston*, and which is found in his work), it appears that 35 of them have a *proper motion* in right ascension.

(369.) In the year 1759, M. *Mayer* observed 80 stars, and compared them with the observations of *Roemer* in 1706. M. *Mayer* is of opinion, that (from the goodness of the instruments with which the observations were made) where the disagreement is at least 10″ or 15″, it is a very probable indication of a proper motion of such a star. He further adds, that when the disagreement is so great as he has found it in some of the stars, amongst which is *Fomahand*, where the difference was 21″ in 50 years, he has no doubt of a proper motion. Dr. *Herschel* following *Mayer*'s judgement of his own and *Roemer*'s observations, has compared the observations, and leaving out of his account all those stars which did not show a disagreement amounting to 10″, he found that 56 of them had a proper motion. From thence he attempts to deduce the motion of the solar system in the following manner.

(370.) If the sun be in motion as well as the stars, the effects will be altered according to their motion, compared with the motion of our sun. Some of them, therefore, from their own proper motions, might destroy, or more than counteract, the effects arising from the motion of the sun. In whatever direction our system should move, it would be very easy to find what effect in latitude and longitude would have taken place upon any star, by means of a celestial globe, by

conceiving the sun to move from the center upon any radius directed to the point to which the sun is moving. Dr. *Herschel* describes the effect thus. Let an arc of 90° be applied to the surface of a globe, and always passing through that point to which the motion of the system is directed. Then whilst one end moves along the equator, the other will describe a curve passing through it's pole, and returning into itself; and the stars in the northern hemisphere, within this curve, will appear to move to the north; and the rest will go to the south. A similar curve may be described in the southern hemisphere, and like appearances will take place.

(371.) Now Dr. *Herschel* first takes the seven stars before mentioned, whose proper motions had been determined by Dr. *Maskelyne*, and he finds, that if a point be assumed about the 77° of right ascension, and the sun to move from it, it will account for all the motions in right ascension. And if, instead of supposing the sun to move in the plane of the equator, it should ascend to a point near to $\lambda$ *Herculis*, it will account for the observed change of declination of *Sirius* and *Arcturus*. In respect to the *quantity* of motion of each, that must depend upon their unknown relative distances; he only speaks here of the *directions* of the motions.

(372.) He next takes twelve stars from the catalogue of 56, whose proper motions have been determined from a comparison of the observations of *Roemer* and *Mayer*, and adds to them *Regulus* and *Castor*; these have all a proper motion in right ascension and declination, except Regulus, which has none in declination. Of these 27 motions, the above-supposed motion of the solar sytem will satisfy 22. There are also some remarkable circumstances in the *quantities* of these motions. *Arcturus* and *Sirius* being the largest, and therefore, probably, the nearest, ought to have the greatest apparent motion; and so we find they have. Also, Arcturus is better situated to have a mo-

tion in right ascension, and it has the greatest motion. Several other facts of the same kind are found also to take place. But there is a very remarkable circumstance in respect to Castor. Castor is a double star; now, how extraordinary must appear the concurrence, that two such stars should both have a proper motion so exactly alike, that they never have been found to vary a second! This seems to point out the common cause, the motion of the solar system.

(373.) Dr. *Herschel* next takes 32 more of the same catalogue of 56 stars, and shows that their motions agree very well with his supposed motion of the solar system. But the motions of the other 12 stars cannot be accounted for upon this hypothesis. In these, therefore, he supposes the effect of the solar motion has been destroyed and counteracted by their own proper motions. The same may be said of 19 stars out of the 32, which only agree with the solar motion one way, and are, as to sense, at rest in the other. According to the rules of philosophising, therefore, which direct us to refer all phænomena to as few and simple principles as are sufficient to explain them, Dr. *Herschel* thinks we ought to admit the motion of the solar system. Perhaps, however, this argument cannot be properly applied here, because there is no new cause or principle introduced, by supposing each star to have a proper motion. Admitting the doctrine of universal gravitation, the fixed stars ought to move as well as the sun. But the sun's motion, as here estimated, cannot be owing to the action of a body upon it which might give it a rotatory motion at the same time, as M. *de la Lande* conjectures; because a body acting on the sun, to give it it's rotation about it's axis, would not, at the same time, give it that progressive motion. See Dr. *Herschel*'s Account in the *Phil. Trans.* 1783.

(374.) But it will be proper to consider how far this motion of the solar system agrees with the proper motion of the 35 stars determined by Dr. *Maskelyne*.

Now, upon supposition that the sun moves, as conjectured by Dr. *Herschel*, that motion will account for the motion of 20 of them, so far as regards their directions; but the motion of the other 15 is contrary to that which ought to arise from this supposition. As some of the stars must have a proper motion of their own, even upon the hypothesis of a solar motion, and which probably arises from their mutual attraction, it is very probable that they have all a proper motion from the same cause, but most of them so very small, as not yet to have been discovered. And it might also happen, that such a motion might be the same as that which would arise from the motion of the solar system. Yet it must be confessed, that the circumstance of *Castor*, and the motions, both in right ascension and declination, of many of the stars being such as arise from this hypothesis, with the apparent motion of those stars being greatest which are probably nearest, form a strong argument in it's favour.

## *On the Zodiacal Light.*

(375.) The *Zodiacal Light* is a pyramid of light which sometimes appears in the morning before sun-rise, and in the evening after sun-set. It has the sun for it's basis, and in appearance resembles the *Aurora Borealis*. It's sides are not straight, but a little curved, it's figure resembling a lens edgeways. It is generally seen here about October and March, that being the time of our shortest twilight; for it cannot be seen in the twilight; and when the twilight lasts a considerable time, it is withdrawn before the twilight ends. It was observed by M. *Cassini*, in 1683, a little before the vernal equinox, in the evening, extending along the ecliptic from the sun. He thinks, however, that it has appeared formerly, and afterwards disappeared, from an observation of Mr. *J. Childrey*, in a book published in 1661, entitled, *Britannia Baconia*. He says, that "in the

month of February, for several years, about six o'clock in the evening, after twilight, he saw a path of light tending from the twilight towards the *Pleiades*, as it were touching them. This is to be seen whenever the weather is clear, but best when the moon does not shine. I believe this phænomenon has been formerly, and will hereafter appear always at the above-mentioned time of the year. But the cause and nature of it I cannot guess at, and therefore leave it to the enquiry of posterity." From this description, there can be no doubt but that this was the zodiacal light. He suspects also, that this is what the ancients called *Trabes*, which word they used for a meteor, or impression in the air like a beam. *Pliny*, lib. II. p. 26, says, *Emicant Trabes, quos docos vocant*. *Des Cartes* also speaks of a phænomenon of the same kind. M. *Fatio de Duillier* observed it immediately after the discovery by M. *Cassini*, and suspected that it had always appeared. It was soon after observed by M. *Kirch* and *Eimmart* in Germany. In the year 1707, on April 3, it was observed by Mr. *Derham*, in Essex. It appeared in the western part of the heavens, about a quarter of an hour after sun-set, in the form of a pyramid, perpendicular to the horizon. The base of this pyramid he judged to be the sun. It's vertex reached 15° or 20° above the horizon. It was throughout of a dusky red colour, and at first appeared pretty vivid and strong, but faintest at the top. It grew fainter by degrees, and vanished about an hour after sun-set. This solar atmosphere has also been seen about the sun in a total solar eclipse, a luminous ring appearing about the moon at the time when the eclipse was total.

(376.) M. *Fatio* conjectured, that this appearance arises from a collection of corpuscles encompassing the sun in the form of a lens, reflecting the light of the sun. M. *Cassini* supposed that it might arise from an infinite number of planets revolving about the sun; so that this light might owe it's existence to these bodies,

*de St. Pierre*. Upon this, Mr. *Flamstead* was appointed Astronomer Royal, and an Observatory was built at Greenwich for him; and the instructions to him and his successors were, "That they should apply themselves with the utmost care and diligence to rectify the Tables of the motions of the heavens, and the places of the fixed stars, in order to find out the so-much desired longitude at sea, for the perfecting of the art of navigation."

(378.) In the year 1714, the British Parliament offered a reward for the discovery of the longitude; the sum of 10000*l*. if the method determined the longitude to 1° of a great circle, or 60 geographical miles; of 15000*l*. if it determined it to 40 miles; and of 20000*l*. if it determined it to 30 miles; with this proviso, that if any such method extend no further than 80 miles adjoining to the coast, the proposer shall have no more than half such rewards *. The Act also appoints the First Lord of the Admiralty, the Speaker of the House of Commons, the First Commissioner of Trade, the Admirals of the Red, White, and Blue Squadrons, the Master of Trinity-House, the President of the Royal Society, the Royal Astronomer at Greenwich, the two Savilian Professors at Oxford, and the Lucasian and Plumian Professors at Cambridge, with several other persons, as Commissioners for the Longitude at Sea. The Lowndian Professor at Cambridge was afterwards added. After this Act of Parliament, several other Acts passed, in the reigns of *George* II. and III, for the encouragement of finding the longitude. At last, in the year 1774, an Act passed, repealing all other Acts, and offering separate rewards to any person who shall discover the longitude, either by the lunar method, or by a watch keeping true time, within certain limits, or by any other method. The

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* See *Whiston's* account of the proceedings to obtain this Act, in the Preface to his *Longitude discovered by Jupiter's Planets.*

Act proposes, as a reward for a time-keeper, the sum of 5000*l.* if it determine the longitude to 1°, or 60 geographical miles; the sum of 7500*l.* if it determine the same to 40 miles; and the sum of 10000*l.* if it determine the same to 30 miles, after proper trials specified in the act. If the method be by improved solar and lunar Tables, constructed upon Sir *I. Newton's* theory of gravitation, the author shall be entitled to 5000*l.* if such Tables shall show the distance of the moon from the sun and stars within 15″ of a degree, answering to about 7′ of longitude, after making an allowance of half a degree for the errors of observation. And for any other method, the same rewards are offered as those for the time-keeper, provided it gives the longitude true within the same limits, and be practicable at sea. The commissioners have also a power of giving smaller rewards, as they shall judge proper, to any one who shall make any discovery for finding the longitude at sea, though not within the above limits. Provided, however, that if such person or persons shall afterwards make any further discovery so as to come within the above-mentioned limits, such sum or sums, as they may have received, shall be considered as part of such greater reward, and deducted therefrom accordingly.

(379.) After the decease of Mr. *Flamstead*, Dr. *Halley*, who was appointed to succeed him, made a series of observations on the moon's transit over the meridian, for a complete revolution of the moon's apogee, which observations being computed from the Tables then extant, he was enabled to correct the Tables of the moon's motion. And as Mr. *Hadley* had then invented an instrument by which altitudes could be taken at sea, and also the moon's distance from the sun or a fixed star, Dr. *Halley* strongly recommended the method of finding the longitude from such observations, having found, from experience, the impracticability of all other methods, particularly at sea.

## *To find the Longitude by the Moon's Distance from the Sun, or a fixed Star.*

(380.) The steps by which the longitude is found by this method, are these:

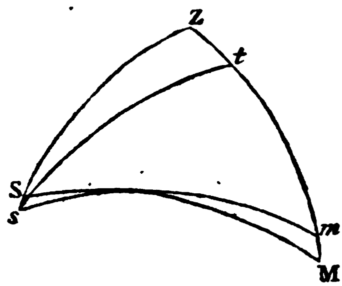
From the *observed* altitudes of the moon and the sun, or a fixed star, and their *observed* distance, compute the moon's *true* distance from the sun or star.

From the *Nautical Almanac*, find the time at Greenwich when the moon was at that distance.

From the altitude of the sun or star, find the time at the place of observation.

The difference of the times thus found, gives the difference of the longitudes.

(381.) To find the *true* distance of the moon from the sun or star by observation, let $Z$ be the zenith, $S$ the apparent place of the sun or a star, $s$ the true place,




$M$ the apparent place of the moon, $m$ it's true place; then the altitudes of $M$ and $s$ being known, by observation, the refractions $Ss$, $Mm$ are known; also $MS$ is known by observation; hence, in the triangle $ZSM$, we know $SM$ the apparent distance, $SZ$, $ZM$, the complements of the apparent altitudes, to find the angle $Z$ (Trig. Art. 239); and then in the triangle $sZm$, we know the angle $Z$, and $sZ$, $mZ$, the comple-

ments of the true altitudes, to find *sm* the *true* distance* (Trig. Art. 233).

Ex. Suppose on June 29, 1793, the sun's apparent zenith distance *ZS* was observed to be 70°. 56′. 24″, the moon's apparent zenith distance *ZM* to be 48°. 53′. 58″, their apparent distance *SM* to be 103°. 29′. 27″, and the moon's horizontal parallax to be 58′. 35″; to find their *true* distance *sm*.

The *true* distance *sm*, computed by the above method, is 103°. 3′. 18″.

(382.) The *true* distance of the moon from the sun or star being thus found, we are next to find the time at Greenwich. For this purpose, the sun or such fixed stars are chosen, as lie in or very near the moon's way, so that, looking upon the moon's motion to be uniform for a small time, the moon may be considered as approaching to, or receding from, the sun or star uniformly. To determine, therefore, the time at Greenwich corresponding to any given true distance of the moon from the sun or star, the *true* distance is computed in the *Nautical Almanac* for every three hours, for the meridian of Greenwich. Hence, considering that distance as varying uniformly, the time corresponding to any other *true* distance may be thus computed: Look into the *Nautical Almanac*, and take out two distances, one next greater and the other next less than the *true* distance deduced from observation, and the difference *D* of these distances gives the access of the moon to, or recess from, the sun or star in three hours; then take the difference *d* between the moon's distance at the beginning of that interval and the distance deduced from observation, and then say, *D* : *d* :: 3 hours : the time the moon is acceding to, or receding from, the sun or star by the quantity *d*; which

* There are shorter methods than this direct one, of computing the true distance, as the reader will see in my *Complete System of Astronomy*; but we here purpose only to explain the principles by which the longitude may be thus found.

added to the time at the beginning of the interval, gives the apparent time at Greenwich, corresponding to the given true distance of the moon from the sun or star.

Ex. On June 29, 1793, in latitude 52°. 12'. 35", the sun's altitude in the morning was, by observation, 19°. 3'. 36", the moon's altitude was observed to be 41°. 6'. 2", the sun's declination at that time was 23°. 14'. 4", and the moon's horizontal parallax 58'. 35"; to find the apparent time at Greenwich.

| | |
|---|---|
| True dist. of ☾ from ☉ by Art. 381. - | 103°. 3'. 18" |
| True dist. by *Naut. Alm.* on June 29, at 3*h*. | 103. 4. 58 |
| True dist. by *Naut. Alm.* on June 29, at 6*h*. | 101. 26. 42 |
| *d* = . . . . . . . . . . . . . . | 0. 1. 40 |
| *D* = . . . . . . . . . . . . . . | 1. 38. 16 |

Hence, 1°. 38'. 16". : 0°. 1'. 40" : 3*h*. :: 0h. 3'. 3", which added to 3*h*. gives 3*h*. 3'. 3", the apparent time at Greenwich, when the *true* distance was 103°. 3'. 18".

(383.) Find the apparent time at the place of observation, by the altitude of the sun (12). Then the difference of the times at Greenwich, and at the place of observation, is the distance of the meridians in time.

(384.) Now to find the apparent time at the place of observation, we have the sun's declination 23°. 14'. 4", it's altitude 19°. 3'. 36", it's refraction 2'. 44", and parallax 8"; hence, it's true altitude was 19°. 1', and therefore it's true zenith distance was 70°. 59'; also, the complement of declination was 66°. 45'. 36"; hence, by Art. 92:

| | | |
|---|---|---|
| 66°. 46'. 56" | - - ar. co. sin. | 0,0367325 |
| 37. 47. 25 | - - ar. co. sin. | 0,2127004 |
| 70. 56. 24 | | |
| 175. 29. 45 | | |
| 87. 44. 52 | - - - - sin. | 9,9996644 |
| 16. 48. 28 | - - - - sin. | 9,4601408 |
| | | 2)19,7092381 |
| | | 9,8546190 |

the cosine of 44°. 18′. 52″, which doubled gives 88°. 37′. 44″, the hour-angle from apparent noon, which in time gives 5*h*. 54′. 31″, the time before apparent noon, or 18*h*. 5′. 29″, on June 28. Hence,

| | | | | | |
|---|---|---|---|---|---|
| Apparent time at place of observ. | June 28, | 18ʰ. | 5′. | 29″ |
| ——————— at Greenwich, | June 29, | 3. | 3. | 3 |
| Difference of meridians in time - - - - | | 8. | 57. | 34 |

Which converted into degrees, gives 134°. 23′. 30″, the longitude of the place of observation west of Greenwich.

### *To find the Longitude by a Time-keeper.*

(385.) Let the *time-keeper* be well regulated, and set to the meridian of Greenwich; then if it neither gain nor lose, it will always show the time at Greenwich. Hence, to find the longitude of any other place, find the mean time from the sun's altitude by Art. 92; and observe, at the instant of taking the altitude, the time by the watch; and the difference of these times, converted into degrees, at the rate of 15° for an hour, gives the longitude from Greenwich. If, for example, the time by the watch, when the altitude was taken was 6*h*. 19′, and the mean time deduced from that altitude was 9*h*. 23′, the difference 3*h*. 4′, converted into degrees, gives 46° the longitude of the place *east* from Greenwich, because the time at the place of observation is *forwarder* than that at Greenwich. Thus the longitude could be very readily determined, if you could depend upon the watch. But as a watch will always gain or lose, before it is sent out, it's gaining or losing every day for some time, a month for instance, is observed; this is called the *rate of going* of the watch, and from thence the *mean* rate of going is thus found.

(386.) Suppose, for instance, I examine the rate of a watch for 30 days; on some of those days I find it has

gained, and on some it has lost; add together all the quantities which it has gained, and suppose they amount to 17″; add together all the quantities which it has lost, and let the sum be 13″; then the difference 4″ is the *mean* rate of gaining for 30 days, which divided by 30, gives 0″,133 for a *mean daily rate* of gaining. Or you may get the mean daily rate thus. Take the *difference* between what the watch was too fast, or too slow, on the first and last days of observation, if it be too fast or too slow on each day; but take the *sum*, if it be too fast on one day and too slow on the other, and divide by the number of days between the observations *. And to find the time at the place of trial at any future period by this watch, you must put down, at the end of the trial, how much the watch is too fast or too slow; then subtract from the time shown by the watch, 0″133 × number of days from the end of the trial, being the exact quantity which it has gained according to the above mean rate of gaining, and you are then supposed to get the true time affected with the error at the end of the trial. This would be all the error, if the watch had continued to gain according to the above rate; and although, from the different temperatures of the air to which the watch may be exposed, and from the imperfection of the workmanship, this cannot be expected, yet, by taking it into consideration, the probable error of the time will be diminished. In watches which are under trial at the Royal Observatory at Greenwich, as candidates for the rewards offered by Parliament for the discovery of the longitude, this allowance of a mean rate, to be applied in order to get the time, is not granted by the Act of Parliament, but it requires that the watch itself should go within the limits assigned; the Commissioners, however, are so indulgent as to grant the applica-

* For further information on this subject, see Mr. Wales's *Method of finding the Longitude at Sea*.

tion of a mean rate, which is undoubtedly favourable to the watches.

(387.) As the rate of going of a watch is subject to vary from so many circumstances, the observer, whenever he goes ashore and has sufficient time, should compare his watch, for several days, with the mean time deduced from the altitude of the sun or a star, by which he will be able to determine it's rate of going. And whenever he comes to a place whose longitude is known, he may correct his watch, and set it to Greenwich time. For instance, if he go to a place known to be 30°. east longitude from Greenwich, his watch should be two hours slower than the time at that place. Find therefore, the time at that place by the altitude of the sun or a fixed star, and correct it by the equation of time, and compare the time so found with the time by the watch when the altitude was taken, and if the watch be two hours slower than the time deduced from observation, it is right; if not, correct it by the difference, and it again gives Greenwich time.

(388.) In long voyages, unless you have sometimes the means of adjusting the watch to Greenwich time, it's error will probably be very considerable, and consequently the longitude deduced from it will be subject to a proportional error. In short voyages, a watch is undoubtedly very useful, and also in long ones, where you have the means of correcting it from time to time. It serves to carry on the longitude from one known place to another, supposing the interval of time not to be very long; or to keep the longitude from that which is deduced from a lunar observation, till you can get another observation. Thus the watch may be rendered of great service in Navigation.

*To find the Longitude by an Eclipse of the Moon, and of Jupiter's Satellites.*

(389.) By an eclipse of the moon. This eclipse begins when the umbra of the earth first touches the

moon, and it ends when it leaves the moon. Having the times calculated when the eclipse begins and ends at Greenwich, observe the times when it begins and ends at any other place; the difference of these times converted into degrees, gives the difference of longitudes. For as the phases of the moon in an eclipse happen at the same instant to every observer, the difference of the times at different places, when any phase is observed, will give the difference of the longitudes. This would be a very ready and accurate method, if the time of the first and last contact could be accurately observed; but the darkness of the penumbra continues to increase till it comes to the umbra, so that, until the umbra actually gets upon the moon, it is not discovered. The umbra itself is also very badly defined. The beginning and end of a lunar eclipse cannot, in general, be determined nearer than 1′ of time; and very often not nearer than 2′ or 3′. Upon these accounts, the longitude, from the observed beginning and end of an eclipse, is subject to a considerable degree of uncertainty. Astronomers, therefore, determine the difference of the longitudes of two places by corresponding observations of other phases, that is, when the umbra bisects any of the spots upon the moon's surface. And this can be determined with a greater degree of accuracy than the beginning and end; because, when the umbra is gotten upon the moon's surface, the observer has leisure to consider and fix upon the proper line of termination, in which he will be assisted by running his eye along the circumference of the umbra. Thus the coincidence of the umbra with the spots may be observed with tolerable accuracy. The observer, therefore, should have a good map of the moon at hand, that he may not mistake. The telescope, to observe a lunar eclipse, should have but a small magnifying power with a great deal of light. The shadow comes upon the moon on the east side, and goes off on the west; but if the telescope invert, the appearances will be contrary.

(390.) The eclipses of *Jupiter's* satellites afford the readiest method of determining the longitude of places at land. It was also hoped that some method might be invented to observe them at sea, and Mr. *Irwin* made a chair to swing for that purpose, for the observer to sit in; but Dr. *Maskelyne*, in a voyage to Barbadoes, under the direction of the Commissioners of Longitude, found it totally impracticable to derive any advantage from it; and he observes, that, "considering the great power requisite in a telescope for making these observations well, and the violence as well as irregularities of the motion of a ship, I am afraid the complete management of a telescope on ship-board will always remain among the desiderata. However, I would not be understood to mean to discourage any attempt, founded upon good principles, to get over this difficulty." The telescopes proper for making these observations, are common refracting ones from 15 to 20 feet, reflecting ones of 18 inches or 2 feet, or the 46-inch achromatic with three object glasses, which were first made by Mr. *Dolland*. On account of the uncertainty of the theory of the satellites, the observer must be settled at his telescope a few minutes before the expected time of an immersion; and if the longitude of the place be also uncertain, he must look out proportionably sooner. Thus, if the longitude be uncertain to 2°, answering to eight minutes of time, he must begin to look out eight minutes sooner than is mentioned above. However, when he has observed one eclipse, and found the error of the Tables, he may allow the same correction to the calculations of the Ephemeris for several months, which will advertise him very nearly of the time of expecting the eclipses of the same satellite, and dispense with his attending so long. Before the opposition of Jupiter to the sun, the immersions and emersions happen on the west side of Jupiter, and after opposition, on the east side; but if the telescope invert, the appearance will be the contrary. Before opposition, the immersions only of the

first satellite are visible; and after opposition, the emersions only. The same is generally the case with respect to the second satellite; but both immersion and emersion are frequently observed in the two outer satellites.

(391.) When the observer is waiting for an emersion, as soon as he suspects that he sees it, he should look at his watch, and note the second, or begin to count the beats of the clock, till he is sure that it is the satellite, and then look at the clock, and subtract the number of seconds which he has counted from the time then observed, and he will have the time of emersion. If Jupiter be 8° above the horizon, and the sun as much below, an eclipse will be visible; this may be determined near enough by a common globe.

(392.) The immersion or emersion of a satellite being observed according to apparent time, the longitude of the place from Greenwich is found, by taking the difference between that time and the time set down in the *Nautical Almanac*, which is calculated for apparent time.

Ex. Suppose the emersion of a satellite to have been observed at the Cape of Good Hope, May 9, 1767, at 10*h*. 46′. 45″ apparent time; now the time in the *Nautical Almanac* is 9*h*. 33′. 12″; the difference of which time is 1*h*. 13′. 33″, the longitude of the Cape *east* of Greenwich in time, or 18°. 23′. 15″.

(393.) But to find the longitude of a place from an observation of an eclipse of a satellite, it is better to compare it with an observation made under some well-known meridian, than with the calculations in the Ephemeris, because of the imperfection of the theory; but where a corresponding observation cannot be obtained, find what correction the calculations of the Ephemeris require, by the nearest observations to the given time that can be obtained; and this correction, applied to the calculation of the given eclipse in the Ephemeris, renders it almost equivalent to an actual observation. The observer must be careful to regulate

his clock or watch by apparent time, or at least to know the difference; this may be done, either by equal altitudes of the sun, or of proper stars; or the latitude being known, from one altitude at a distance from the meridian, the time may be found by Art. 92.

(394.) In order the better to determine the difference of longitudes of two places from corresponding observations, the observers should be furnished with the same kind of telescopes. For at an immersion, as the satellite enters the shadow, it grows fainter and fainter, till at last the quantity of light is so small that it becomes invisible, even before it is immersed in the shadow; the instant, therefore, that it becomes invisible will depend upon the quantity of light which the telescope receives, and it's magnifying power. The instant, therefore, of the disappearance of a satellite will be later the better the telescope is, and the sooner it will appear at it's emersion. Now the immersion is the instant the satellite is wholly gotten into the shadow, and the emersion is the instant before it begins to emerge from the shadow; if, therefore, two telescopes show the disappearance or appearance of the satellite at the same distance of time from the immersion or emersion, the difference of the times will be the same as the difference of the true times of their immersions and emersions, and therefore will show the difference of longitudes accurately. But if the observed time at one place be compared with the computed time at another, then we must allow for the difference between the apparent and true times of immersion or emersion, in order to get the true time where the observation was made, to compare with the true time from computation at the other place. This difference may be found, by observing an eclipse at any place whose longitude is known, and comparing it with the time by computation. Observers, therefore, should settle the difference accurately by the mean of a great number of observations thus compared with the computation, by which means the longitude will

be ascertained to a much greater accuracy and certainty. After all this precaution, however, the different states of the air at different times, and also the different states of the eye, will introduce a small degree of uncertainty; the latter case may perhaps, in a great measure, be obviated, if the observer will be careful to remove himself from all warmth and light for a little time before he makes the observation, that the eye may be reduced to a proper state; which precaution the observer should also attend to, when he settles the difference between the apparent and true times of immersion and emersion. Perhaps also the difference arising from the different states of the air might, by proper observations, be ascertained to a considerable degree of accuracy; and as this method of determining the longitude is of all others, the most ready, no means ought to be left untried to reduce it to the greatest certainty.

# TABLE I.

*For converting Degrees, Minutes and Seconds, into Time, at the Rate of 360 Degrees for 24 Hours.*

| Deg. Min. | Hou. Min. | Min. Sec. | Deg. Min. | Hou. Min. | Min. Sec. | Sec. | Dec. of Sec. |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 4 | 30 | 2 | 0 | 1 | ,066 |
| 2 | 0 | 8 | 40 | 2 | 40 | 2 | ,133 |
| 3 | 0 | 12 | 50 | 3 | 20 | 3 | ,2 |
| 4 | 0 | 16 | 60 | 4 | 0 | 4 | ,266 |
| 5 | 0 | 20 | 70 | 4 | 40 | 5 | ,333 |
| 6 | 0 | 24 | 80 | 5 | 20 | 6 | ,4 |
| 7 | 0 | 28 | 90 | 6 | 0 | 7 | ,466 |
| 8 | 0 | 32 | 100 | 6 | 40 | 8 | ,533 |
| 9 | 0 | 36 | 200 | 13 | 20 | 9 | ,6 |
| 10 | 0 | 40 | 300 | 20 | 0 | 10 | ,666 |
| 20 | 1 | 20 | | | | | |

# TABLE II.

*For converting Time into Degrees, Minutes, and Seconds, at the Rate of 24 Hours for 360 Degrees.*

| Hou. | Deg. | Min. Sec. | Deg. Min. | Min. Sec. | Dec. of Sec. | Sec. |
|---|---|---|---|---|---|---|
| 1 | 15 | 1 | 0 | 15 | ,1 | 1,5 |
| 2 | 30 | 2 | 0 | 30 | ,2 | 3,0 |
| 3 | 45 | 3 | 0 | 45 | ,3 | 4,5 |
| 4 | 60 | 4 | 1 | 0 | ,4 | 6,0 |
| 5 | 75 | 5 | 1 | 15 | ,5 | 7,5 |
| 6 | 90 | 6 | 1 | 30 | ,6 | 9,0 |
| 7 | 105 | 7 | 1 | 45 | ,7 | 10,5 |
| 8 | 120 | 8 | 2 | 0 | ,8 | 12,0 |
| 9 | 135 | 9 | 2 | 15 | ,9 | 13,5 |
| 10 | 150 | 10 | 2 | 30 | | |
| 11 | 165 | 20 | 5 | 0 | | |
| 12 | 180 | 30 | 7 | 30 | | |
| 16 | 240 | 40 | 10 | 0 | | |
| 20 | 300 | 50 | 12 | 30 | | |